

# A Stylized Model of Boom and Bust Housing Cycles

Roberto Dieci <sup>a</sup> and Frank Westerhoff <sup>b</sup>

<sup>a</sup> Department of Mathematics for Economics and Social Sciences  
University of Bologna, Italy  
*roberto.dieci@unibo.it*

<sup>b</sup> Department of Economics, University of Bamberg, Germany  
*frank.westerhoff@uni-bamberg.de*

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## **Shiller's (2005, 2007a, 2007b, 2008) evidence**

Boom and bust home price cycles appear since centuries.

Current boom-bust cycle seems to dwarf anything seen before.

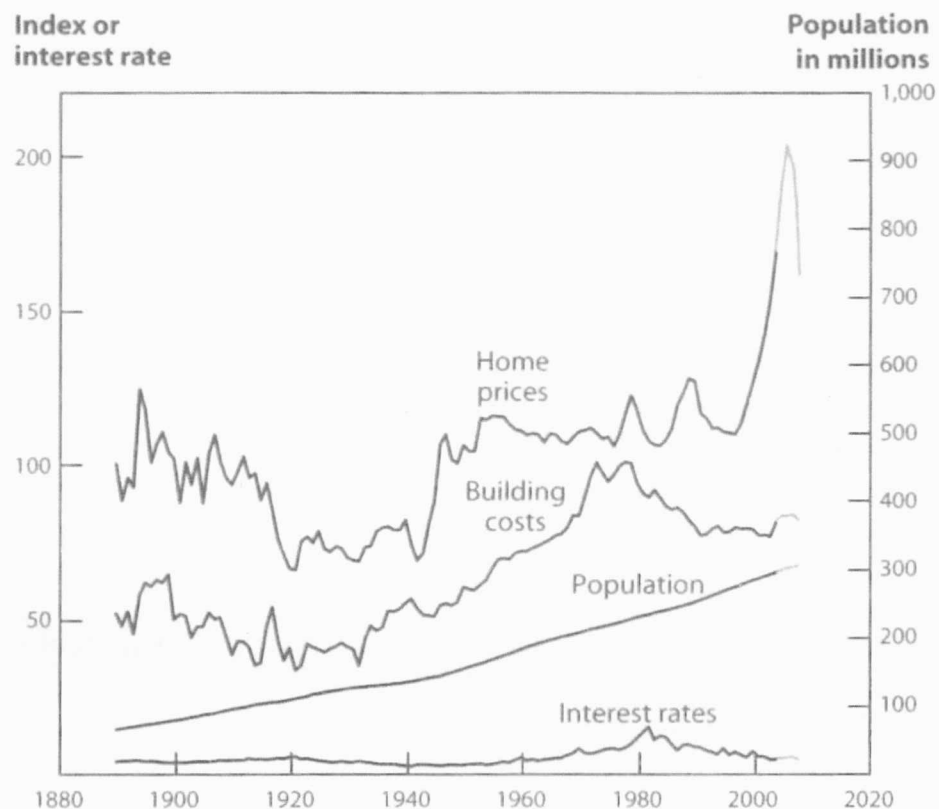
Countries currently included: Australia, Canada, China, France, India, Ireland, Italy, Korea, Russia, Spain, the United Kingdom, and the United States.

Example 1: Real home prices in London nearly tripled from 1996 to 2008.

Example 2: Real home prices increased in Las Vegas by 49 percent in 2004.

Example 3: Real home prices increased the United States by 85 percent between 1997 and 2006.

Want more details? See the next four figures!



**Figure 2.1.**

U.S. Real Home Prices, 1890–2008, along with Building Costs, Population, and Long-Term Government Bond Interest Rates, annual 1890–2008. *Source:* From Robert J. Shiller, *Irrational Exuberance*, 2nd Edition (Princeton, N.J.: Princeton University Press, 2005), p. 13, updated here, with updates shown in gray. Home price index is shown quarterly for 2007-I to 2008-I.

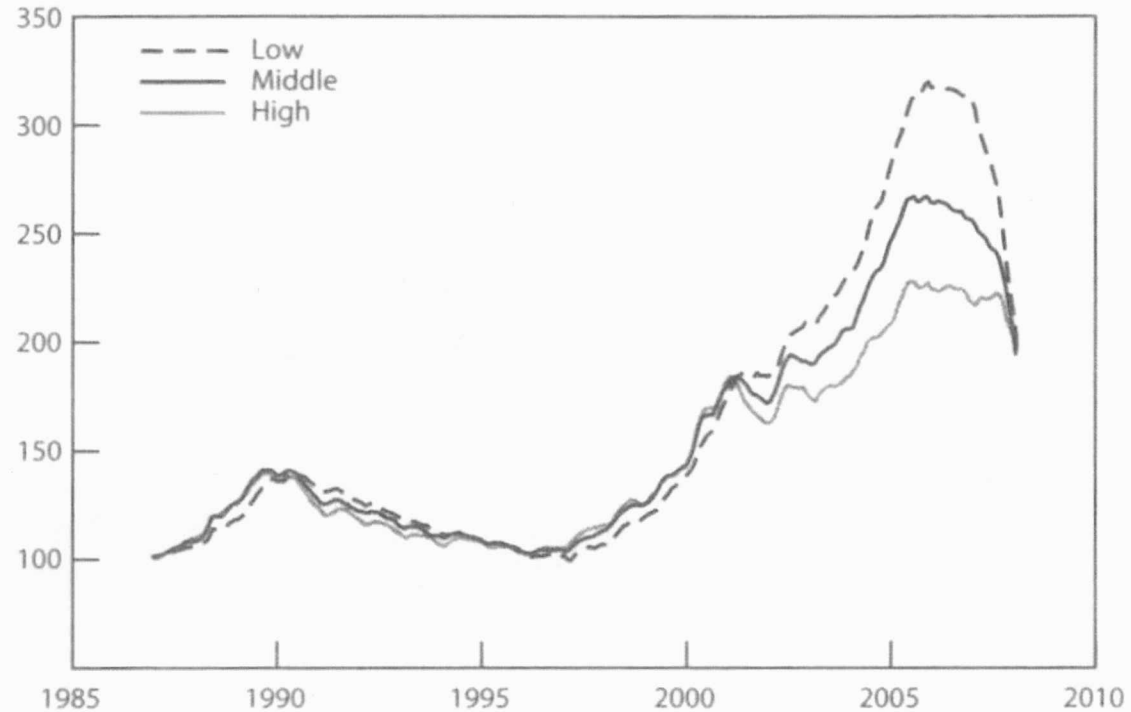
**Real home  
price index  
(1987 = 100)**



**Figure 2.2**

Real Home Prices in a Sample of Cities, Monthly, January 1983 to March 2008. *Source:* Author's calculations using data from [www.homeprice.standardandpoors.com](http://www.homeprice.standardandpoors.com) and [www.bls.gov](http://www.bls.gov).

