Natural Selection in Markets

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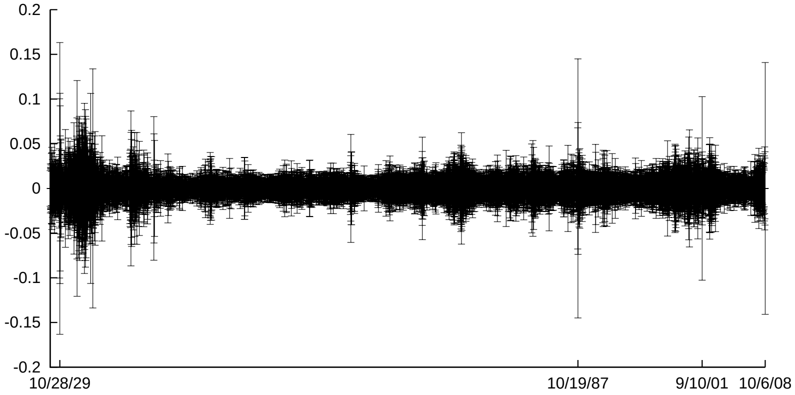
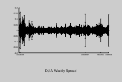
Understand This!



DJIA Weekly Spread



We view them [derivatives] as time bombs both for the parties that deal in them and the economic system . . . In our view . . . derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal.
– Warren Buffett, Berkshire Hathaway Annual Report, 2002

> *Institutions are herding animals. We watch the same indicators and listen to the same prognostications. Like lemmings, we tend to move in the same direction at the same time. And that, naturally, exacerbates price movements.*
> *– WSJ, 17 Oct 1989*

This is from a trader.

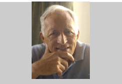
- Psychological explanations of herding.
- Social cognition explanations of herding.
- Michaely's fund-manager incentives paper describes a rational source of herding.
- Information cascades.

It doesn't take herding to get volatility.

The Representative Consumer

1. The Lucas critique
2. The representative consumer as an equilibrium construct
3. Welfare problems
4. Behavioral representative consumers

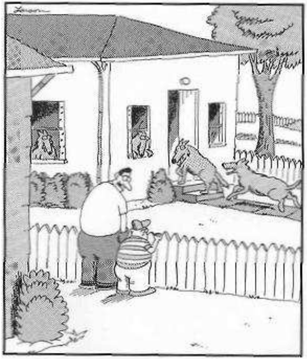
An Important Question



Why do GE?

- GE is not a theory of markets, it is an outcome function.
- Its utility as such is an empirical question.
- Alan's research agenda is interesting.
  - Model specific market processes
  - See if GE does well
  - If not, what does better
  - Is there a small class of outcome functions which span most markets (if not, we're sunk).

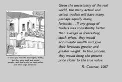
Evolution and Economics I



"I know you miss the Wainrights, Bobby, but they were weak and stupid people—and that's why we have wolves and other large predators."

Given the uncertainty of the real world, the many actual and virtual traders will have many, perhaps equally many, forecasts. . . If any group of traders was consistently better than average in forecasting stock prices, they would accumulate wealth and give their forecasts greater and greater weight. In this process, they would bring the present price closer to the true value.

R. Cootner, 1967



The classic 'market efficiency through selection' argument. It gives an evolutionary justification for **rational expectations**. Also Fama (65):

> *. . . dependency in the noise generating process would tend to produce 'bubbles' in the price series. . . If there are many sophisticated traders in the market, however, they will be able to recognize situations where the price of a common stock is beginning to run up above its intrinsic value. If there are enough of these sophisticated traders, they may trend to prevent these 'bubbles' from ever occurring.*

And how do we tell who is sophisticated?

> *A superior analyst is one who gains over many periods of time are consistently greater than those of the market.*

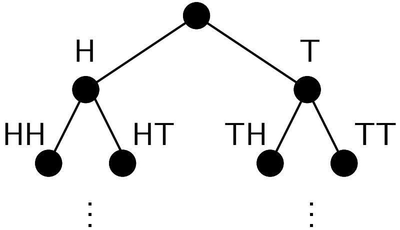
Note:

- Chicago circularity.
- Collapse to homogeneity. Correct pricing comes from uniformity of beliefs, which is enforced by the market. Is this plausible?

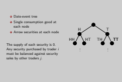
Competitive Exchange Economies

- Date-event tree
- Single consumption good at each node
- Arrow securities at each node

The supply of each security is 0. Any security purchased by trader i must be balanced against security sales by other traders j.



