

Price and Quantity Competition Markets : an Ecological Approach

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- 1) **Context, questions and methods**
- 2) Behaviors and evolutionary dynamics
- 3) Convergence and Cooperative behavior
- 4) Equilibrium under other hypothesis
- 5) Conclusion

Studied market and possible issues

- We study a market in which prices are defined by the suppliers. They also pre-produce the quantities they are going to sell. Our interest goes to the way they interact.
 - Ex : Fruit markets.
- For such markets, there is no Nash Equilibrium for homogeneous goods [Alger 79]
- Could evolutionary dynamics give a result on convergence ?
- It has been shown that evolutionary processes can converge towards an unexpected equilibrium [Vega Redondo 97]

Studied market and possible issues

(2)

- What would be the possible convergences of an evolutionary process for our markets ?
 - Chaotic fluctuations
 - Periodic fluctuations
 - Competitive equilibrium
 - Cooperative equilibrium : cartel

- Can we find conditions for cooperative equilibrium ?

Cartel : a cooperative equilibrium

- Cartel equilibrium is a form of cooperative equilibrium on the market. It maximises the collective output and each agent can choose to deviate from the cartel, inflicting losses to its peers.
- => to study the emergence and maintenance of cartelistic coalition, it is possible to use the results about cooperation that have been shown in Evolutionary game theory.

Evolutionary Game Theory

- A theory to analyse population dynamics.
[Smith 81][Axelrod 84]
- Some key results about the evolution of cooperation.
- In this theory, cooperation is often treated as the C behavior of a Prisoner's Dilemma

	C	D
C	R=3	S=0
D	T=5	P=1

Evolutionary game theory

- Evolutionary game theory studies the evolution of populations where each individual plays according to one of the behaviors of the matrix.
 - For example, for a population of individuals playing the PD, each agent adopt the behavior C or the behavior D.

Evolutionary game theory

- Population dynamics (replicator dynamics)
 - Each individual meets every over one of the population of a randomly chosen part of the population. By doing so, each individual gets a score.
 - For each behavior, it is possible to compute an average score : the average score of all agents which have adopted this behavior
 - A behavior which average score is lesser than the average score of all behaviors tend to disappear
 - A behavior which average score is greater than the average score of all behaviors tend to spread.

$$x_i^{t+1} - x_i^t = x_i^t \left((AX_t)_i - X_t^T A X_t \right) \quad [\text{Haufbauer 98}]$$

Evolutionary game theory

- From a population of agents of behaviors C and D, as D is a dominant behavior, the population converges towards only D behaviors. The simple cooperative behavior is eliminated.
- Cooperative behaviors can survive upon different conditions :
 - **Reacting agents interacting repeatedly** [Axlerod 84] For a TFT behavior : if its current opponent start being aggressive, it reacts playing D. On the other side, if its current opponent has played C, TFT plays C. In a population with TFT, D and C behaviors, complex patterns can be shown. Cooperation can survive through cyclic variations.
 - **Spatial repartition** with multiple dynamics depending on the underlying graph [Nowak 93]

- Generalize the results from Evolutionary Game Theory by adapting its models / dynamics to our market.
- A period for the market :
 - Each agent determines its price and quantity before market interaction : (p_i, q_i)
 - Price and quantity are modified with a noise $N(0;0.02)$
 - Agents meet demand on the market. Some of them are rationed : they don't sell all they produced.
 - Agents adapt with the information on profit, unsold quantities they got on the market
- Periods are repeated. Each agent earns a global profit over the periods.
- In general, we study markets of $K=10$ agents

Characterize interactions -coordination

- To evaluate the level of cooperation, we study the evolution of two statistics :

- Coordination :
$$\sum_{i=1}^K \frac{q_i}{D(p_i)} - 1$$

If all agents play on the demand curve, coordination is 0.

When coordination the 0 it is possible for an agent to play the cartel.

If they all produce beyond the demande curve
coordination > 0

Characterize interactions -cooperation

- Cooperation :

$$\frac{\textit{effective profit} - \textit{competitive profit}}{\textit{cartel profit} - \textit{competitive profit}}$$

- Cooperation = 1 => cartel/cooperative equilibrium

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Behaviors

- We retain only behaviors with memory one : ie behaviors only use the information from the previous period.
- Three kinds of behaviors for three possible market issues :
 - Cooperative behaviors : these behaviors collectively reach the cooperative equilibrium.
 - Price-taking behaviors : these behaviors always produce a quantity such that their marginal cost equals their price. They differ in the way they choose their price.
 - Strategic behaviors : these behaviors try to give the best answer to the behaviors adopted by their peers.

