Growth fluctuations and market efficiency

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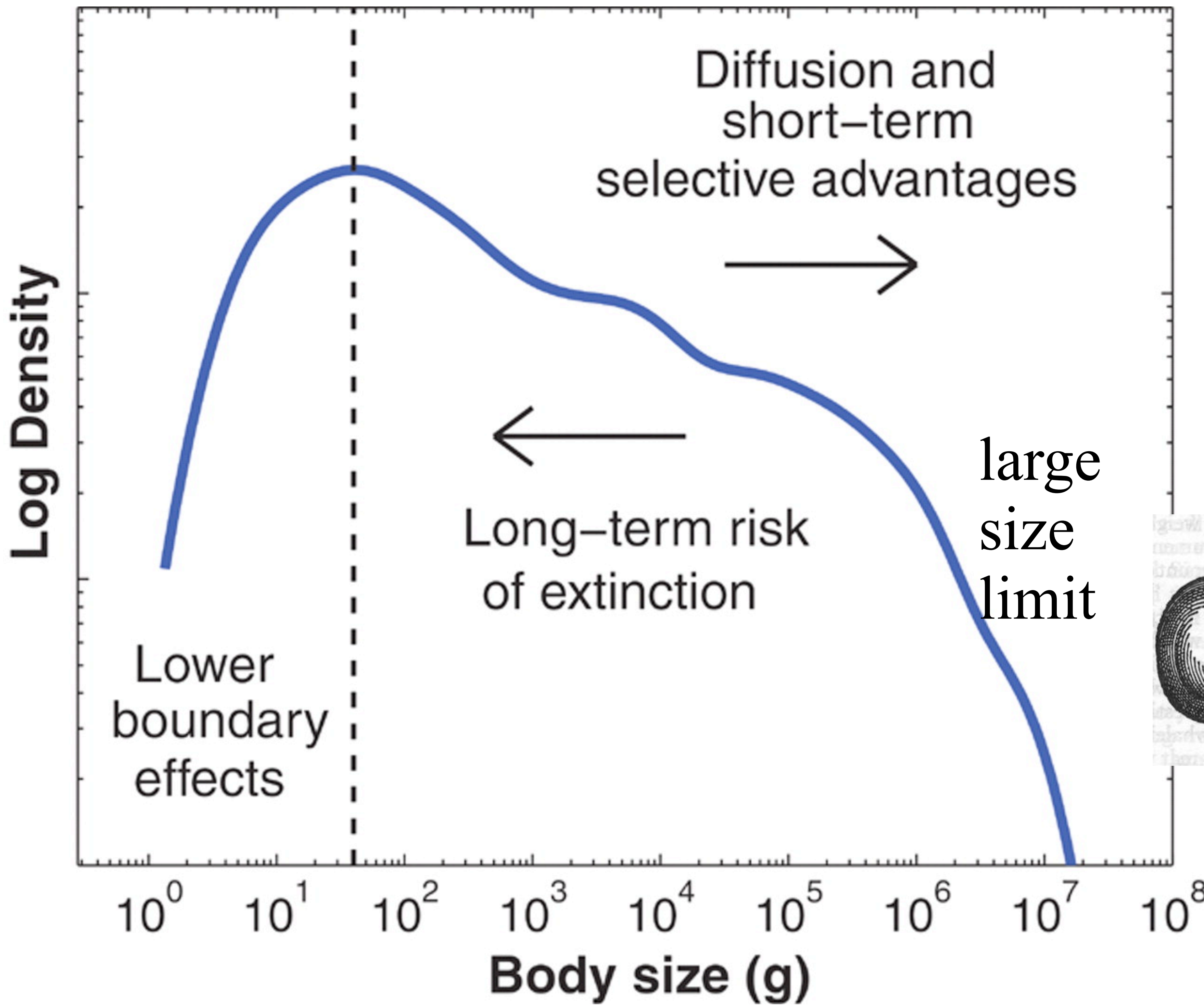
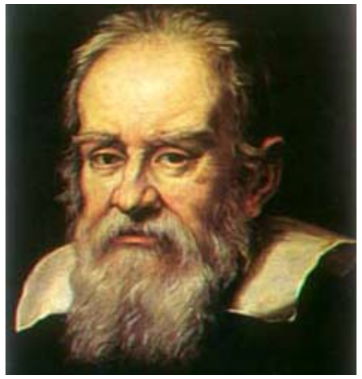
Philosophy

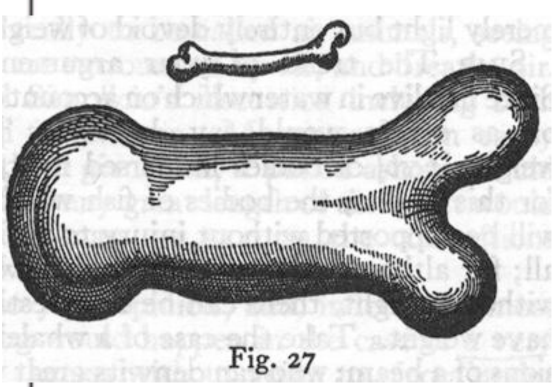
- Pick low-hanging fruit first
- Find quantitative laws that are as general as possible. Make sharp, falsifiable predictions.
- Evolutionary finance will catch on only when it makes clear theoretical predictions, in agreement with data, that cannot be made through neoclassical methods.

Fig. 1. Smoothed species body-size distribution of 4002 Recent terrestrial mammals [data from (21)], showing the three macroevolutionary processes that shape the relative abundances of different sizes



large
size
limit





square
cube
law

A. Clauset et al., Science 321, 399 -401 (2008)

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Science
AAAS

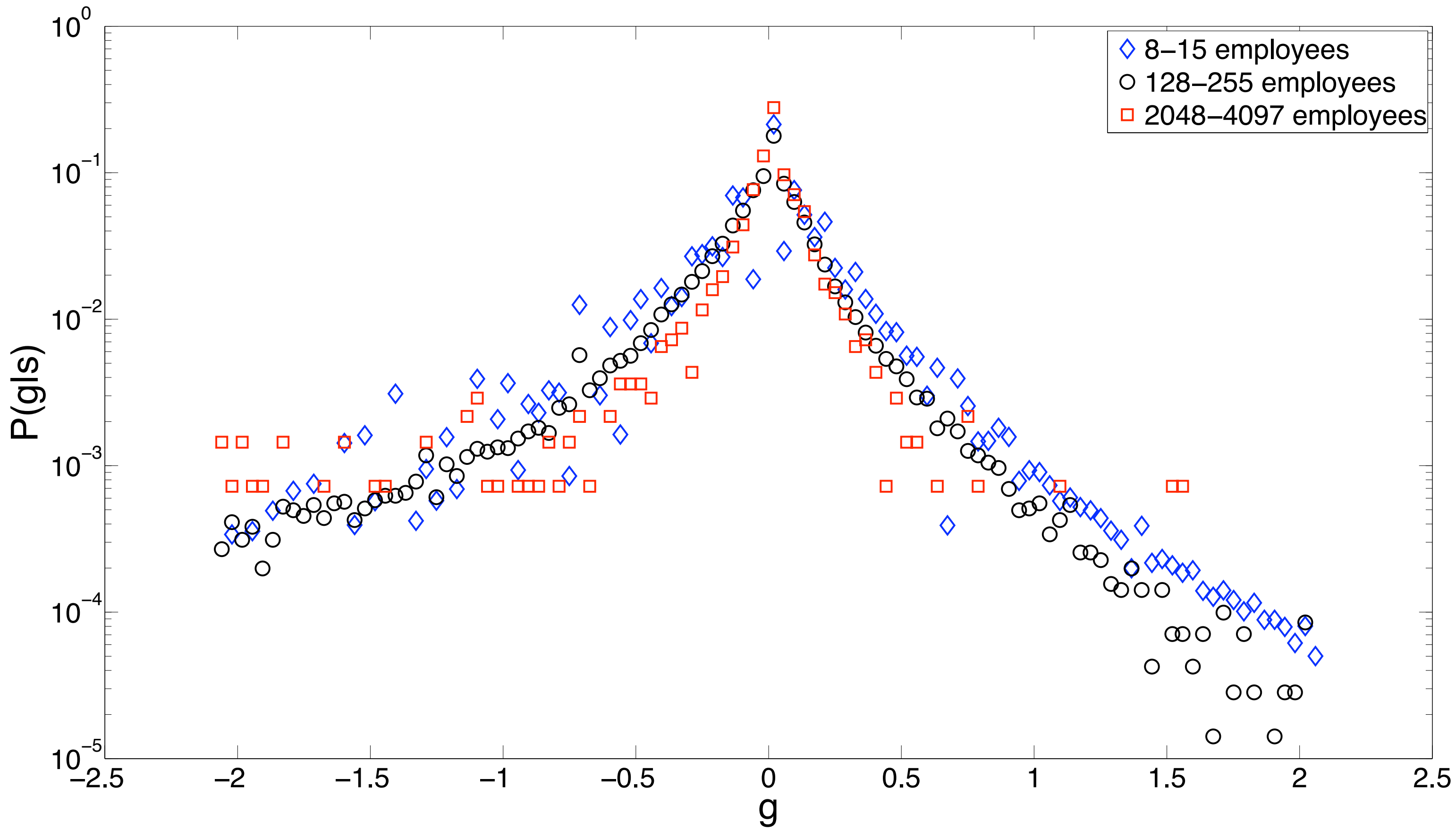
Evolution?