Competitive Exchange Economies



Our economies are built on the usual date-event tree, described on this slide.

At each node, say H, there is a single consumer good, and two Arrow securities, HH and HT.

Assets are in 0 net supply. If you're not used to this, think insurance, or horse races.
If we had firms, this models stock markets as follows. Ownership of the profit stream is endowed, and these streams can be traded. Arrow securities in this case measure net trades.

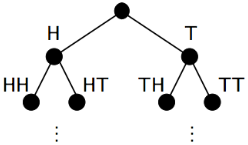
c A *consumption plan* $\{c_t\}_{t=1}^{\infty}$, $c_t : \sigma^t \mapsto \mathbf{R}_{++}$.

e^i Trader i's *endowment*. ...

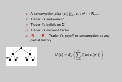
ρ^i Trader i's beliefs on Σ.

β_i Trader i's discount factor.

$u^i : \mathbf{R}_+ \to \mathbf{R}$ Trader i's payoff to consumption at any partial history.



$$U_i(c) = E_{\rho^i}\left\{\sum_{t=0}^{\infty} \beta_i^t u_i\big(c_t(\sigma^t)\big)\right\}$$



A *consumption plan* is any map from the nodes of the date event tree to positive amounts of the consumption good. Thus e^i is the *consumption plan* representing trader *i*s endowment. Notice that all components of all plans are (by definition) measurable with respect to the right things.

Preferences here are the usual intertemporally additively separable type. Each such preference order is characterized by a *payoff function* u_i, a *discount factor* β_i, and beliefs ρ^i. The selection question is, which of these factors is important for selection, and how do they trade off.

Explain how to move resources from HH to TT.

To describe a CE, there are prices at each state. Traders know the prices that will be realized at each state (prices are part of the state description) but disagree on probabilities with which states are realized. Consumers have a **budget constraint**. Demand is derived from EU maximization. Prices (the states which can be realized) are those in which markets clear — net demand equals 0.

Assumptions

A.1. The payoff functions u_i are C^1, strictly concave, strictly monotonic, and satisfy an Inada condition at 0.

A.2. Each trader has a strictly positive endowment at every partial history, and the aggregate endowment is uniformly bounded, below away from zero and from above.

A.3. At every partial history, each trader i believes all truly possible states to be possible: $\rho_i \gg 0$

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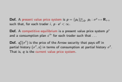
1. The Inada conditions are not necessary. Any trader who does not satisfy the Inada condition will almost surely vanish in finite time. That is, the probability that $c_t^i > 0$ infinitely often is 0.
2. David's assumptions from yesterday.

Def. A present value price system is $p = \{p_t\}_{t=0}^{\infty}$, $p_t : \sigma^t \mapsto \mathbf{R}_{++}$ such that, for each trader i, $p \cdot e^i < \infty$.

Def. A competitive equilibrium is a present value price system p^* and a consumption plan c^{i*} for each trader such that ...

Def. $q_t^s(\sigma^t)$ is the price of the Arrow security that pays off in partial history (σ^t, s) in terms of consumption at partial history σ^t. That is, q is the current value price system.



As was already said, competitive paths exist and inherit all the properties of optimal paths. We want to talk about the evolution of current value prices — the value of a date t asset in units of date t consumption.

Existence comes from standard arguments.

Multiple Survivors

Limit log MU ratios:

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} = t\left(\log \frac{\beta_j}{\beta_i} - \sum_s \rho_s \Big(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j}\Big)\right)$$

$$- \sum_s \big(n_t^s(\sigma) - t\rho_s\big)\left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j}\right)$$

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Multiple Survivors

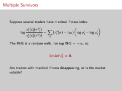
Suppose several traders have maximal fitness index.

$$\log \frac{u_i'\left(c_t^i(\sigma^t)\right)}{u_j'\left(c_t^j(\sigma^t)\right)} = \sum_s \left(n_t^s(\sigma) - t\rho_s\right)\left(\log \rho_s^j - \log \rho_s^i\right)$$

The RHS is a random walk. $\limsup \text{RHS} = +\infty$, so

$$\mathbf{\liminf c_t^i = 0}.$$

Are traders with maximal fitness disappearing, or is the market volatile?



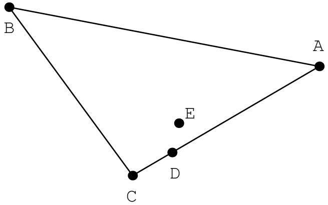
In this extended example, the log Marginal Utility ratio is a random walk. The difference in fitness is the mean of the walk. Different maximal-fitness types can exist because of a trade-off between accuracy of beliefs and discount factors, or because beliefs can be equally wrong (in the entropy sense) in different ways. With multiple maximal-fitness types, the random walks of log MU ratios has 0 drift.

