## Optimal Transport from Lebesgue to Poisson

We study couplings $q^\bullet$ of the Lebesgue measure and the Poisson point process $\mu^\bullet$ – i.e. the set of measure-valued random variables $\omega \mapsto q^\omega$ s.t. for a.e. $\omega$ the measure $q^\omega$ on $\mathbb{R}^d \times \mathbb{R}^d$ is a coupling of $\mathfrak{L}^d$ and $\mu^\omega$ – and we ask for a minimizer of the asymptotic mean $L^p$-transportation costs for any given $p \in (0, \infty)$

$$\mathfrak{c}_\infty \; = \; \inf_{q^\bullet \in \mathfrak{Q}} \; \limsup_{n\to\infty} \; \frac{1}{\mathfrak{L}^d(B_n)} \mathbb{E}\left[\int_{\mathbb{R}^d \times B_n} |x-y|^p \, dq^\bullet(x,y)\right]$$

with $B_n := [0, 2^n)^d \subset \mathbb{R}^d$. On of our main results states that the asymptotic mean $L^p$-transportation costs are finite if $d \geq 3$, or if $d = 2$ and $p < 1$, or if $d = 1$ and $p < 1/2$.

Moreover, in any of these cases we prove that there exist a unique translation invariant coupling which minimizes the asymptotic mean $L^p$-transportation costs. The unique optimal coupling $q^\omega$ can be represented as $(Id, T^\omega)_* \mathfrak{L}^d$ for some map $T^\omega : \mathbb{R}^d \to \mathrm{supp}(\mu^\omega) \subset \mathbb{R}^d$.

In the case $p = 2$, actually, $T^\omega = \nabla\varphi^\omega$ for some convex function $\varphi^\omega : \mathbb{R}^d \to \mathbb{R}$ and the pre-image $U(\xi) = (T^\omega)^{-1}(\{\xi\})$ of any $\xi \in \mathrm{supp}(\mu^\omega)$ is a convex polytope of volume 1.

*This is joint work with Martin Huesmann (University of Bonn).*