- Darwin: Descent + variation + selection
- How does this differ in financial markets?
- What is role of investor choice? Analogue to Galileo's square-cube law?
- Goal: Theory for allometry in finance
- Style of modeling used here is old in economics, e.g. Gibrat (30's), Simon (50's).

Outline

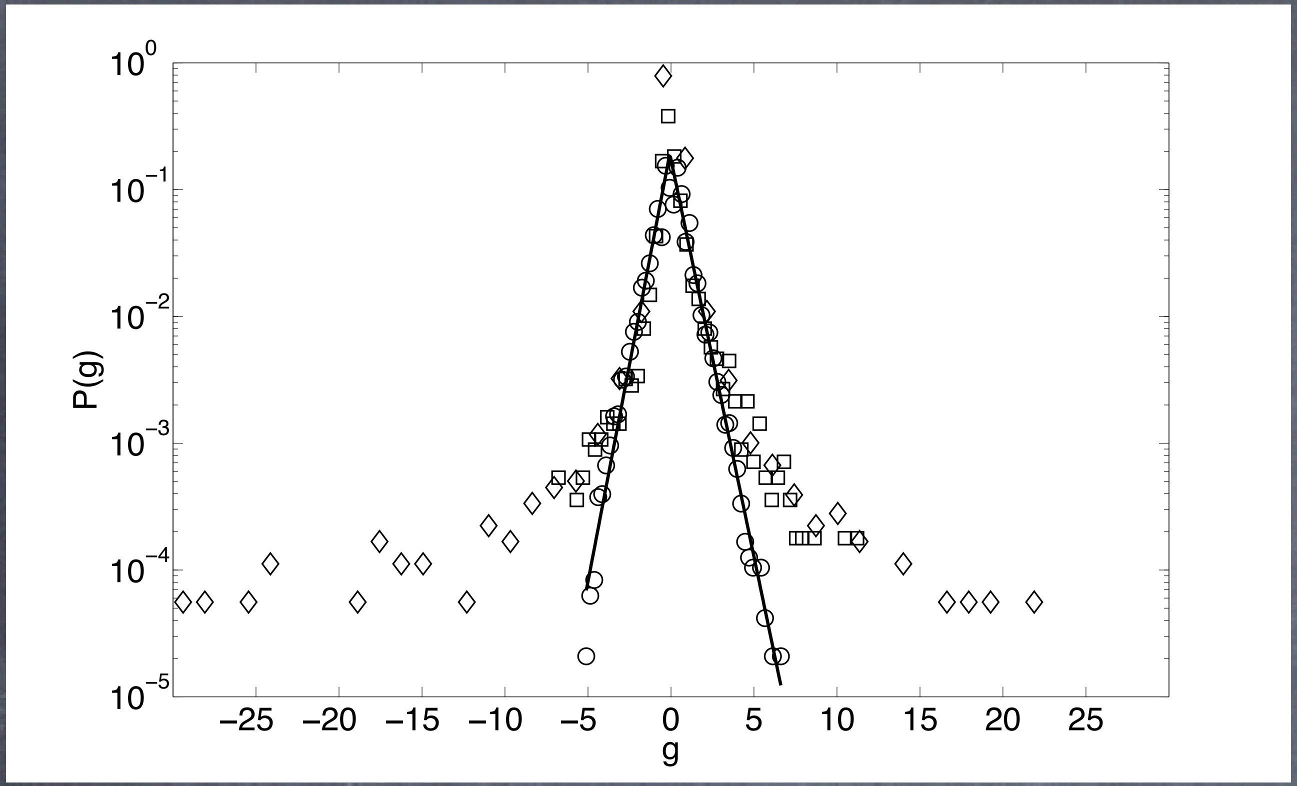
- Theory for growth fluctuations of firms, bird populations, mutual funds, (GDP, city size, ...
- Theory for size of mutual funds
- Role of skill and transaction costs?