Price taking behaviors

These agents play according to the rule $p=C^m$ they differ on the way they choose a price.

=> They collectively converge towards competitive equilibrium.

- PP1 : Always plays the competitive equilibrium
- PP2 : Plays the average price of the precedent periode on the market
- PP3 : Chooses the price of the agent that has obtained the more profit in the precedent period
- PP4 : we note x the coordination for the last periode, the new price is :

$$\overline{p}_{t-1} * \left(1 - \frac{x_{t-1}}{K} \right)$$

the price decreases if coordination is over 0, increases otherwise

Cooperative behaviors

These behaviors play on or close to the demand curve, so that coordination is at most 0 (except for little random variations).

They collectively converge towards cooperative equilibrium (Cartel)

- Coop1 : Always plays cooperative equilibrium
- Coop2 : It plays on the demand curve, if rationed it increases its price by 5% otherwise it lowers its price by 5%
- Coop3 : If rationed it increases its price by 5% et keep on producing the same quantity, otherwise : it lowers its price by 5% and play according to the demand curve.
- Coop4 : If **global rationing** is under 5%, it plays the cooperative equilibrium, otherwise, it plays a form of MaxMin.
 - To choose a MaxMin, the agent consider all possible prices. For each of them, it considers the profit it would earn given the other agents play the same price and produce at marginal cost. The agent choose the price that maximize its profit.

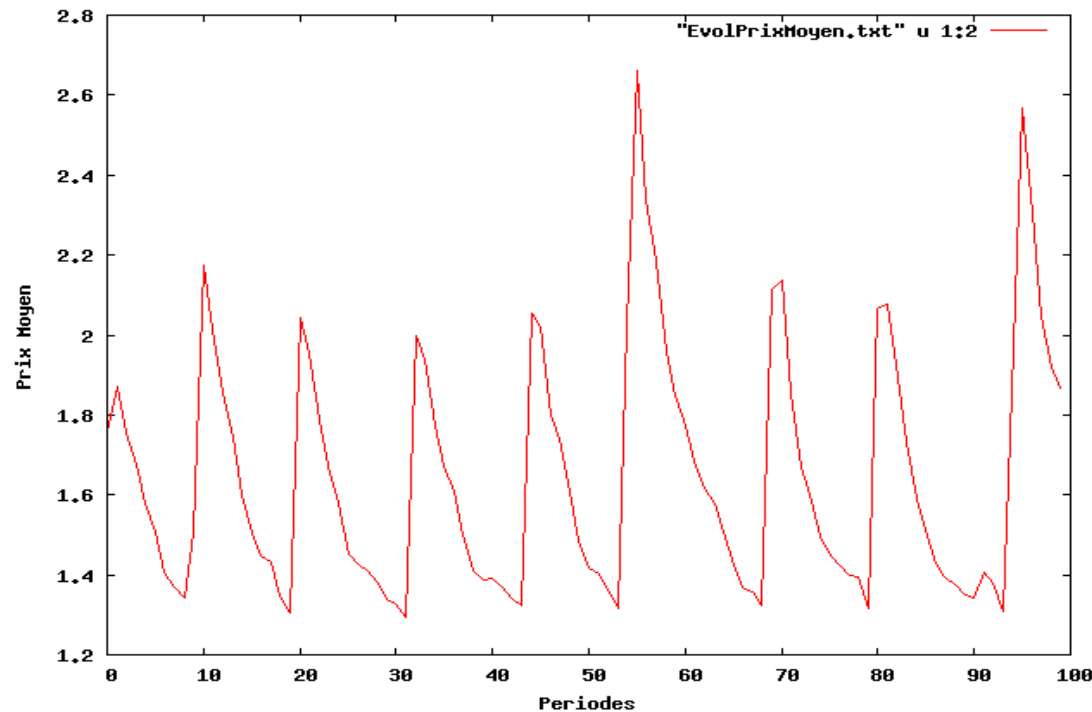
This agent can be considered as a form of non-aggressive TFT

Strategic behaviors (1)

These behaviors try to find the best answer to the observed choices of their peers.

