Evolution and Market Behavior: Wealth Dynamics and Learning

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The Market is Smart

Ultimately, the economy always follows the stock market so, somehow, the stock market knows.

– Ted Andros, Wall Street Plus

Given the uncertainty of the real world, the many actual and virtual traders will have many, perhaps equally many, forecasts. . . If any group of traders was consistently better than average in forecasting stock prices, they would accumulate wealth and give their forecasts greater and greater weight. In this process, they would bring the present price closer to the true value..

– Paul Cootner, 1967

The Market is Mad

traders are not machines guided by silicon chips; they are impressionable and imitative; they run in flocks and retreat in hordes.

– Roger Lowenstein,
When Genius Failed:
The Rise and Fall of LTCM

They (economists) turned a blind eye to the limitations of human rationality that often lead to bubbles and busts; to the problems of institutions that run amok; to the imperfections of markets especially financial markets that can cause the economys operating system to undergo sudden, unpredictable crashes...

– Paul Krugman
New York Times 2009

What Do We Know About Prices?

Complete markets and SEU traders implies—nothing interesting about prices.

Simple Example.

- One Trader with Log utility
- SEU implies Bayesian learning, but it does not otherwise restrict beliefs.
- Any stochastic process of beliefs is possible.
- Any stochastic process of Arrow security prices is possible.

Structure on prices must come from structure on beliefs.

Representative consumer.

How Markets Learn

Three mechanisms for market learning:

- Markets share information among traders, and traders come to learn what others know from market prices.
- Markets select for traders with the most accurate beliefs.
- Markets balance the beliefs of differentially informed traders.

What We Do

Our analysis builds on Blume and Easley (2006), Sandroni (2000) and Blume and Easley (1992).

- Infinite horizon exchange economy
- Complete markets, Arrow securities
- Heterogeneous subjective expected utility maximizers

We characterize Pareto optimal consumption paths and their supporting prices.

First, examine (simple) iid world with iid beleifs.

The Environment:

- S States, $\{1, \ldots, \mathtt{s}\}$.
- Σ Paths, $\sigma = (\sigma_0, \ldots)$. $\sigma^t = (\sigma_0, \ldots, \sigma_t)$.
- n_t^s State counts, $n_t^s = \#\{\tau \leq t : \sigma_\tau = s\}$.
- ρ True probability distribution on S; State process is iid.

Markets are Complete:

At each node in the date-event tree there is a single consumption good and one security (in 0 supply) for each state. Security s available at partial history σ^t pays off one unit of consumption good in partial history (σ^t, s) at date $t+1$, and 0 otherwise.

Traders:

- c A *consumption plan* $\{c_t\}_{t=1}^{\infty}$, $c_t : \sigma^t \mapsto \mathbf{R}_{++}$.
- e^i Trader i's *endowment*. . . .
- ρ^i Trader i's iid beliefs on S.
- β_i Trader i's discount factor.
- $u^i : \mathbf{R}_+ \to \mathbf{R}$ Trader i's payoff to consumption at any partial history.

$$U_i(c) = E_{\rho^i}\left\{\sum_{t=0}^{\infty} \beta_i^t u_i\big(c_t(\sigma^t)\big)\right\}$$

Assumptions

A.1. The payoff functions u_i are C^1, strictly concave, strictly monotonic, and satisfy an Inada condition at 0.

A.2. Each trader has a strictly positive endowment at every partial history, and the aggregate endowment is uniformly bounded, below away from zero and from above.

A.3. At every partial history, each trader i believes all truly possible states to be possible: $\rho_i \gg 0$

Pareto Optimality

If c^* is a Pareto optimal allocation of resources, then there is a strictly positive vector of welfare weights $(\lambda^1, \dots, \lambda^I)$ such that c^* solves

$$\begin{aligned} \max_{(c^1,\dots,c^I)} \quad & \sum_i \lambda^i U_i(c^i) \\ \text{such that} \quad & \sum_i c^i - e \leq \mathbf{0} \\ & \forall t, \sigma \; c^i_t(\sigma^t) \geq 0 \end{aligned}$$

where $e_t = \sum_i e^i_t$.

For all t there is a random variable $\eta_t : \sigma^t \mapsto \mathbf{R}_{++}$ such that

$$\lambda^i \beta_i^t u_i'\big(c_t^i(\sigma^t)\big) \prod_s (\rho_s^i)^{n_t^s(\sigma)} - \eta_t(\sigma^t) = 0$$

almost surely, and

$$\sum_i c_t^i(\sigma^t) = e_t(\sigma^t)$$

Selecting Bettors

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} = t \log \frac{\beta_j}{\beta_i} + \sum_s n_t^s(\sigma) \log \frac{\rho_s^j}{\rho_s^i}$$

This is something to which the law of large numbers can be applied.