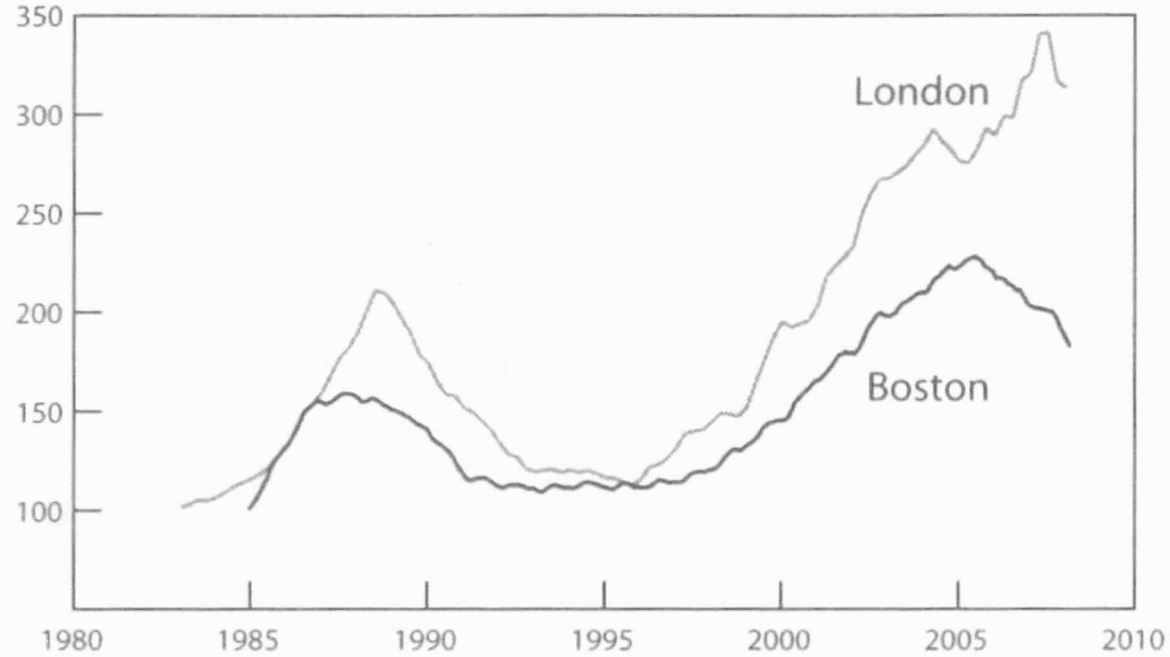
**Real home  
price index  
(1987 = 100)**



**Figure 2.3**

Real San Francisco Metro Area Home-Price Indices by Price Tier, Monthly, January 1987 to March 2008. *Source:* Author's calculations using data from [www.homeprice.standardandpoors.com](http://www.homeprice.standardandpoors.com) and [www.bls.gov](http://www.bls.gov).

**Real  
price index  
(1983, 1985 = 100)**



**Figure 2.4**

Real Greater London and Greater Boston Home Prices.

*Source:* London prices quarterly 1983-I to 2008-I are from the Halifax House Price Index, divided by the U.K. Retail Prices Index. U.S. prices monthly January 1983 to March 2008 are from the S&P/Case-Shiller Home Price Indices, divided by the Consumer Price Index.

## Shiller's impression

Dramatic boom-bust home price cycles hard to explain with standard economic thinking

- Economic fundamentals (population growth, construction costs, interest rates, real rents, ...) do not match up with the observed home price increases
- Little long-term trend in comparison to the amplitude of price fluctuations in the short / medium term: prices rise when optimism prevails, but a crash is set in motion when prices get too high
- Irregularities in boom-bust housing price cycles (e.g. Greater London, 2004-05: a 6% downturn, that was supposed to mark the end of a bubble, reverted into a new period of growing prices)

## Shiller's impression

The boom of the early 2000s across cities and countries suggests that *something very broad and general has been at work*. This development cannot be linked to factors specific to any of these markets

*Speculative thinking* among investors, the use of *heuristics* such as *extrapolative expectations*, *market psychology* in the form of optimism and pessimism, *herd behavior* and social contagion of new ideas (new era thinking), and *positive feedback dynamics* are elements that play an important role in determining housing prices



## What are we going to do now?

The goal of our paper is to develop a simple model of a speculative housing market.

- Demand for houses depends (also) on expected future prices.
- Agents use simple (extrapolative and regressive) rules to predict prices.
- Agents switch between rules (with respect to market circumstances).

Nonlinear model may generate boom-bust housing price cycles.

Our approach is inspired by recent work on agent-based financial market models, see, e.g., Hommes (2006), LeBaron (2006), Chen (2008), Lux (2009) and Westerhoff (2009) for surveys.

## The model without speculation

Linear price adjustment function

$$P_{t+1} = P_t + a(D_t - S_t), \quad a > 0, \quad a = 1.$$

Total demand (desired stock) for houses

$$D_t = D_t^R + D_t^S, \quad \text{with } D_t^S = 0 \text{ for the moment.}$$

Real demand for houses

$$D_t^R = b - cP_t, \quad b, c > 0.$$

Total supply (stock) of houses

$$S_t = S_{t-1} - (1-d)S_{t-1} + eP_t, \quad 0 < 1-d < 1, \quad e > 0.$$

## Analytical results

Define  $Z_{t+1} = S_t$ . We obtain a two-dimensional linear map

$$\begin{cases} P_{t+1} = (1-c-e)P_t - dZ_t + b \\ Z_{t+1} = eP_t + dZ_t \end{cases}.$$

The model's unique fixed point is

$$\bar{Z} = \frac{e}{1-d}\bar{P} \quad \text{and} \quad \bar{P} = \frac{(1-d)b}{e+c(1-d)}.$$

Let us call  $\bar{P}$  the fundamental value.

