Individual Evolutionary Learning in the double auction market with full or limited information

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Set of questions

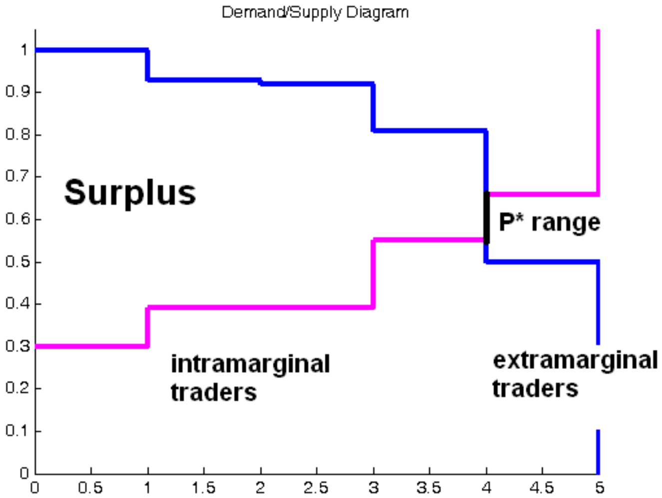
- What makes markets efficient?
  - Efficient (Walrasian) allocation
  - Efficient (Walrasian) price
- What is the role of agents behavior?
  - Fully rational behavior
  - Bounder rationality (learning)
  - Zero intelligence
- What is the role of market design?
  - Type of auction
    - ⋆ Call auction (CA)
    - ⋆ Continuous Double Auction (CDA)
  - Availability of information
    - ⋆ Open book (past actions are observed)
    - ⋆ Closed book (past actions are not observed)
- What agents learn?
- How behavior interacts with design (AP, JEDC 2009)?

Motivation

- Double auctions are popular mechanisms (secondary markets, stock exchanges)
- Double auctions are hardly tractable from game-theoretic perspective
- Yet in many experiments convergence to the equilibrium was observed (starting from Smith (1962))

- Call auction
  - reasonable strategy to bid/ask own valuations/costs
- Continuous double auction
  - not clear intuition about a strategy
    Friedman (JEBO, 1991) - Bayesian learning
    Gjerstad, Dickhaut (GEB, 1998) - boundedly rational surplus maximization

CA vs CDA


Demand/Supply Diagram
Surplus
P* range
intramarginal traders
extramarginal traders
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Motivation: Existing Literature on CDA

- Gode and Sunder (1993, JPE, 1997, QJE)
  - CDA, ZI agents, budget constraints
  - *Conclusion*: CDA market mechanism alone leads to efficient allocation and (sometimes) price

Critique:

- Gjerstad and Shachat (2007)
  - Individual Rationality (budget constraints) is not a part of market mechanism
  - Other measures of convergence may lead to different conclusions
- LiCalzi and Pellizarri (2008)
  - Crucial role of *resampling* -assumption, i.e., after each transaction Gode and Sunder force agents to submit new bids/asks
  - No convergence without resampling
  - Only sophisticated learning as in (Gjerstad and Dickhaut, 1998) leads to efficient allocation and price

Learning Models

- Major types:
  - belief based models - learn past decisions of opponents and play best response
  - reinforcement learning models - play strategy that behaved well in the past (Erev and Roth, 1998)
  - Hybrid - EWA, Experience Weighted Attraction (Camerer and Ho, 1999)

Evidence of reinforcement learning from estimating experimental data. Caviet: possible heterogeneity creates bias in estimation (Wilcox, Econometrica, 2006)

Individual Evolutionary Learning (Arifovic and Ledyard, 2007)

- reinforcement learning with experimentation
- "belief based model", where best response on the past period is learned in evolutionary way
- adaptation of genetic algorithms to economic decision

Market description

- Buyers
  - consume 1 unit of commodity, extracting given value V_b
- Sellers
  - endowed with 1 unit of commodity which costs C_s
- Buyers submit bids, sellers submit asks according to IEL
- Repeated trade over certain number of periods
- Fixed environment: costs and values do not change
- Mechanisms and Information