Bettor Asymptotics

How far are i's beliefs from the truth?

Relative Entropy $\qquad I_i = \sum_s \rho_s \log \frac{\rho_s}{\rho_s^i}$

Limit MU ratios:

$$\frac{1}{t} \log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} \to \log \frac{\beta_j}{\beta_i} + \sum_s \rho_s \left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j} \right)$$
$$= (\log \beta_j - I_j) - (\log \beta_i - I_i)$$

Define the fitness index $\mathrm{f}_i = \log \beta_i - I_i$.

Assume A.1–3. If $\mathbf{f_i} < \mathbf{max_j}\, \mathbf{f_j}$, then trader **i** vanishes.

Next, what does this result imply about prices?

Def. A present value price system is $p = \{p_t\}_{t=0}^{\infty}$, $p_t : \sigma^t \mapsto \mathbf{R}_{++}$ such that, for each trader i, $p \cdot e^i < \infty$.

Def. A competitive equilibrium is a present value price system p^* and a consumption plan c^{i*} for each trader such that ...

Existence is due to Peleg and Yaari (1970), and the first welfare theorem is elementary.

Def. $q_t^s(\sigma^t)$ is the price of the Arrow security that pays off in partial history (σ^t, s) in terms of consumption at partial history σ^t. That is, q is the current value price system.

Correct Asset Pricing?

A.4. The aggregate endowment is history-independent. $e_t(\sigma) \equiv e \gg 0$. For interpretative purposes only.

Proposition: Assume A.1–4. Then on each path σ at each date t and for all $\epsilon > 0$ there is a $\delta > 0$ such that if $|c_t^i(\sigma^t) - e| < \delta$, then $||q_t(\sigma^t) - \rho^i|| < \epsilon$. If all traders have identical beliefs ρ', for all dates t and paths σ, $q_t^s(\sigma^t) = \rho'$.

If all traders have correct beliefs, then the price at σ^t of the asset that pays off in state s at date $t+1$ is ρ_s, the probability state s occurs.

Implications of Selection for Asset Pricing

Assume A.1–4.

If there is a unique trader $\mathbf{i}$ with maximal survival index $\mathbf{f_i}$ among the trader population, then market prices converge to $\boldsymbol{\rho}^i$ almost surely.

- If $\boldsymbol{\rho}^i$ is correct then correct asset pricing emerges.
- If there is a common discount factor then market prices converge to the $\boldsymbol{\rho}^i$ closest in relative entropy to $\boldsymbol{\rho}$.

- IID beliefs—generalized, beliefs can follow any stochastic process, but the fitness index and pricing implications are harder to interpret.
- Bayesians—any SEU trader is a Bayesian. The market selects among Bayesians who have the truth in the support of their prior according to the dimension of the support of the prior.
- Bayesians—The market selects for Bayesians. If there is a Bayesian with the truth in the support of his prior, then any learning rule which is not asymptotically Bayesian is driven out of the market.
- IID world—generalized to arbitrary stochastic processes on states. But note that even in the IID world endowments need not be IID.

- SEU traders—analysis applies to any rules that can be generated by SEU.
- example with log traders—all traders have log utility, a common discount factor and beliefs that are measurable functions of partial histories. If some trader has correct beliefs then all traders whose beliefs do not converge (fast enough) to the correct beliefs vanish, and prices converge to correct prices.
- portfolio rules—all traders invest fraction β of their wealth in the market. Their portfolio rules are measurable functions of partial histories. If there is trader whose protfolio rule is the one generated by correct beliefs then....

Limitations and Extensions

- Complete markets—results can change with incomplete markets. Traders with correct beliefs and maximal discount factor can be driven out of the market.
- Bounded economy—generalizable, although now curvature of utility affects the survival index.
- Multiple Survivors—possible and can generate complex dynamics.

The Bayesian Market

$u^i(c) = \log c$, identical discount factors.

$$\log \frac{c_t^i(\sigma^t)}{c_t^j(\sigma^t)} = \log \frac{\lambda_i}{\lambda_j} + \sum_s n_t^s(\sigma^t) \log \frac{\rho_i(s)}{\rho_j(s)}$$

Consumption shares evolve as do Bayesian posteriors.

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