(Common sense) Properties of  $\bar{P}$

- $\bar{P} > 0$  (and thus also  $\bar{Z} > 0$ )
- $b \uparrow \rightarrow \bar{P} \uparrow$ ,  $e, c, d \uparrow \rightarrow \bar{P} \downarrow$

The fixed point is globally asymptotically stable if

$$2 - \frac{e}{1+d} - c > 0, \quad \rightarrow \text{not always true}$$

$$c(1-d) + e > 0, \quad \rightarrow \text{always true}$$

$$1 - d + cd > 0. \quad \rightarrow \text{always true}$$

(Common sense) implications for stability domain of  $\bar{P}$

- $b \uparrow \rightarrow$  no impact on stability domain
- $c \uparrow \rightarrow$  stability domain  $\downarrow$
- $d \uparrow \rightarrow$  stability domain  $\uparrow$
- $e \uparrow \rightarrow$  stability domain  $\downarrow$

## The model with speculation

Speculative demand due to extrapolation

$$D_t^E = f(P_t - \bar{P}), \quad f > 0.$$

Speculative demand due to mean reversion

$$D_t^R = g(\bar{P} - P_t), \quad g > 0.$$

Total speculative demand

$$D_t^S = W_t D_t^E + (1 - W_t) D_t^R.$$

Relative impact of extrapolative demand

$$W_t = \frac{1}{1 + h(P_t - \bar{P})^2}, \quad h > 0.$$

## Model summary

Linear price adjustment function

$$P_{t+1} = P_t + a(D_t - S_t)$$

Total demand (desired stock) for houses

$$D_t = D_t^R + D_t^S$$

Total speculative demand

$$D_t^S = W_t D_t^E + (1 - W_t) D_t^R$$

Speculative demand due to mean reversion

$$D_t^R = g(\bar{P} - P_t)$$

Total supply (stock) of houses

$$S_t = S_{t-1} - (1 - d)S_{t-1} + eP_t$$

Real demand for houses

$$D_t^R = b - cP_t$$

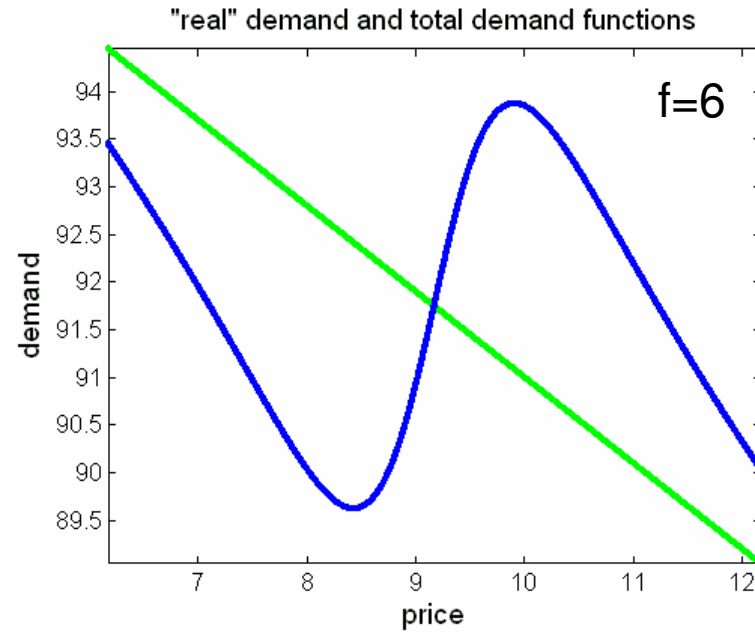
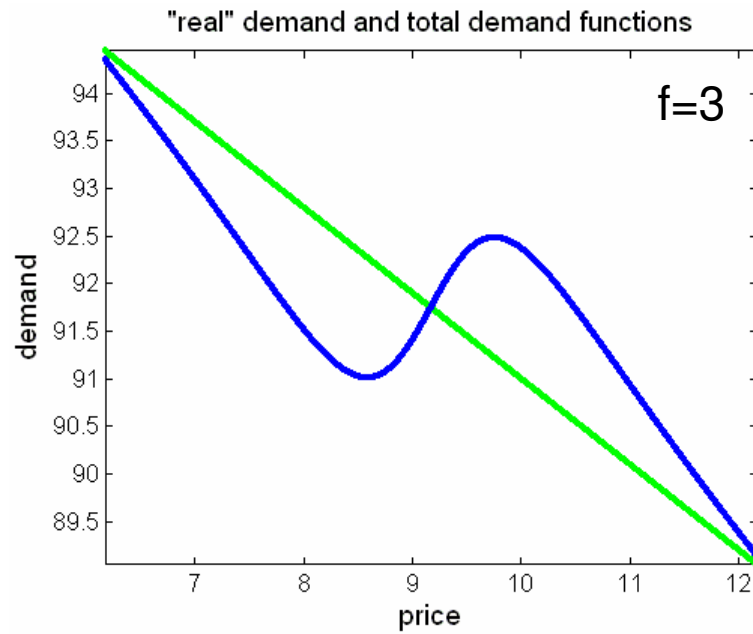
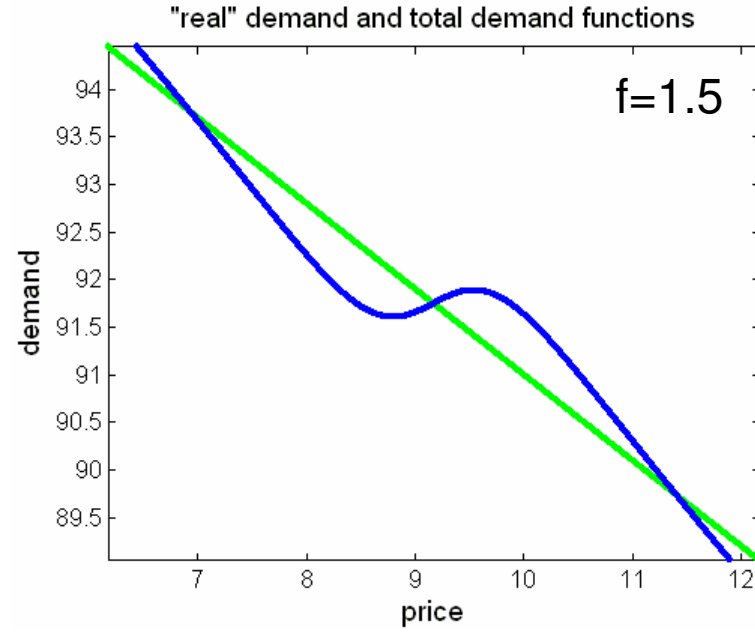
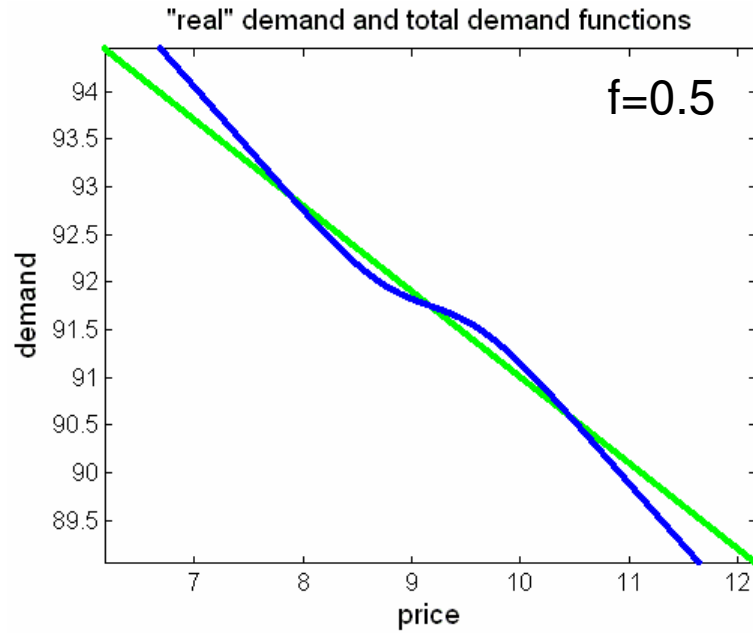
Speculative demand due to extrapolation