They collectively converge towards Edgeworth cycles

- Strat 1 : It computes the profit it would earn by playing the higher non rationed price and producing at marginal cost. It compares it with playing at cartel given coordination from precedent period. It chooses the more profitable move



Strategic behaviors (2)

- Strat 2 : if its rationing is under 5%, it increases its price by 5%, produces the same quantity. Otherwise, it plays as Strat1 would play.
- Strat 3 : if its rationing is under 5%, it plays as strat1. Otherwise it decreases its price by 5% and produces the same quantity.
- Strat 4 : It predicts what a Strat 1 would choose and plays the best answer to Strat1.

Evolutionary dynamics

- The population is initialized with the $N=12$ behaviors previously defined.
- For each behavior, there are E_i agents which adopt it : $(E_1 \dots E_N)$. For the simulations we present here, $1000 = E_1 = E_2 = \dots = E_N$.
- For each generation :
 - All the agents are randomly distributed in $\frac{\sum_{j=1}^N E_j}{K} = 1200$ markets
 - During $T=100$ periods the agents interact on the markets. Each agent accumulates a certain amount of profit depending of its behavior and of the behaviors of his peers.
 - For each market, the behaviors are modified according to their relative performance : the best behavior of the market spread out while the worst tend to disappear.

Evolutionary Dynamics (2)

- Example for a three agents market :