	Open Book	**Closed Book**
Call Auction		
Continuous Double Auction		

CDA - random order of arrival, bids/asks do not depend on current state of the book

Individual Evolutionary Learning

- Each agent has an own finite **pool of strategies** (ask/bid prices)
  - Initially strategies are randomly drawn (within bounds of costs/valuations)
  - The pool is evolving over time
- A strategy from a pool is used with some **probability**
  - Probabilities are based on counterfactual analysis: those which *would* give higher payoff are reinforced
- Pool is always **evolving**
  - *Experimentation (mutation)* with certain (small) probability a strategy in the pool is replaced with a *new strategy*
    - ⋆ drawn around the old strategy according to some distribution
  - *Replication (reinforcement)* - replace a strategy from the pool with another randomly selected strategy *from the pool* if the latter performs better than the former

Individual Evolutionary Learning - Counterfactual analysis

The probability π_s to select a strategy depends on "foregone" payoff U_s

$$\pi_s = \frac{U_s}{\sum_i U_i}$$

Foregone payoff U_s for buyers as a function of *counterfactual* bid:

$$U_s = \begin{cases} V - P^* & \text{if trade occurs} \\ 0 & \text{otherwise} \end{cases}$$

P^* - *counterfactual* price
Closed book: trade occurs if $bid > P^*$
Open book: can recalculate the whole book

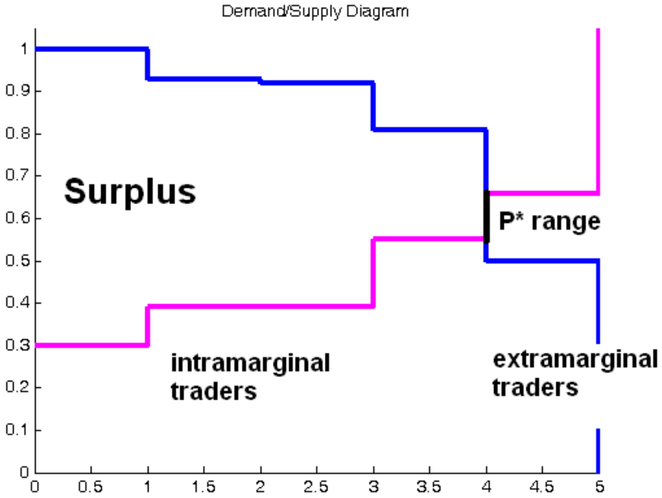
Closed vs Open book

- CA
  - closed book - P^* is price of last round
  - open book - P^* calculated **changing** own bids and holding strategies of others **fixed**
- CDA
  - closed book - P^* is average price of all transactions of last round
  - open book - P^* calculated **changing** own bids and holding the strategies of others and the order of arrival **fixed**

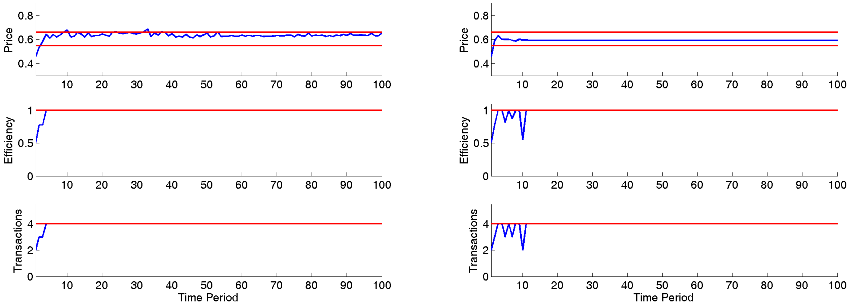
Set-up

Parameter	Symbol	Value (Range)
Interval of valuation/costs	$[0, \eta]$	$[0, 1.2]$
Number of strategies in a pool	J	100
Number of buyers and sellers	$B = S$	5
Probability of experimentation	ρ	0.03
Distribution of experimentation	$\mathcal{P}(0, \sigma^2)$	$N(0, 0.01^2)$
Individual Rationality constraint	IR	*enforced*