$$D_t^E = f(P_t - \bar{P})$$

Relative impact of extrapolative demand

$$W_t = \frac{1}{1 + h(P_t - \bar{P})^2}$$

$b=100$   $c=0.9$   $d=0.95$   $e=0.5$   $g=0.3$   $h=1$



## Analytical results

Define  $\pi_t = P_t - \bar{P}$  and  $\zeta_t = Z_t - \bar{Z}$ . We obtain a two-dimensional nonlinear map

$$\begin{cases} \pi_{t+1} = (1-c-e)\pi_t + \frac{f\pi_t - gh\pi_t^3}{1+h\pi_t^2} - d\zeta_t \\ \zeta_{t+1} = e\pi_t + d\zeta_t \end{cases}$$

The map has up to three fixed points. For  $\pi$  we find

$$\bar{\pi}_1 = 0 \quad \text{and} \quad \bar{\pi}_{2,3} = \pm \sqrt{\frac{(1-d)(f-c)-e}{h(e+(1-d)(c+g))}} \quad (\text{if } f > c + e/(1-d) > 0).$$

In addition, we get

$$\bar{\zeta}_{1,2,3} = \frac{e}{1-d} \bar{\pi}_{1,2,3}.$$



The fixed point ( $\bar{\pi}_1 = 0, \bar{\zeta}_1 = 0$ ) is locally asymptotically stable if

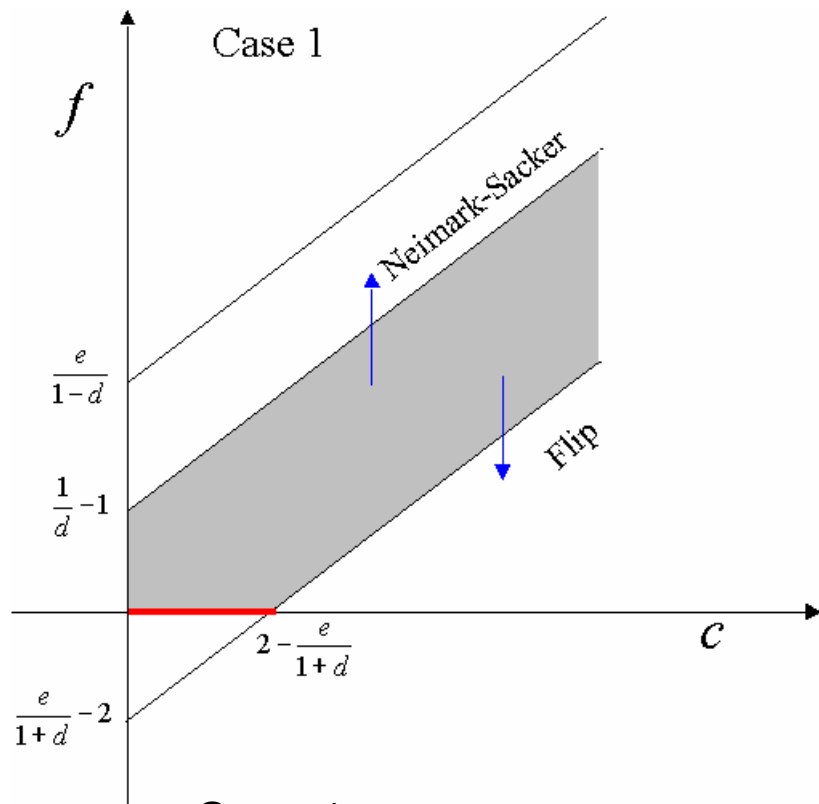
$$f > c + \frac{e}{1+d} - 2, \quad \rightarrow \text{(subcritical) flip bifurcation}$$

$$f < c + \frac{e}{1-d}, \quad \rightarrow \text{(supercritical) pitchfork bifurcation}$$

$$f < c + \frac{1}{d} - 1. \quad \rightarrow \text{(supercritical) Neimark-Sacker bifurcation}$$

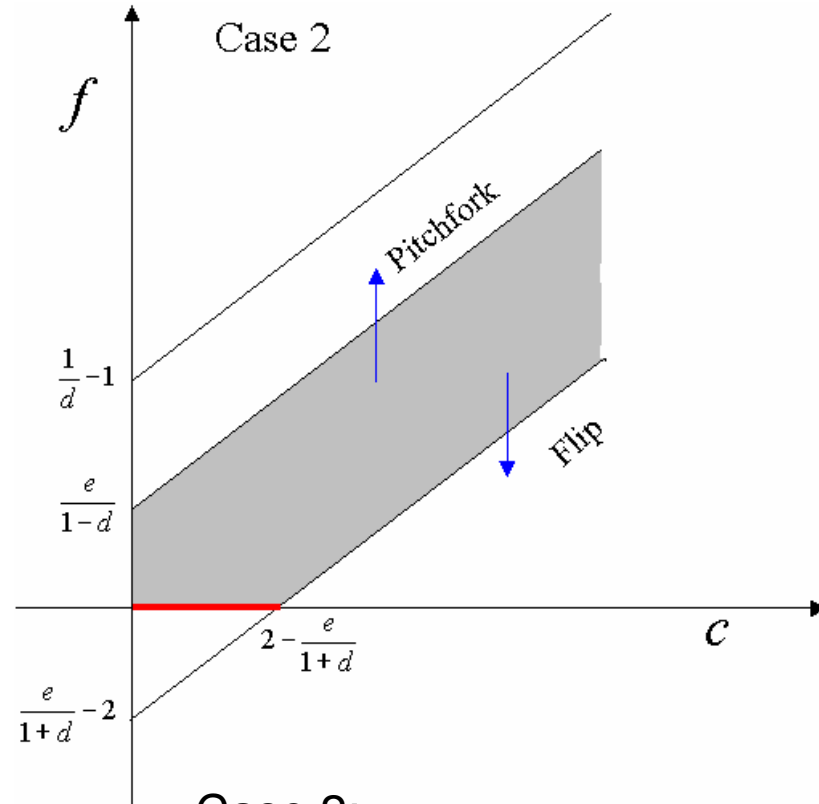
### Interpretation

- Speculation may be stabilizing or destabilizing.
- Three different types of bifurcations are present.