Profits on the Market

Agent 1 – behavior 2. Profit : 4000

Agent 2 – behavior 3. Profit : 1000

Agent 3 – behavior 4. Profit : 0

Shares of total profit

Share for agent 1 : $4/5$

Share for agent 2 : $1/5$

Share for agent 3 : 0

Redistribution des effectifs

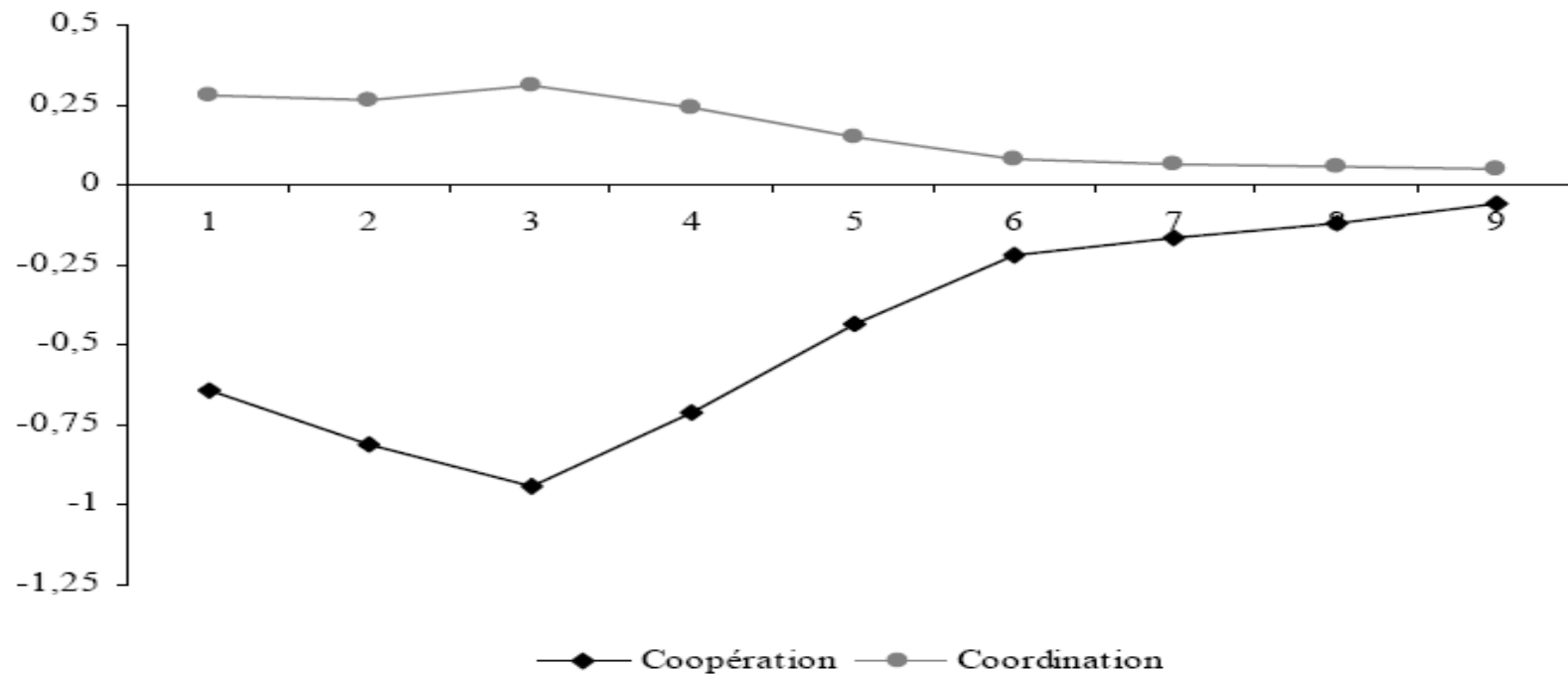
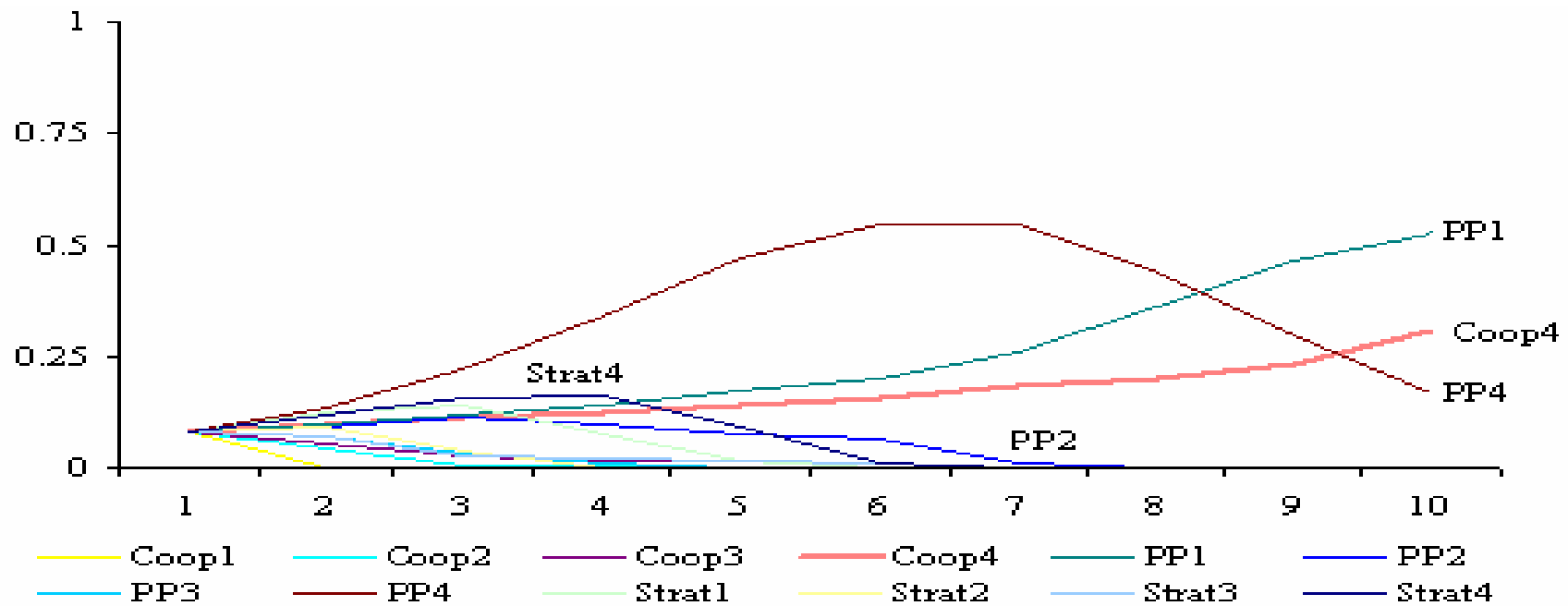
Behavior 2 : $3 \cdot 4/5 \Rightarrow$ 2 Agents of behavior 2

