Existence of an Optimal Transport

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We consider the question of the existence of an optimal transport map for the W_p-Wasserstein distance in the extremal cases $p = +\infty$ and $p = 1$. Our technique for proving such an existence result in those cases is based on the coupling of a simple but powerfull regularity argument and the geometric constraint imposed by the cyclical-monotonicity of the support of an optimal transport plan. We first illustrate this technique for the case $p = +\infty$, and then show how it generalizes to the case $p = 1$ which corresponds to the classical Monge problem.