Case 1:

- high elasticity of supply, or
- low depreciation rate



Case 2:

- low elasticity of supply, or
- high depreciation rate

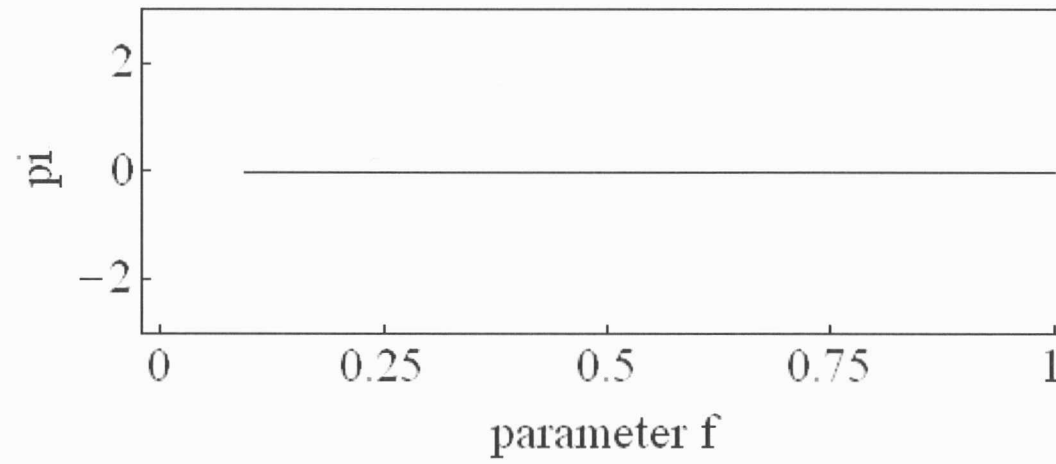
Which of the two scenarios occurs depends only on parameters  $d$ ,  $e$ , associated with the supply-side of the economy

## More economic intuition on local stability / bifurcations

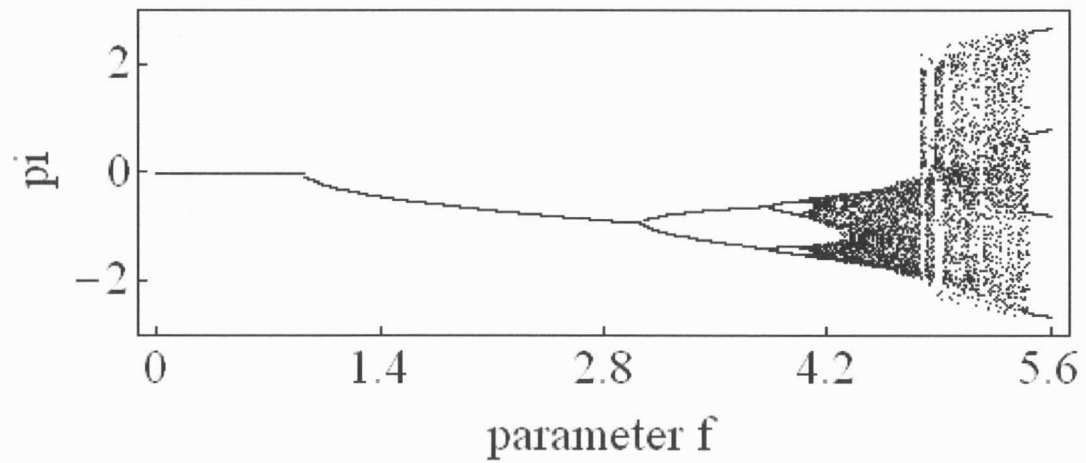
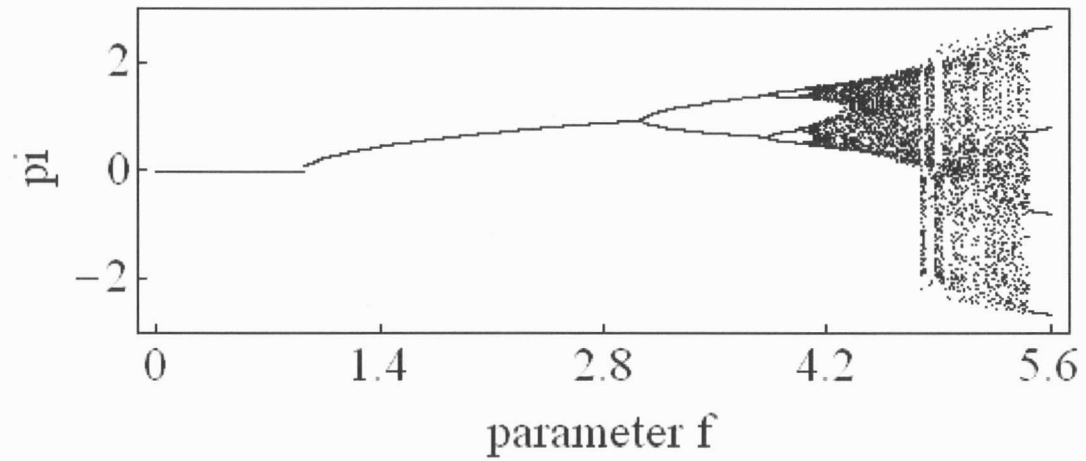
- If (real) demand and supply parameters are such that the model without speculation is stable, then the model with speculative demand is destabilized by sufficiently large values of extrapolation parameter  $f$ . In the opposite case, however, a sufficiently large value of parameter  $f$  may stabilize the model.
- Under a relatively elastic (inelastic) supply curve, the fundamental steady state bifurcates via Neimark-Sacker (Pitchfork) bifurcation.
- Stabilizing impact of ‘real’ demand: for any parameter selection, the stability region is wider if ‘real’ demand schedule is more sloped (provided that the slope is not too large)