The existence of multiple maximal-fitness types is non-generic. Why study it?

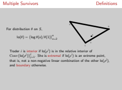
- The phenomenon is robust in more complicated models.
- In some cases with a unique long-run survivor type, the drift of the random walk can be sufficiently slow relative to the speed of the large scale fluctuations in the walk that the asymptotic analysis of the walk is useful. In any event, short run properties of the walk can be translated into statements about short-run price behavior.

For distribution θ on S,

$$\mathsf{lo}(\theta) = \big(\log\theta(s)/\theta(1)\big)_{s=2}^{\mathbf{s}}$$



Trader i is interior if $\mathsf{lo}(\rho^i)$ is in the relative interior of $\mathrm{Conv}\{\mathsf{lo}(\rho^j)\}_{j=1}^{I}$. She is extremal if $\mathsf{lo}(\rho^i)$ is an extreme point, that is, not a non-negative linear combination of the other $\mathsf{lo}(\rho^j)$, and boundary otherwise.



The long-run behavior of the process with multiple survivors is determined by these log-odds ratios. Of course it does not matter how the normalization is done (which state).

Back one slide: The stochastic process $(n_t^s - t\rho_s)_{s=1}^{\mathrm{s}}$ is an $\mathrm{s} - 1$-dimensional random walk, to which the linear functionals $(\log \rho_1^i, \ldots, \log \rho_{\mathrm{s}}^i)$ are applied. After messing around, these terms turn out to be $(n_t^s - t\rho_s)_{s=2}^{\mathrm{s}} \cdot \mathrm{lo}(\rho^i)$ plus a constant. We want to compare this with $(n_t^s - t\rho_s)_{s=2}^{\mathrm{s}} \cdot \mathrm{lo}(\rho^j)$ to see how big or how small it is. This is simple geometry: Where in walk-space is $\mathrm{walk}_t \cdot \big(\mathrm{lo}(\rho^i) - \mathrm{lo}(\rho^j)\big)$ large? How often is that region of walk-space visited?

If $s \leq 3$,

- All maximally fit traders survive.
- For extremal traders, $\limsup_t c^i_t/e = 1$.

If $s > 3$,

- Interior traders vanish.
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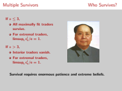
Survival requires enormous patience and extreme beliefs.

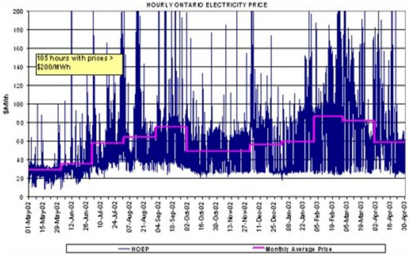


The long run survivor properties and their price implications follow from the recurrence properties of the walk, and these depend upon the state space. Extreme traders have consumption shares which are infinitely often near 1, but not boundary or interior traders. This is because whenever the random walk benefits a boundary or interior trader, it benefits an extreme type even more.

Assume no social risk.

Correct asset Pricing:

$$p(\sigma^t) = \rho.$$



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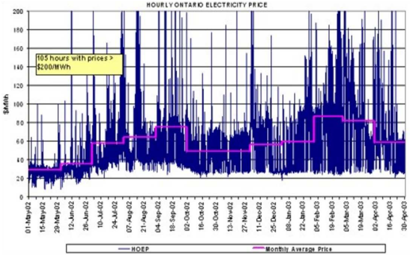
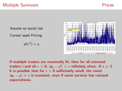


If multiple traders are maximally fit, then for all extremal traders i and all $\epsilon > 0$, $|q_t - \rho^i| < \epsilon$ infinitely often. If $s > 3$ it is possible that for $\epsilon > 0$ sufficiently small, the event $|q_t - \rho| < \epsilon$ is transient, even if some survivor has rational expectations.



When trader *i*'s consumption share is near 1, prices of the Arrow securities are near his beliefs — the price of a bet on horse *s* is approximately the probability that *i* assigns to *s* winning the race. So asset prices are volatile. They return infinitely often to the region of any extreme traders' beliefs.