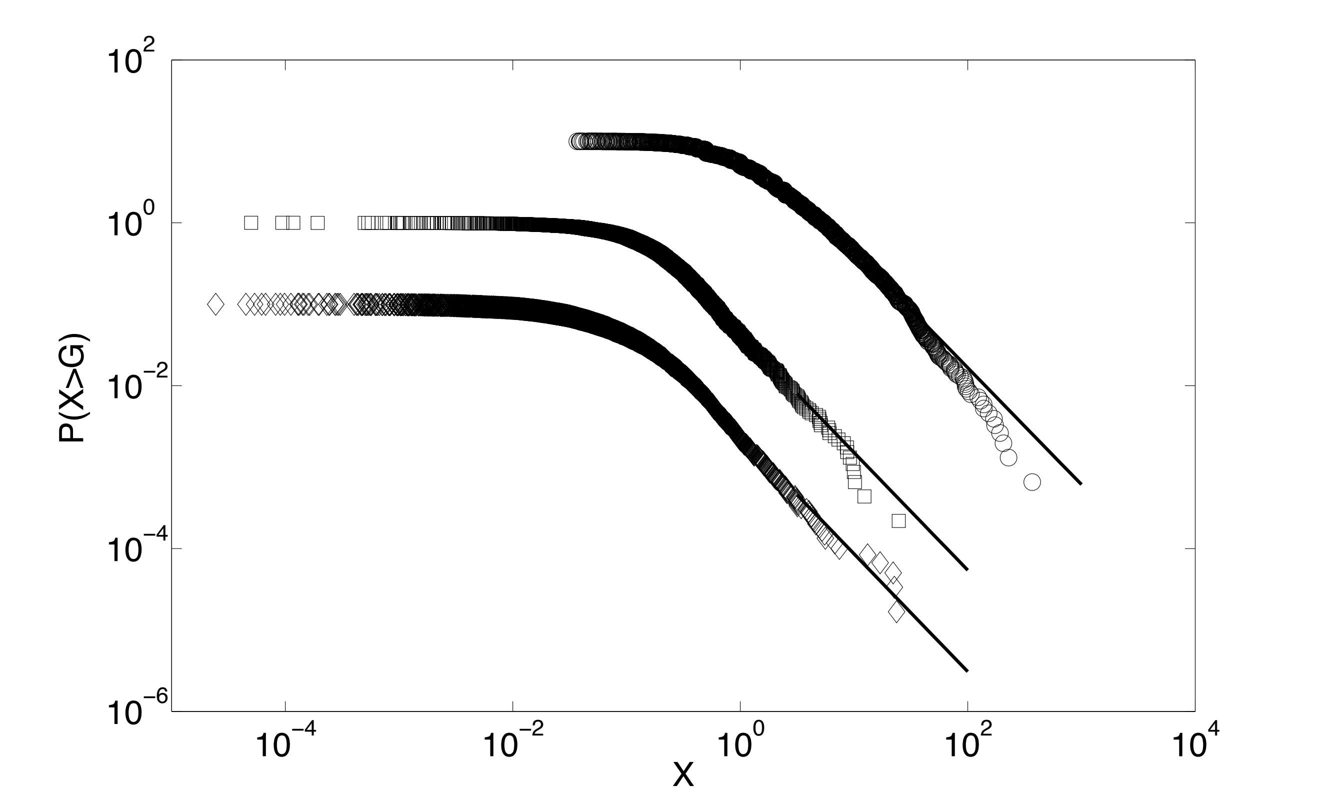
Empirical Observations of Firm Growth

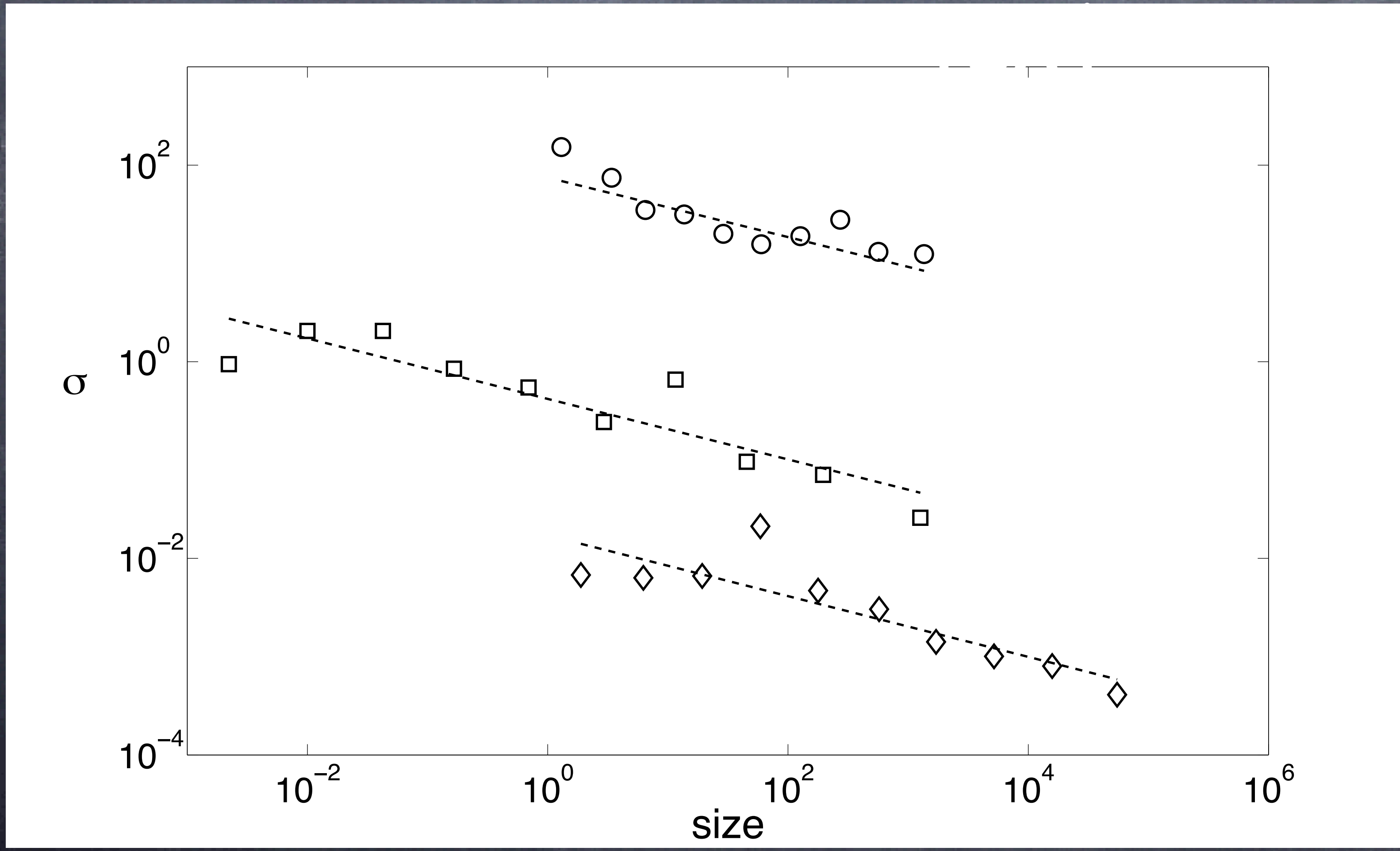


The yearly growth rate distribution for firms between the years 1998 to 1999.
Size was measured by the number of employees as extracted from the Census data base.

- The empirical growth rate distribution $P(g)$ where $g_t = \log(s_{t+1}/s_t)$ is close to an asymmetric Laplacian (double exponential) distribution in the body.







$$\sigma \sim N^{-\beta}$$

birds

firms (sales)

mutual funds

Fluctuation scaling in various systems

- Business firms $\beta \approx 0.15$ (Amaral et al ,1997).
- GDP of countries $\beta \approx 0.15$ (Lee et al ,1998).
- R&D expenditure of 719 american universities $\beta \approx 0.25$ (Plerou et al ,1999).
- Growth of scientific output in either institutions or countries $\beta \approx 0.35$ (Matia et al ,2005).
- Metropolitan area populations (Rozenfeld et al ,2008) $\beta \approx 0.2$ in the US, $\beta \approx 0.27$ in GB and $\beta \approx 0.19$ in Africa.
- Mutual funds investor money growth $\beta \approx 0.2$ (Schwarzkopf and Farmer, 2008).

Why is this interesting?

- Independence of subunits implies $\beta = 1/2$
- Perfect correlation implies $\beta = 0$
- Suggests a nontrivial collective phenomenon
- Phenomena are interesting in and of themselves.

Existing Models

- Subunits of the firm can be created and remerged into the firm and the growth of the subunits is uncorrelated (Amaral et al ,1998).
- Firms have a random number subunits, that is model a random partitioning into uncorrelated subunits. (Sutton ,2001).
- Firms diversify into several uncorrelated markets with an algebraic dependence on firm size (Bottazzi and Secchi ,2003).
- Wyart and Bouchaud: Model for firms, subunit sizes fat tailed, CLT
- <u>All these models concentrate on the structure of the firm!</u>