## Numerical results

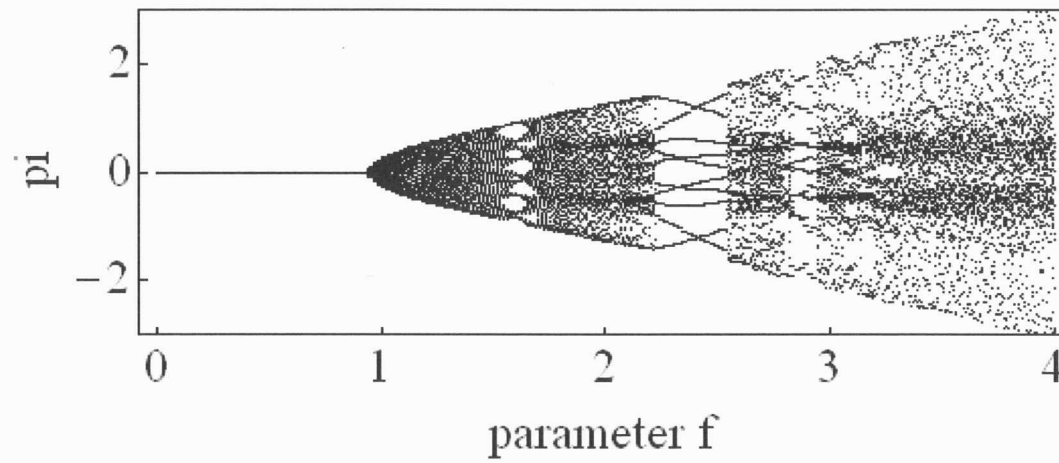
The (subcritical) flip bifurcation: speculation in stabilizing.



The (supercritical) pitchfork bifurcation: speculation is destabilizing.

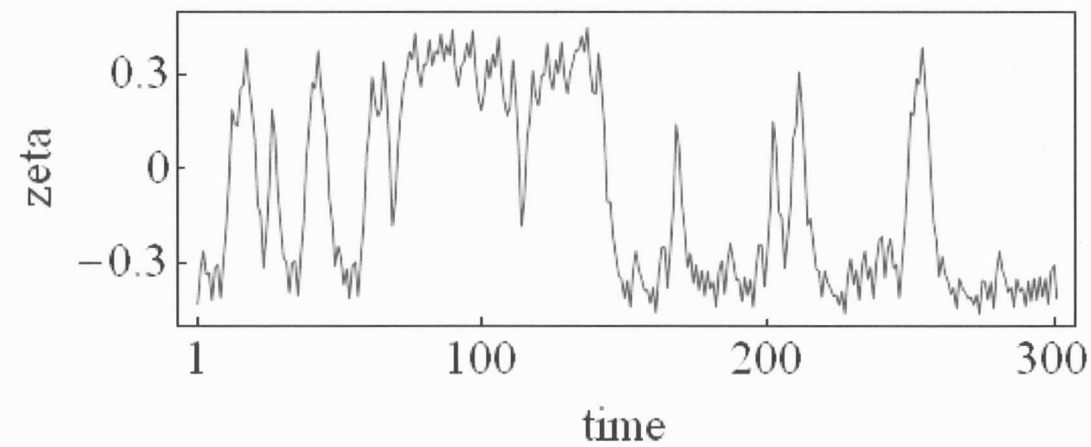
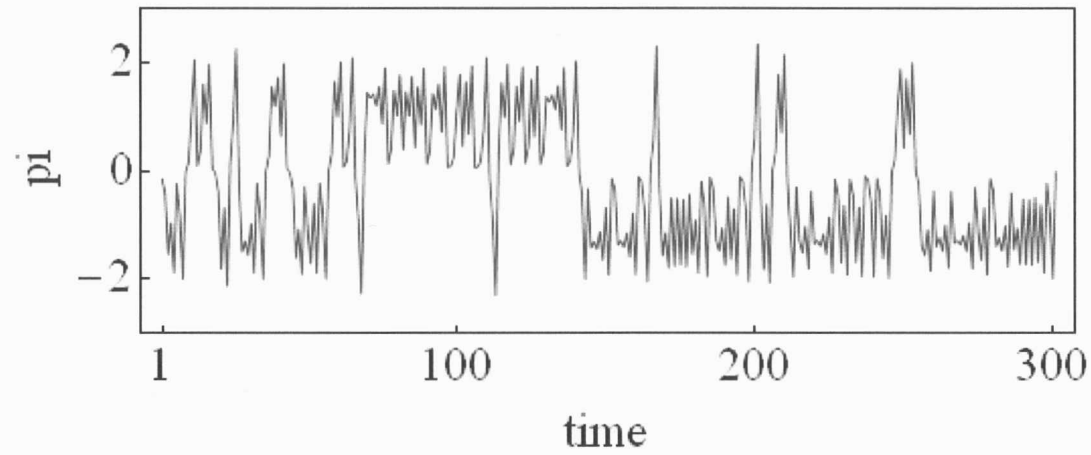


The (supercritical) Neimark-Sacker bifurcation: speculation is destabilizing.