Behavior 1 : $3 \cdot 1/5 \Rightarrow$ 1 Agent of behavior 1

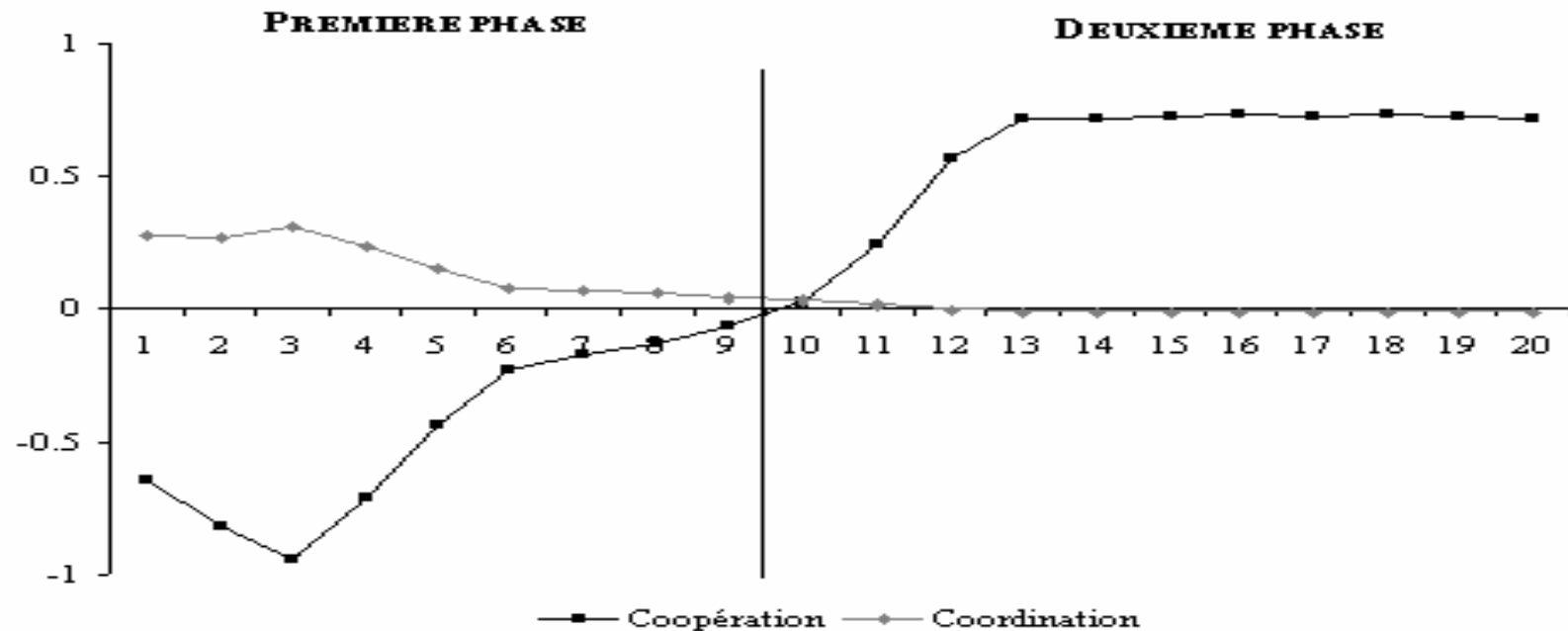
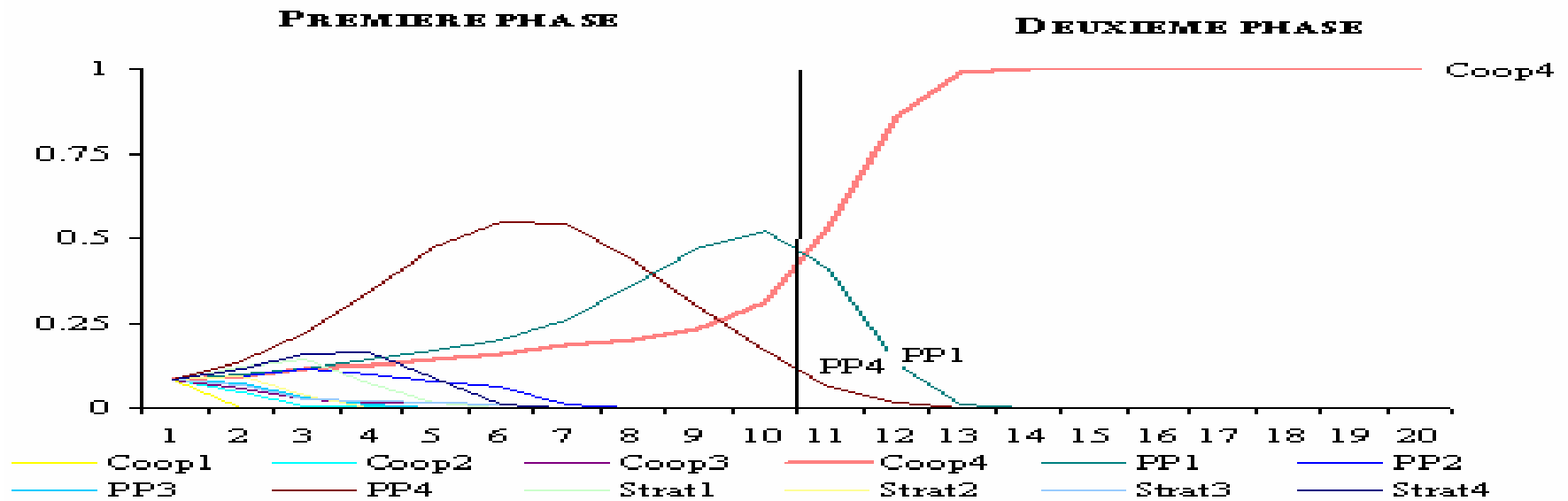
Behavior 4 : 0 \Rightarrow 0 Agent of behavior 4