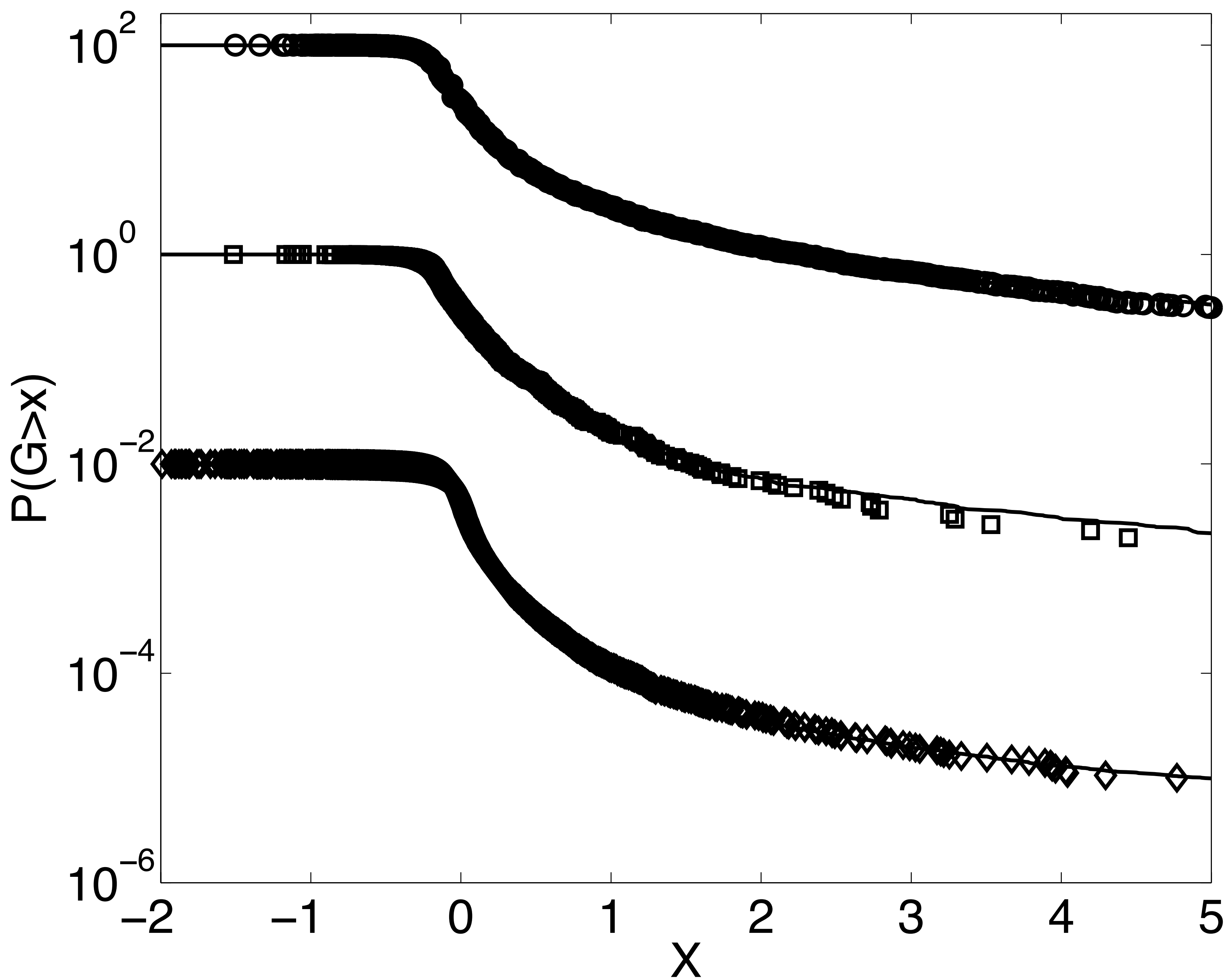
Our theory

- Divide the entity into N subunits
- At each time step, each subunit replicates, generating k_j new subunits drawn from a distribution p(k).
- $$N_{t+1} = \sum_{j=1}^{N_t} k_j$$

$$G_t = \frac{N_{t+1} - N_t}{N_t} = \frac{\sum_{j=1}^{N_t} k_{jt}}{N_t} - 1$$

- If sufficiently uncorrelated, CLT implies P(G | N) is Levy-stable.
- Beware of correlations!

Comparison to Levy-stable distribution



The Levy correctly predicts the scaling of the standard deviation with size

$$\sigma_G^2 \sim N_t^{(4-2\gamma)/(\gamma-1)}$$

year	β	$\hat{\gamma}$	γ
NABB	0.35 ± 0.08	2.54 ± 0.09	2.62 ± 0.08
Mutual funds	0.29 ± 0.03	2.41 ± 0.06	2.44 ± 0.07
Firms	0.31 ± 0.07	2.45 ± 0.32	2.49 ± 0.01

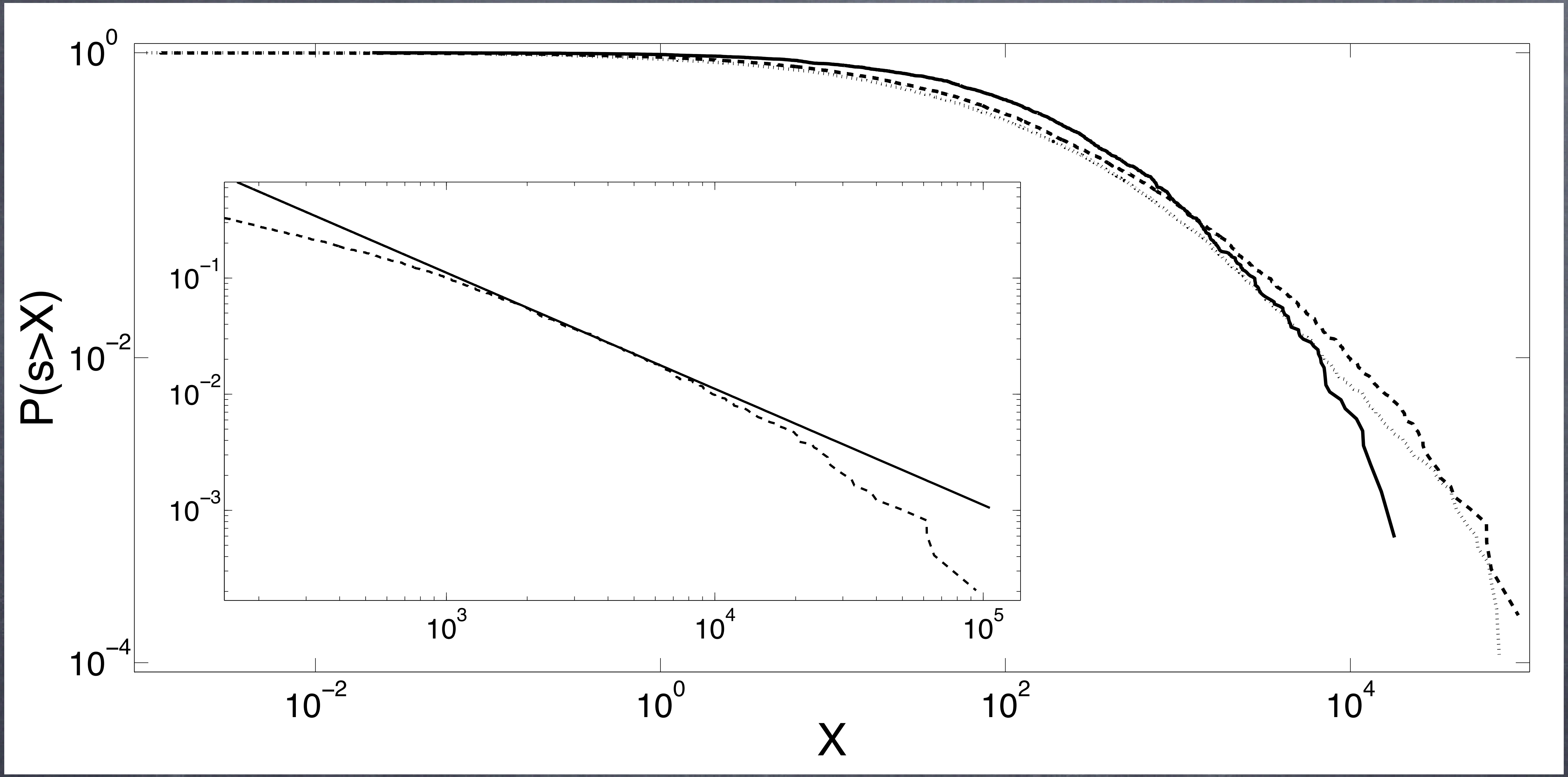
From fluctuations to size

- What determines the size of firms?
- What is the size distribution?
- How does it evolve in time?

U.S. Mutual funds

- 1/4 of holdings in U.S. stock market. 10 trillion dollars, 1/4 of household assets. Important influence in trading: Funds as big as $100B
- Are mutual funds different than other types of firms? Do transaction cost and investor sentiment play a role? Market efficiency?

Empirical distribution of mutual fund size



- s = size of fund
- Inset: comparison to power law
- Gabaix et al (QJE, 2005): Claim this is a power law.