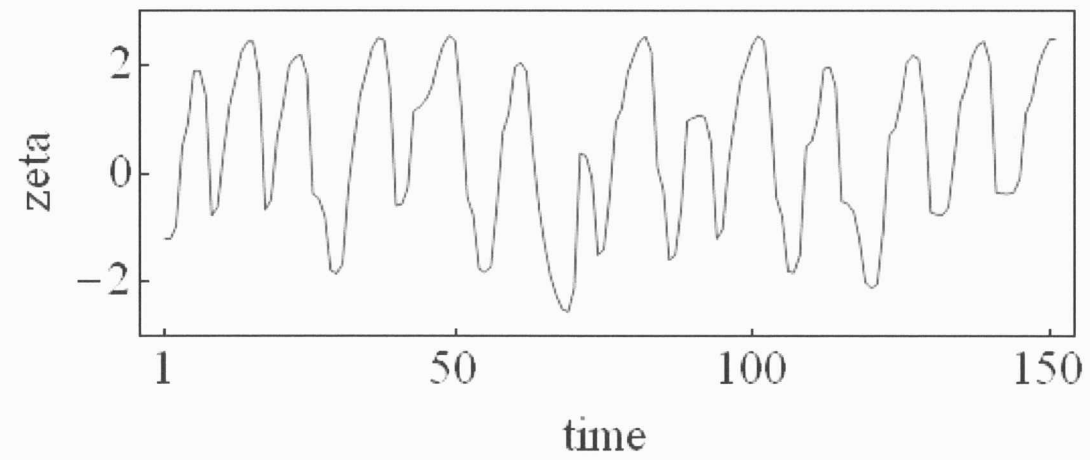
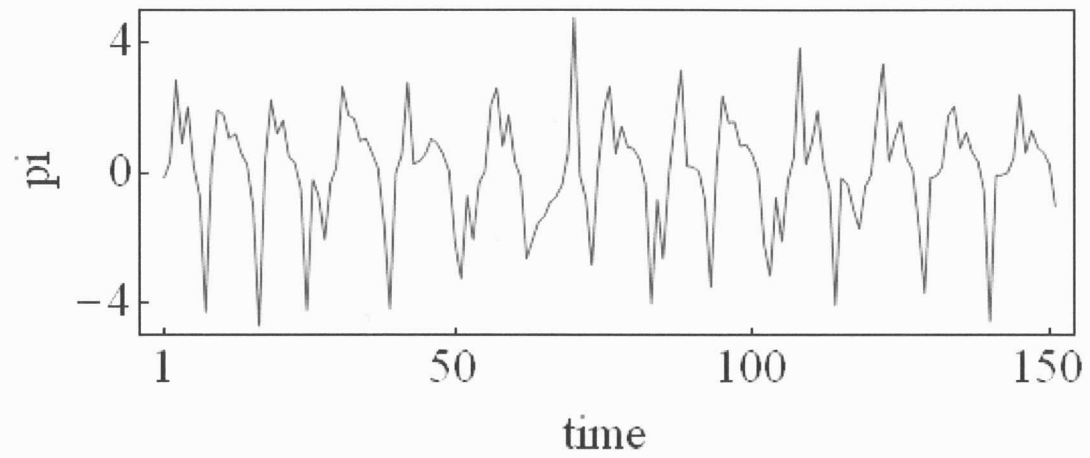


## Some time series examples

Dynamics “after” the pitchfork bifurcation: bull and bear markets.



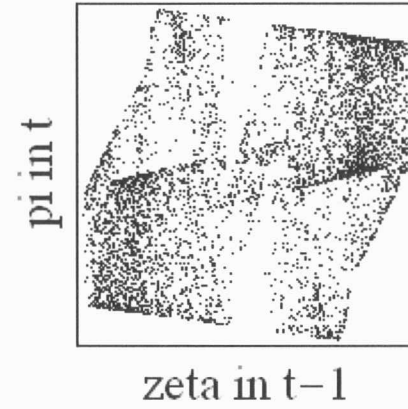
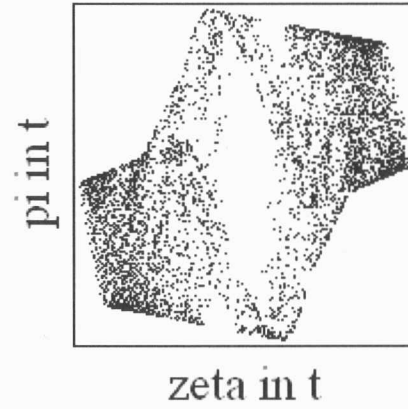
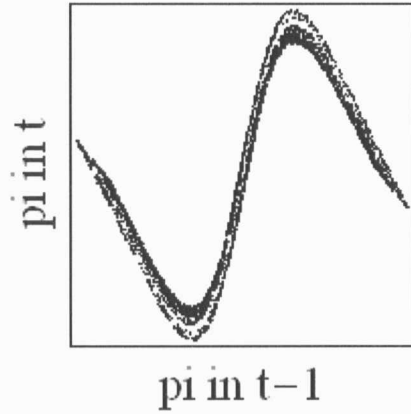
Dynamics “after” the Neimark-Sacker bifurcation: boom and bust cycles.



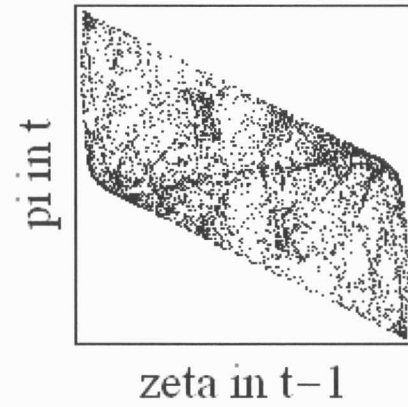
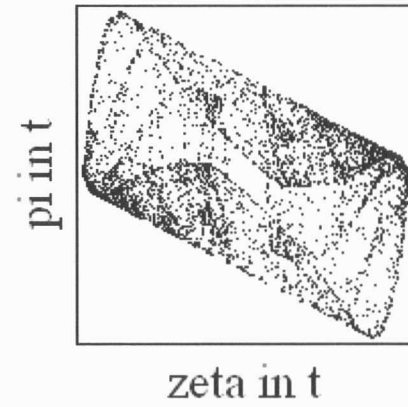
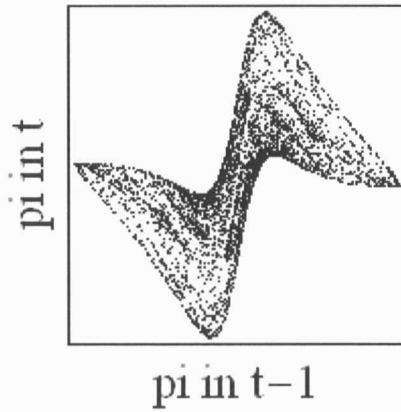


# Comparison in phase/state space

PFB



NSB



## **The stylized story of the model – at least a part of it**

Suppose that prices are slightly above the fundamental value.

Majority of agents is optimistic and expects a price increase.

Demand for houses increases and prices are pushed upwards for some time.

Market appears more and more overvalued and agents switch to mean reversion expectations.