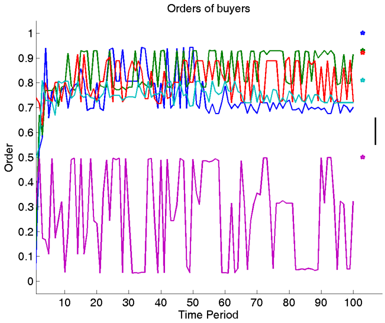
Benchmark: Walrasian market clearing

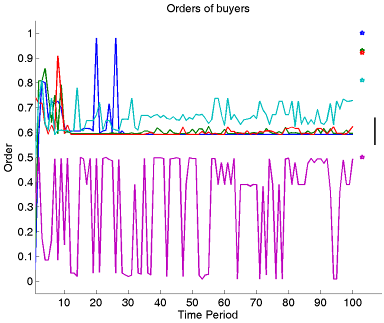


Call Auction: Close vs Open book

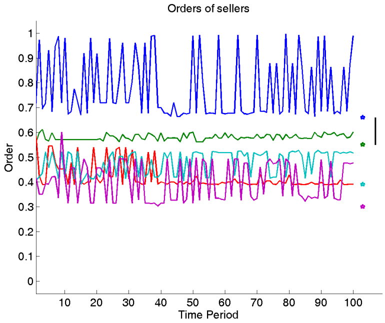


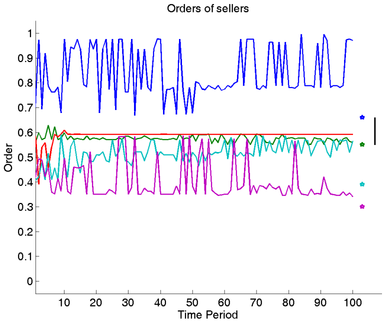
CA, Individual Strategies Buyers: Close vs Open book





CA, Individual Strategies Sellers: Close vs Open book

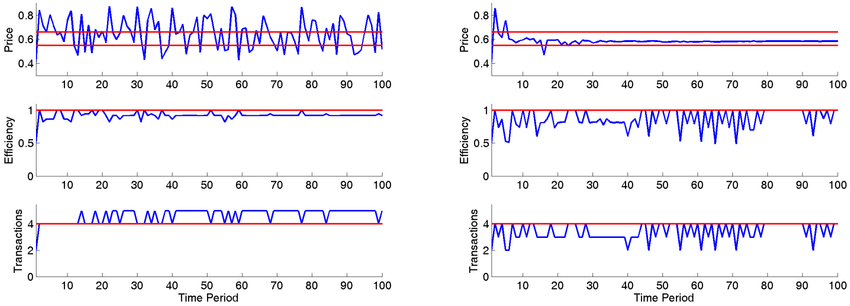




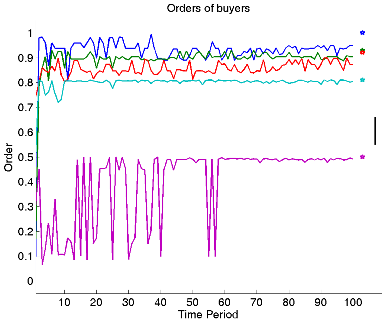
Call Auction: Summary

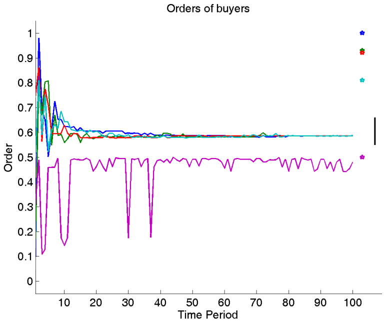
- high allocative efficiency in the long-run
- longer convergence for **open book**
  - traders do not take into account that others are also learning
- correct price discovery
- much more price stability for **open book**
  - marginal traders play their "best responce" on others' strategies
- traders do not learn to submit their own evaluations