Our conclusion:

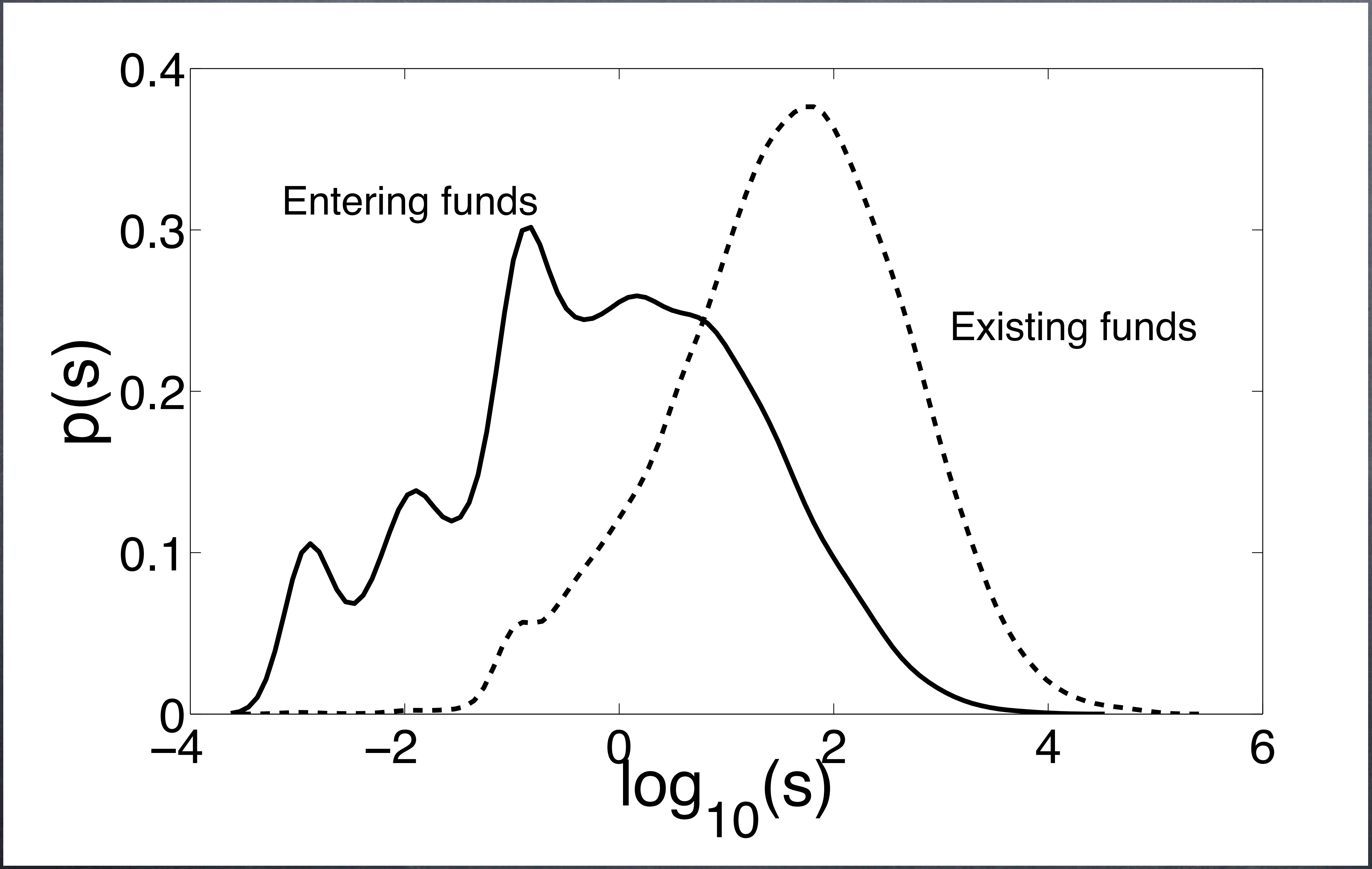
- Based on careful statistical testing over 17 years of CRSP data, with 30K firms in each year, maximum likelihood, K-S tests, Q-Q plots.
- The tail of the mutual fund size distribution is far closer to log-normal than a power law. Highly statistically significant result.

Why?

Mutual funds

- We investigated the growth dynamics as an Entry/Exit process where funds just diffuse in size.
- Mutual fund size distribution evolves from a log normal to a power law.
- We observed no market impact affect.
- We observed no size limit.

Entering vs. existing funds



The exit/entry process

- Based on our empirical investigation:
- Funds enter with a constant rate ν and are created with a size s with probability $f(s)$.
- Each fund exits with a rate λ .

Growth decomposition

- We modeled size change as

$$\Delta_s(t) = \frac{s(t+1) - s(t)}{s(t)}.$$

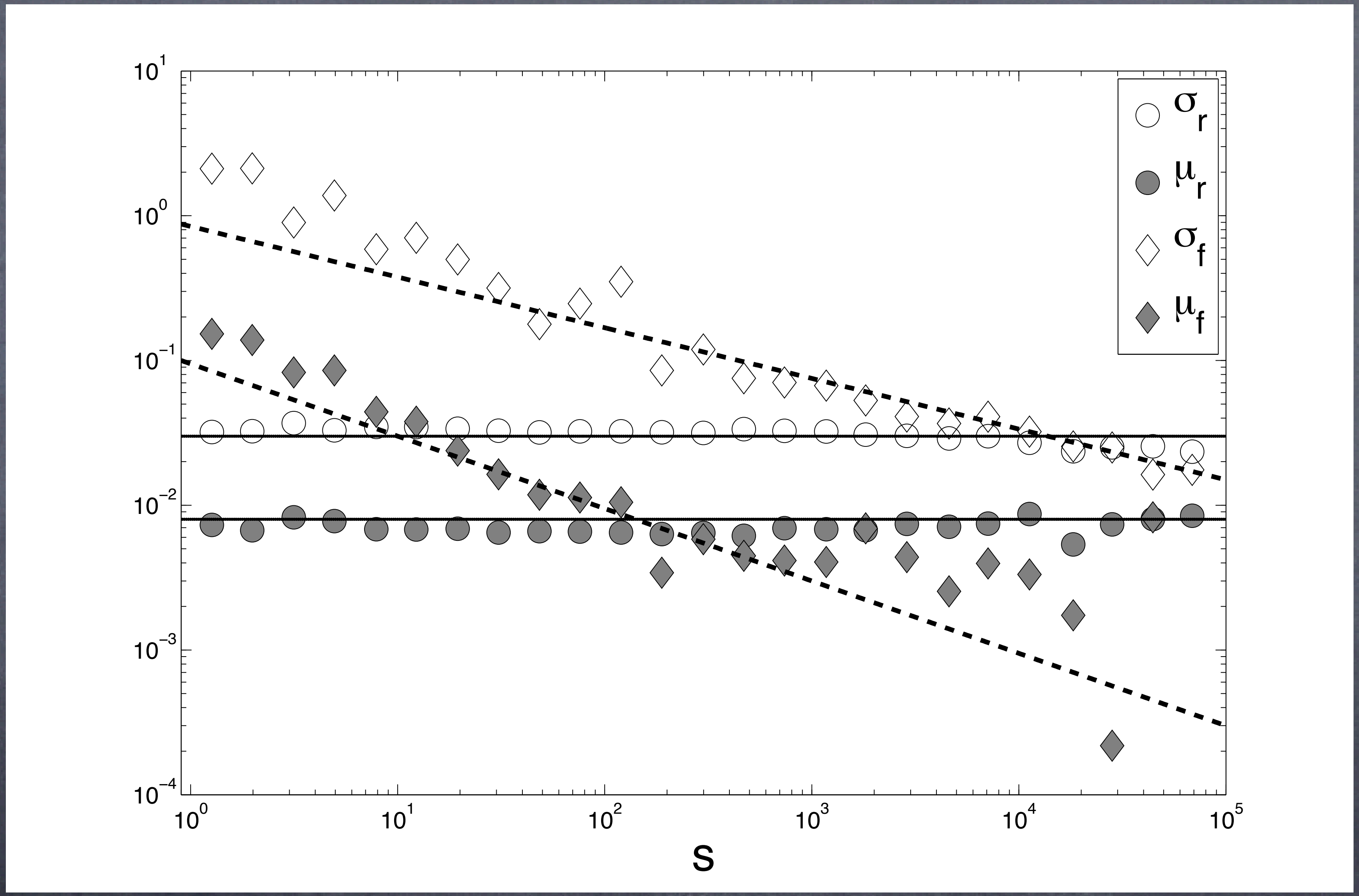
the size change can be decomposed to performance and investor money flux