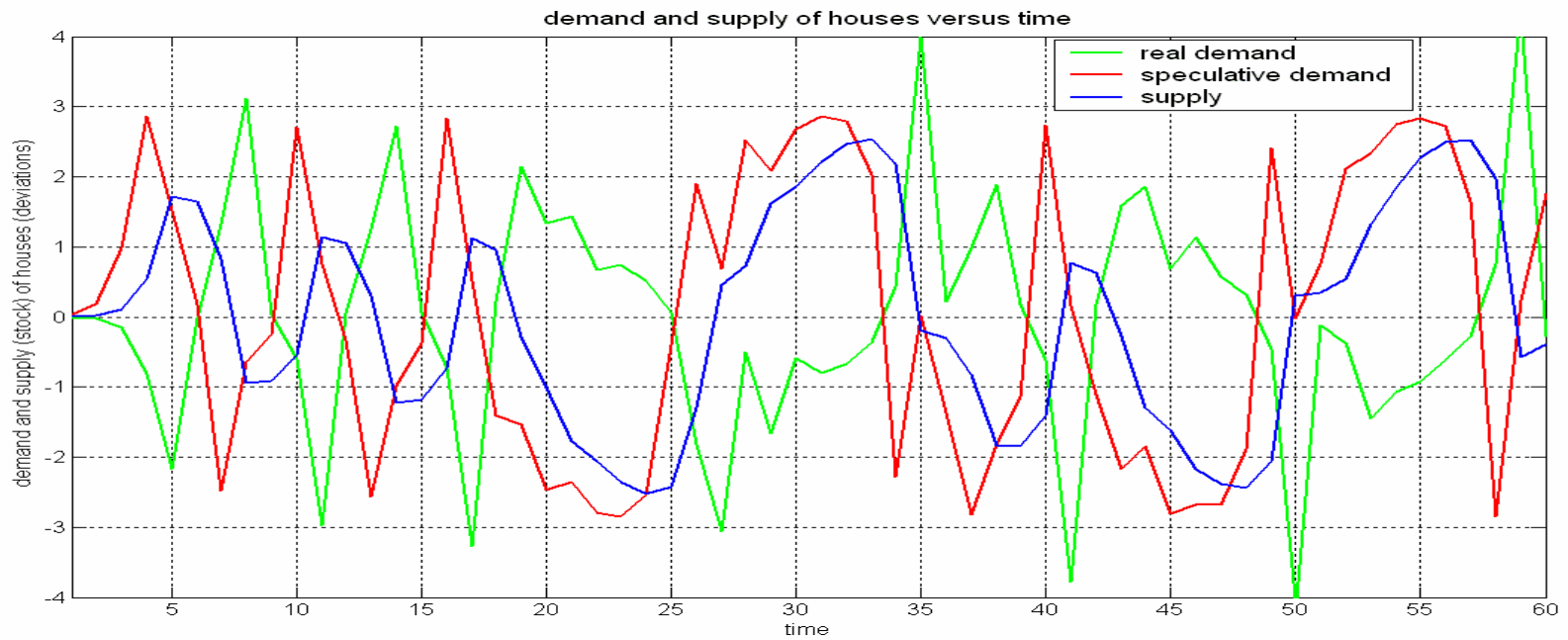
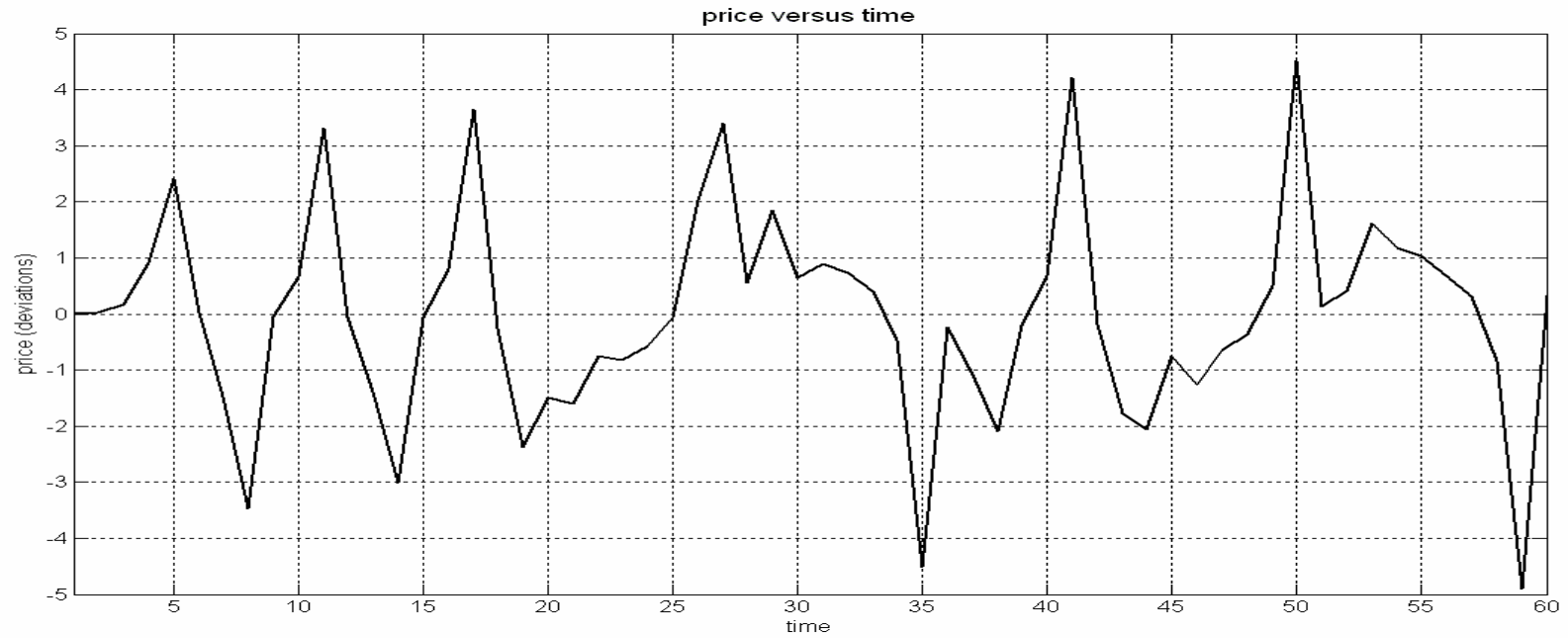
Some kind of adjustment towards the fundamental value sets in.

A strong adjustment may lead to a crash, otherwise the rally continuous after a price dip.

Also the real part of the model impacts on the dynamics.

As long as housing prices are high, new constructions increase the stock of houses.

During a downwards movement, demand for houses may thus be considerably lower than supply of houses, amplifying any price reduction.



## **Conclusions**

- Model/story/view about the mechanism of formation of housing prices
- Speculative thinking among investors modelled via the impact of different types of heuristics and endogenous changes in market sentiment
- Speculative demand is in general destabilizing, two different bifurcation-routes to 'boom-bust' scenarios
- Combination of real and speculative forces appears to be a key factor for generating intricate bubbles and crashes

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