CDA: Close vs Open book

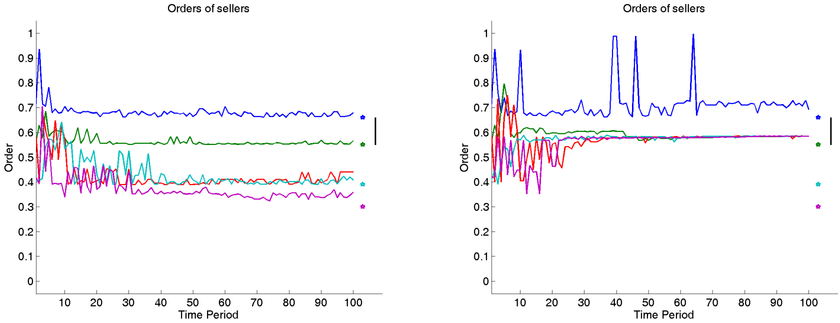


CDA, Individual Strategies Buyers: Close vs Open book





CDA, Individual Strategies Sellers: Close vs Open book



CDA: Summary

- for the **closed book**:
  - traders learn to submit their own evaluations
    - traders have no influence on price
    - order of arrival is random $\Rightarrow$ high price volatility
    - traders avoid do not trade, and shift their orders towards valuations
  - no price discovery, high volatility
  - overtrading $\Rightarrow$ lower allocative efficiency (than in CA)
- for the **open book**:
  - inframarginal traders coordinate on the order submission
    - traders have an influence on price
    - they try to extract a maximum surplus $\Rightarrow$ buyers bid low, sellers ask high
    - traders avoid to lose an opportunity to trade and to trade at lower prices
  - much more price stability, correct price discovery
  - undertrading $\Rightarrow$ lower allocative efficiency (than in CA)

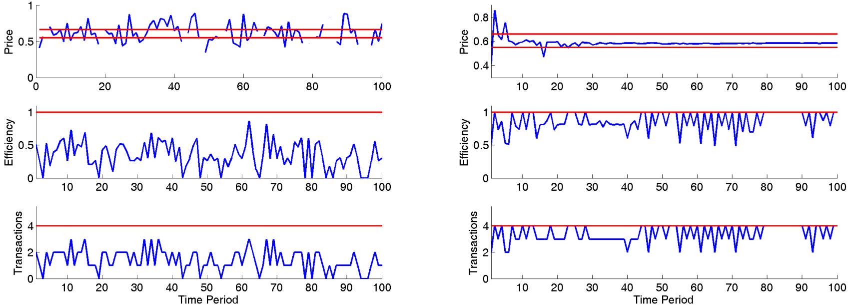
Major Findings

- Open book design leads to more stable price over time in both CA and CDA
- Closed book design is more efficient in terms of surplus under CDA
- Learning
  - agents "coordinate" their bids/asks under open CDA
  - agents "learn" their costs/valuations under closed CDA

Robustness of results

- Experimentation (mutation)
  - Normal and uniform distributions give similar results
  - higher probability of experimentation leads to faster convergence
  - larger variance gives larger deviations in price and efficiency
  - the efficiency of closed auction is more stable under higher mutations
- Replication
  - very important for learning
  - at least 50% of strategies need follow replication
- Strategy pool - need relatively large strategy pool
- Violation of individual rationality (budget constraints))
  - higher volatility of price, lesser efficiency longer converges (more effect on open book CDA)
  - agents are able to eliminate violating strategies

CDA: Zero intelligent vs open book



Conclusions and Extensions

- CA, CDA + other mechanisms, e.g. Market maker
- Information: open/closed, vary more
- Fixed environment to endogenous valuations (BH, AP)
- CDA: strategic timing
- Single item to multiple items
- Compare with other learning, e.g. EWA
- Closer look at data and estimation