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Convergence – 1st phase



Convergence - 2nd phase



Convergence

- The population converges towards the cooperative issue with all agents adopting the behavior Coop4.
- Unexpected result : tournaments would have underline the importance of price taking behaviors.
- The evolution has been re-tested for various runs : qualitative properties are conserved.

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Other hypothesis, stability of the result

- From our initial simulation, we have tested several variations :
 - Numerical variations of the parameters
 - Changes in the rationality of the agents
 - Changes in the selection process

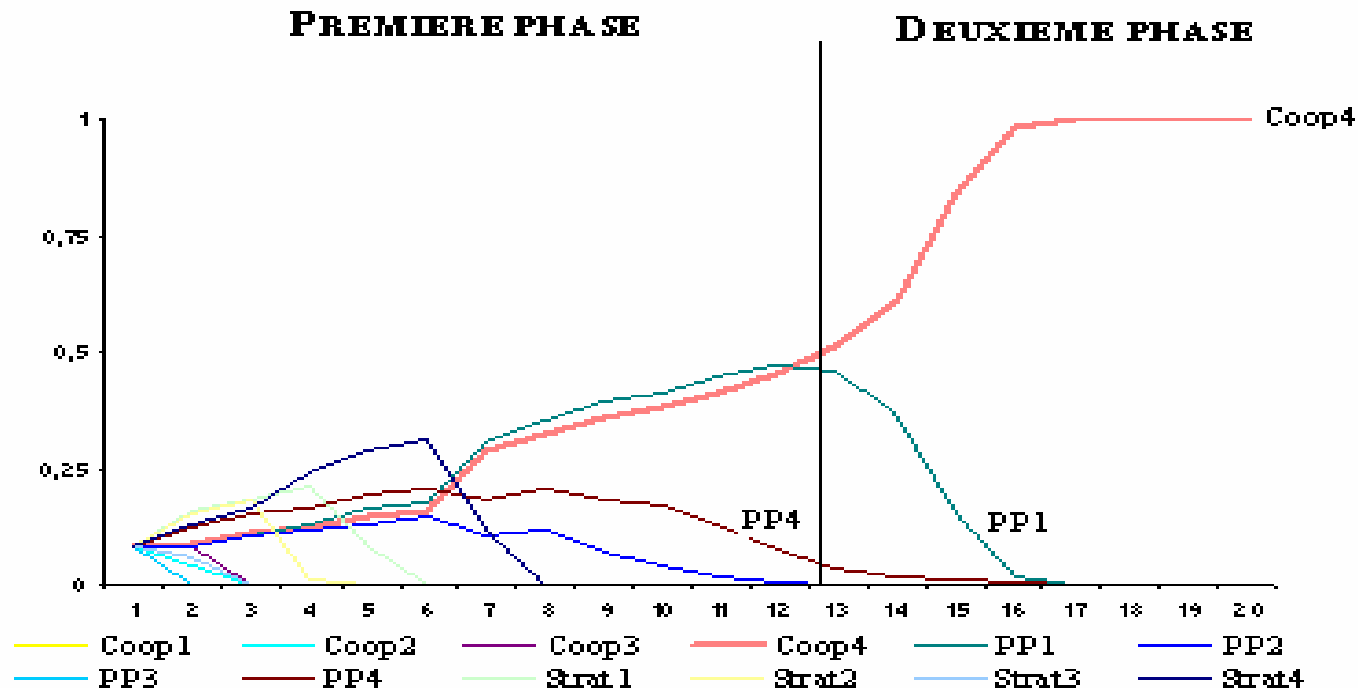
Numerical variations of the parameters

From the initial simulation that has been presented, it is possible to test the variation of some parameters :