$$\Delta_s(t) = \Delta_f(t) + \Delta_r(t).$$

and the return (performance) is given by

$$\Delta_r(t) = \frac{NAV(t+1) - NAV(t)}{NAV(t)}$$

Growth decomposition



Fokker-Planck equation

PDE for number density of funds with size $\omega = \log(s)$

$$\frac{\partial}{\partial t} n(\omega, t) = \nu f(\omega, t) - \lambda n(\omega, t) - \frac{\partial}{\partial \omega}[\mu(\omega) n(\omega, t)] + \frac{\partial^2}{\partial \omega^2}[D(\omega) n(\omega, t)]$$

$$D(\omega) = \sigma^2(\omega)/2$$

where the mean μ and variance σ depend on size as

$$\sigma_s(s) = \sigma_0 s^{-\beta} + \sigma_\infty$$

$$\mu_s(s) = \mu_0 s^{-\alpha} + \mu_\infty.$$

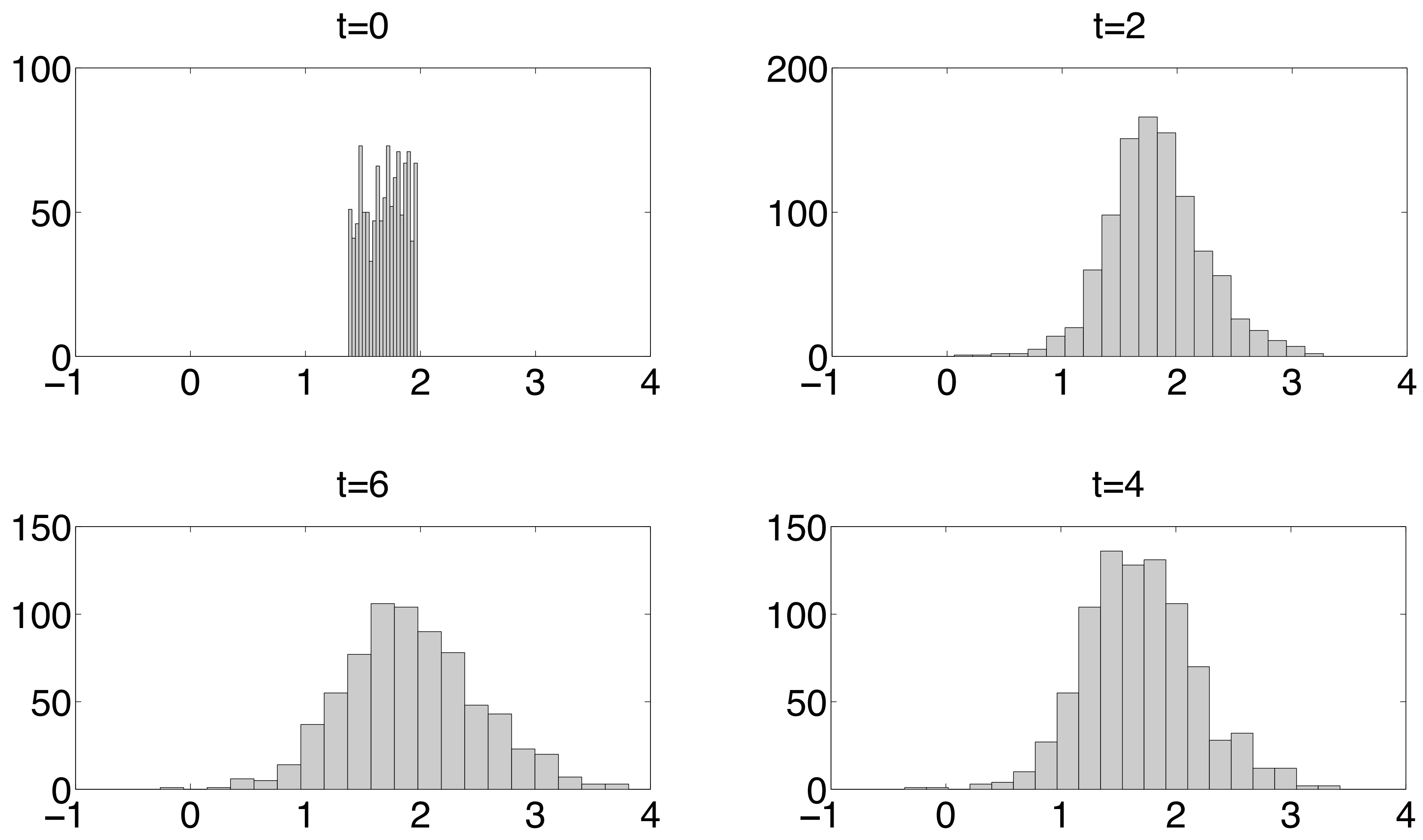
Asymptotic solution for t-> infinity

- When $\mu_\infty > 0$ solution power law with $\alpha \approx 1$
- When $\mu_\infty \to 0$ solution is a stretched exponential (all moments exist)
- Mutual funds have $\mu_\infty > 0$ because of market efficiency: Are evolving toward a power law

Time dependent solution with constant mu and sigma

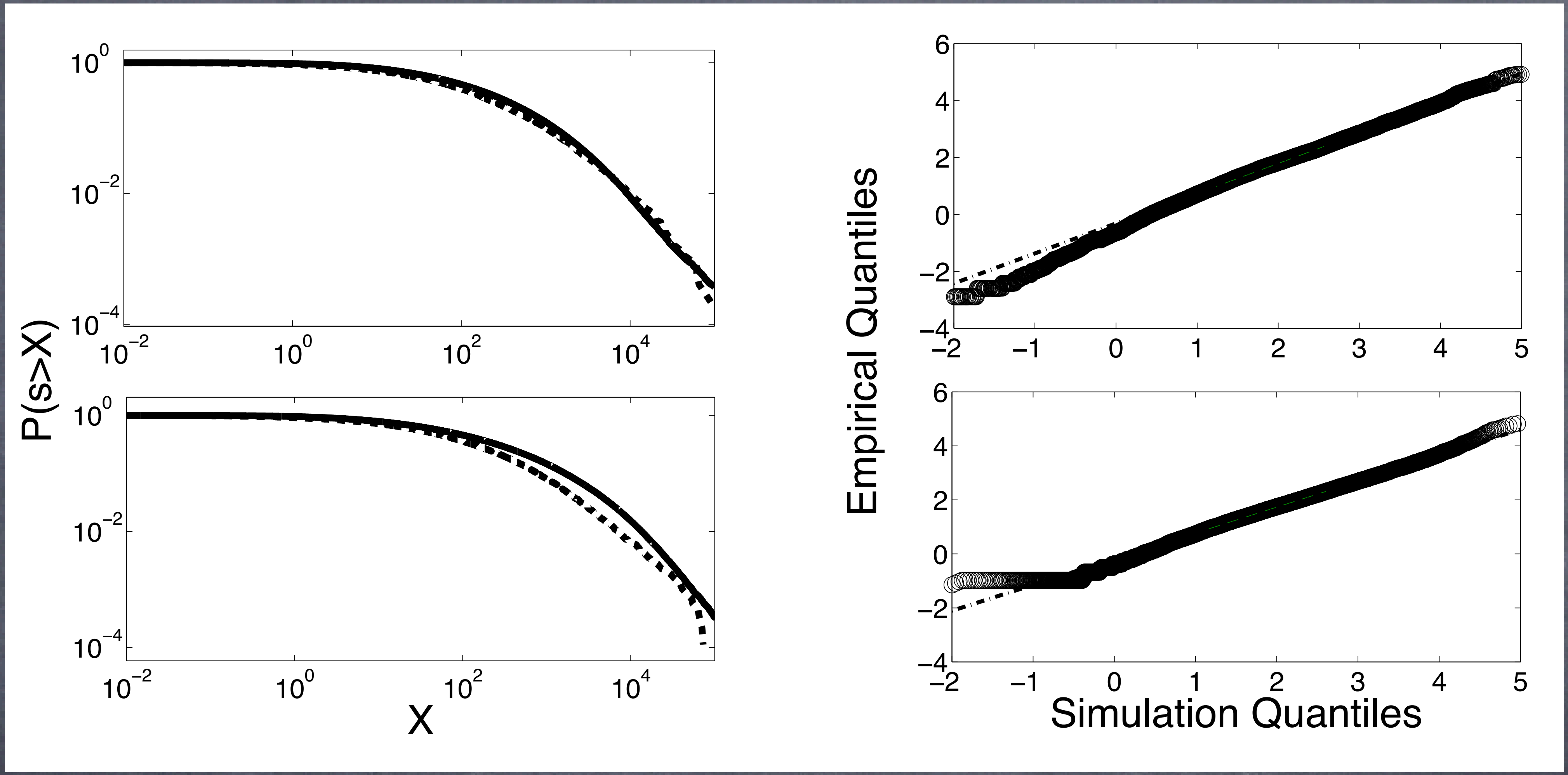
- Time scale to achieve equilibrium is very slow
- With scale dependent diffusion, more than a century for observed tail to become power law
- Mutual industry is young
- In the meantime the tail is a log-normal
- Long tailed growth fluctuations do not alter log-normality of diffusion.