The picture is of minute-by-minute electricity prices on an Ontario energy market. I chose it to illustrate price volatility, but experts tell me that, in a sense, this picture doesn't do the job. Apparently, even at small time scales, demand is fractal, so these prices reflect volatility in demand rather than some internally driven (rather than exogenously driven) volatility.

The model we've built here is for illustrative purposes only. We do not claim that real asset-market price volatility looks like the volatility that comes from a random walk. Our claim is smaller; that, contrary to intuition, long-run price volatility can occur even in the most conventional of equilibrium models, as soon as one takes agent heterogeneity seriously.

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A.2. Each trader has a non-zero endowment.

A.3. At every partial history, each trader i believes all truly possible states to be possible: $\rho_i \gg 0$

$$\log \frac{u_i'\left(c_t^i(\sigma^t)\right)}{u_j'\left(c_t^j(\sigma^t)\right)} = t \log \frac{\beta_j}{\beta_i} + \sum_s n_t^s(\sigma) \log \frac{\rho_j(s)}{\rho^i(s)}$$

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)/u_i'\big(e_t(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)/u_j'\big(e_t(\sigma^t)\big)} =$$

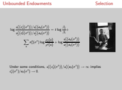
$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)/u_i'\big(e_t(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)/u_j'\big(e_t(\sigma^t)\big)} = t \log \frac{\beta_j}{\beta_i} +$$

$$\sum_s n_t^s(\sigma^t) \log \frac{\rho_j(s)}{\rho^i(s)} + \log \frac{u_j'\big(e_t(\sigma^t)\big)}{u_i'\big(e_t(\sigma^t)\big)}$$

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)/u_i'\big(e_t(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)/u_j'\big(e_t(\sigma^t)\big)} = t \log \frac{\beta_j}{\beta_i} +$$

$$\sum_s n_t^s(\sigma^t) \log \frac{\rho_j(s)}{\rho^i(s)} + \log \frac{u_j'\big(e_t(\sigma^t)\big)}{u_i'\big(e_t(\sigma^t)\big)}$$

Under some conditions, $u_i'\big(c_t^i(\sigma^t)\big)/u_i'\big(e_t(\sigma^t)\big) \to \infty$ implies $c_t^i(\sigma^t)/e_t(\sigma^t) \to 0$.



A quick analysis of the B&E 06 type. The RHS is something to which laws of large numbers applies.

Normalizing by the MU of the endowment controls the denominator.

This insight carries over into production models.

CRRA Utility:

$$u_(c) = \frac{1}{\gamma} c^{\gamma}, \qquad \gamma < 1.$$

Stochastic Geometric Endowment Process:

$$e_0 \text{ given.}$$

$$e_t = \tilde{r}_t e_{t-1}; \quad \tilde{r}_t \text{ iid on } (0, \infty), \quad \mathrm{E}\, \tilde{r}_t = r.$$

$$\mathrm{f}_i = \log \beta_i - I_i(\rho, p_i) + \gamma_i r.$$

CARA Utility:

$$u_(c) = -\frac{1}{\gamma} \exp -\gamma c, \qquad \gamma > 0.$$

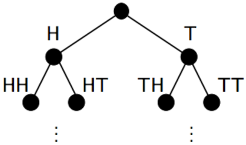
Endowment process is a random walk with mean drift $\eta > 0$ and a reflecting barrier at some $\delta > 0$.

$$\mathrm{f}_i = \log \beta_i - I_i(\rho, p_i) + \gamma_i \eta.$$

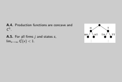
Firms

A.4. Production functions are concave and C^1.

A.5. For all firms j and states s, $\lim_{x \to \infty} f_s^j(x) < 1$.

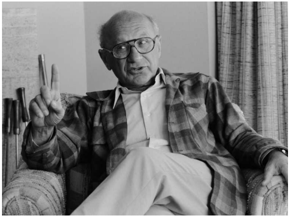


Firms



- Endowments are only initial stocks of consumption goods.
- Private ownership economy
- One input, many outputs
- Price of shares: Value of production plan in consumption good at current node.
- Normalize by maximal possible output.
- FOC are consumer FOC plus firm FOC. So selection analysis proceeds as before.

Evolution and Economics I



Confidence in the maximization-of-returns hypothesis is justified by evidence of a very different character. . . . Let the apparent immediate determinant of business behavior be anything at all. . . Whenever this determinant happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources . . . The process of "natural selection" thus helps to validate the hypothesis — or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival.

Friedman, 1953

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Friedman, 1953

Here Friedman uses an evolutionary argument to justify the hypothesis of profit maximization. We have worked on this.

Finis