- Variation of the standard deviation of the noise introduced on prices and quantity
 - $N(0;0.02)$ to $N(0;0.01)$
 - $N(0;0.02)$ to $N(0;0.04)$
 - => Only changes the intermediate maxima for the price taking behavior
- Variation of the sensibility of the behaviors and their reactivity
 - => Only quantitative variations on the result.

Variations of the evolutionay dynamics – Variation of K (1)

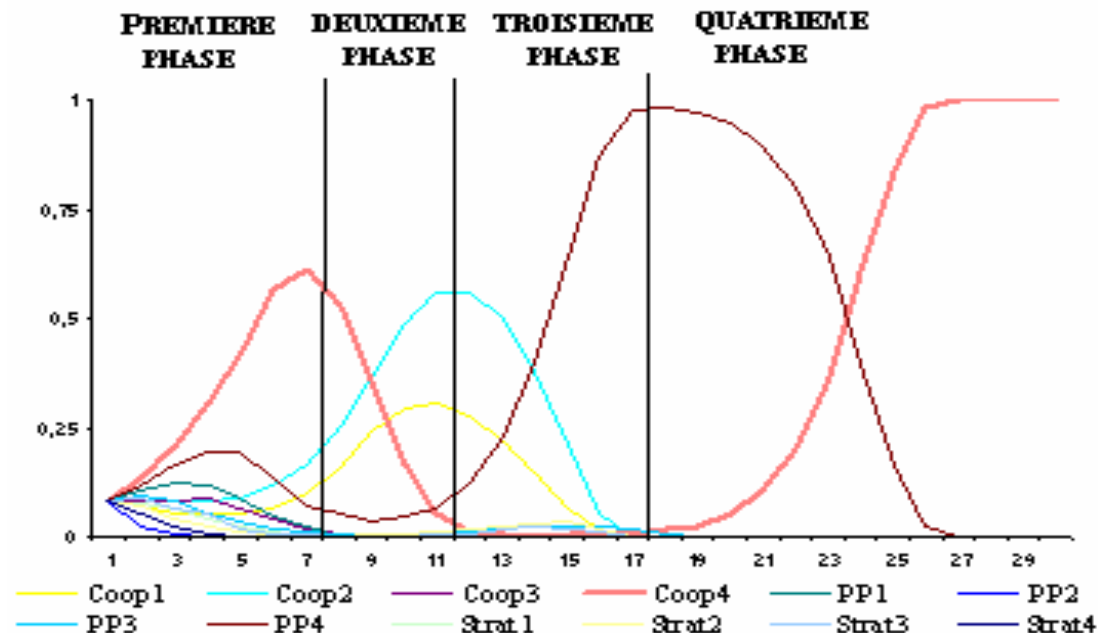
- For the initial simulation, each market is of size $K=10$. Even when $K=100$, the global result doesn't change :



- Opportunist behavior and price taking behavior survive longer here

Variations of the evolutionary dynamics – Variation of K (2)

- For the initial simulation, each market is of size $K=10$. When $K=2$, dynamics is quite different :



- As Coop 4 are quite resistant, the probability of an all-Coop4 issue is very strong as long as the total population is very important.

Variations of the evolutionary dynamics – Asynchronous selection

- At each generation, only one market is created with K randomly chosen agents.
- The population evolves =>
 - At the end of the simulation, Coop4 are 80% of the population, strategic behaviors are 20% of the population

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Conclusion

- Unexpectedly, cooperative behaviors are selected by evolution.
- This result holds against multiple variations but the number of parameters remains important. Upon the set of memory one agents we explored, we didn't find a behavior to modify the result
- A morale ? If demand doesn't have any negociation power, offer naturally converges towards the wealthiest options

Future Work

- Explore the stability of the observed convergence and the importance of each hypothesis, including the evolutionary dynamics.
- All the hypothesis under which cooperative behaviors survive are candidate for explaining the emergence of cartelistic coalition. See Evolutionary Game Theory.
- The present work is being coupled with an experimental approach. Interaction of students versus artificial behaviors is studied to find behavioral patterns which could be used to conceive new agents.