Size change



- Log changes are exponential and will converge to a normal
- Approximate as a log-normal distribution

Comparison to real data



- The model is compared to the empirical distribution at different time horizons. The left column compares CDFs from the simulation (full line) to the empirical data (dashed line). The right column is a QQ-plot comparing the two distributions. In each case the simulation begins in 1991 and is based on the measured parameters. The first row corresponds to the years 1991-1998 and the second row to the years 1991-2005 (in each case we use the data at the end of the quoted year).

Size distribution model Implications

- We modeled the size dynamics without worrying about market impact.
- Size distribution is lognormal->Zipf.
- There is no apparent size limit.
- Investor behavior? Only enters through money flow. Does market impact enter?

How can we reconcile results with market impact?

- Two approaches are currently advocated:

  Berk and Green: rational, skill + impact

  Fama and French: “no skill, no impact”.

Rational approach

- Berk and Green [2004] proposed a rational model

  Managers possess skill α to create before transaction performance $R_t \sim N(\alpha, \sigma_\alpha)$

  The after transaction cost performance is given as

  $$r_{t+1} = R_{t+1} - \frac{C(q_t)}{q_t} - f$$

  f represents the fund fees and $C(q) = c_1 q^{1+\beta}$ is the impact function.

Rational approach

- Investors are Bayesian updaters and they estimate

$$\phi_t = E[R_{t+1} | R_t, R_{t-1}, \ldots]$$

and invest such that

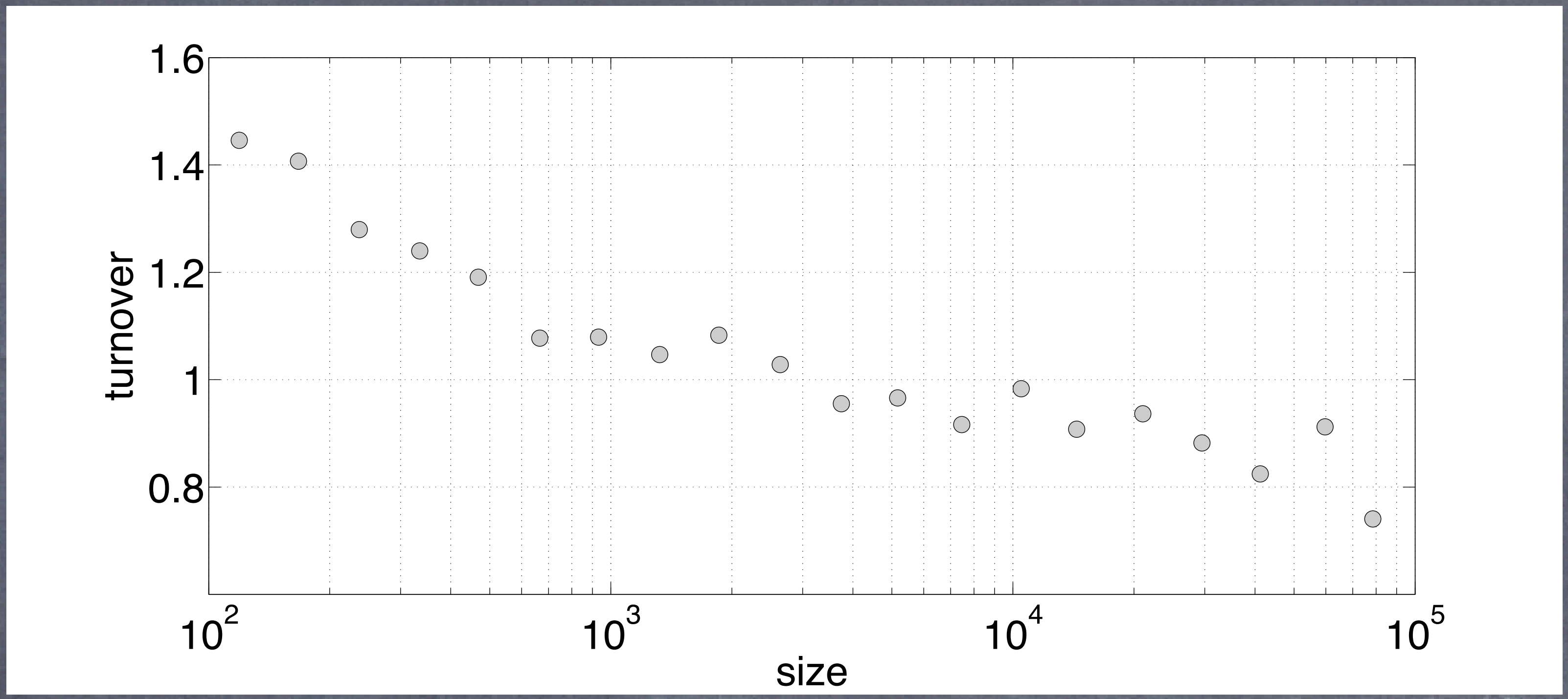
$$0 = E_t[r_{t+1}] = \phi_t - \frac{C(q_t)}{q_t} - f$$

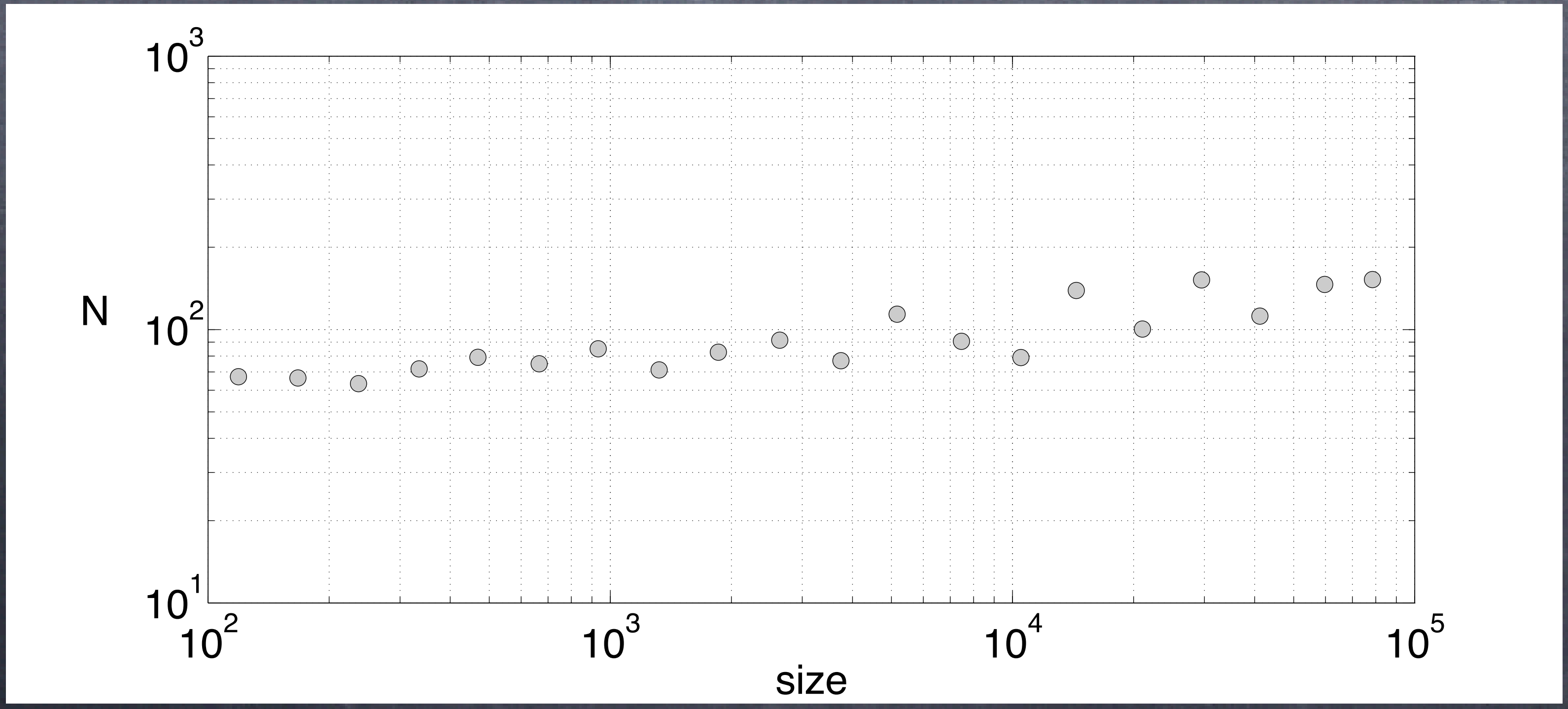
Investors choose the optimal size at each time

Berk and Green do not agree with data

- Empirical estimates of market impact show not enough money to saturate funds
- Distribution of skill has to be log-normal: Dist. of skill = dist. of fund size
- Returns are constant but standard deviation decreases with size: Sharpe ratio increases with size. Why invest in smaller funds?
- Predicts size is independent of fund age

Large funds do not optimize trading very much





No skill approach

- Fama and French [2009] estimated after transaction cost over-performance.
- After transaction performance is size independent and below market.
- Assume no market impact: “equilibrium accounting”
- Conclude skill is narrowly distributed around 0.
- Agrees with our model. But impact must increase with size! (for large funds, at least 80 basis points)

No skill approach

- Equilibrium accounting hypothesizes no impact: You win some you lose some.
- Even under standard market clearing, liquidity providers gain at the expense of liquidity takers.
- Mutual funds act as liquidity takers -> pay impact!

Our (possible) resolution

- Berk and Green assume investors have infinitely deep pockets
- We assume investors allocate a given amount to mutual fund industry each year
- Allocate money across funds based on size change, relative performance

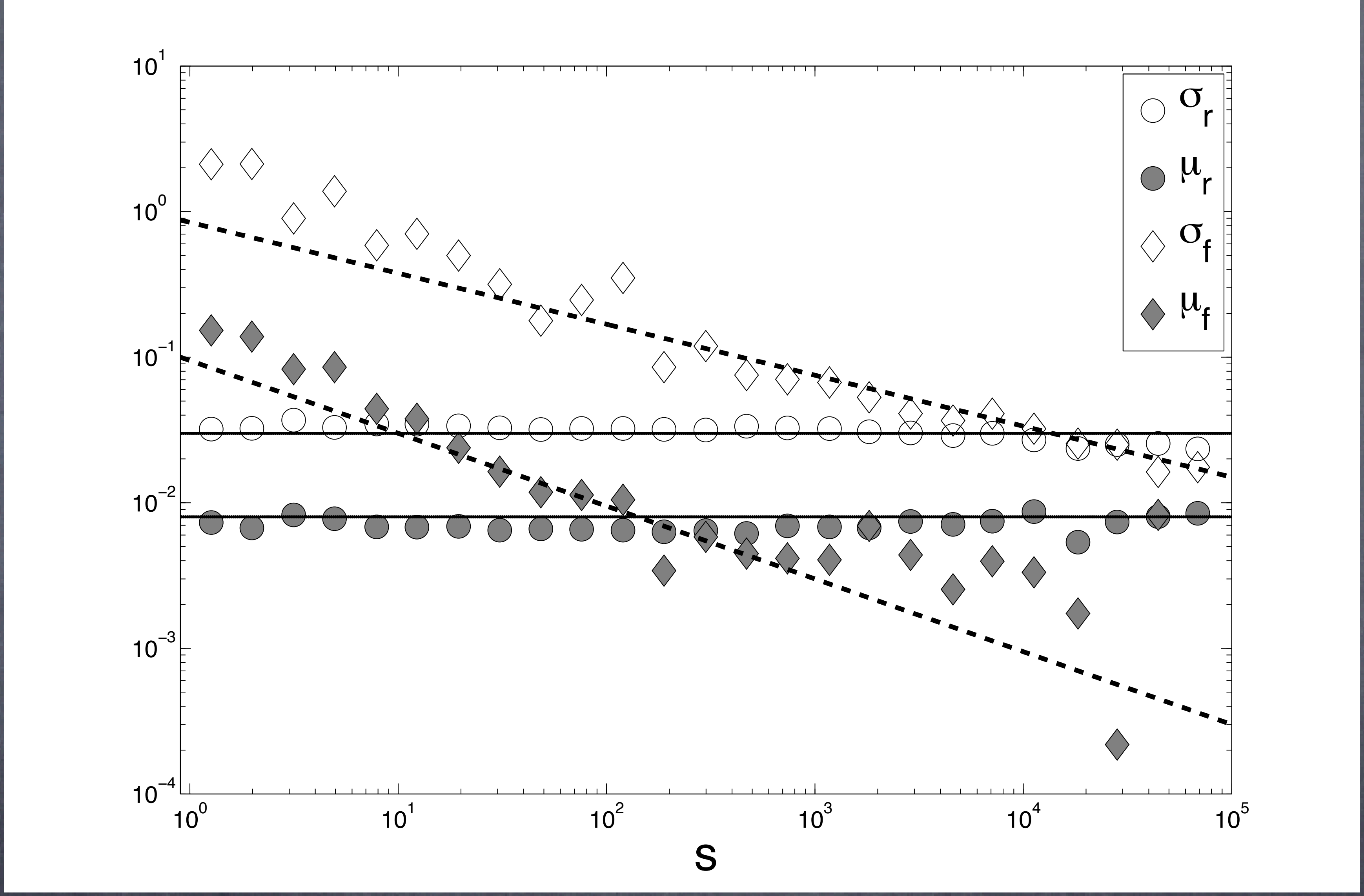
Implications of new approach

- Both $E_t[r_{t+1}]$ and $Var_t[r_{t+1}]$ are size independent.
- Sharp ratio is size independent!
- Investor flux decays with size as

$$\Delta_f \sim q^{-\beta/(1+\beta)}$$

- In agreement with observations of growth dynamics.

Growth decomposition



The average monthly return and its volatility, and the money flux and its volatility, are plotted as a function of the fund size (in millions) for the year 2005. The data are binned based on size, using bins with exponentially increasing size. The average monthly return is compared to a constant return of 0.008 and the monthly volatility is compared to 0.03. The average monthly flux is compared to a slope of -0.5 and the money flux volatility is compared to a linear slope of -0.35.

Conclusions

- Anomalous growth fluctuations explained by CLT. But why are increments fat tailed?
- Mutual fund size explained by diffusion, entry, exit. Diffusion is really slow!
- Mutual fund growth fluctuations differ from other firms due to market efficiency. Asymptotic solution is power law for mutual funds, stretched exponential for other firms.
- Investors do not have infinitely deep pockets.

Conclusions: Big picture

- Descent with variation + selection
- Funds "reproduce" by performance, money flows, with random variations
- Firms are selected based on performance
- Performance -> bigger size.
- Tradeoff between diffusion (blind luck + marketing) and skill for largest funds?
- Hypothesis: Selection slow acting, weak, noisy. We are a long way from the long run!
- Structure vs. strategy: Here structure dominates -- strategy enters but is secondary.
- Note huge diversity, heterogeneity of outcomes
- No assumptions on microstructure, minimal behavioral assumptions