

On the extremality, uniqueness and optimality of transference plans

Stefano Bianchini

Abstract

We consider the following standard problems appearing in optimal mass transportation theory:

- when a transference plan is extremal,
- when a transference plan is the unique transference plan concentrated on a set A ,
- when a transference plan is optimal.

We study these three problems with a general approach:

1. choose some necessary conditions, depending on the problem we are considering;
2. find a partition into sets B_α where these necessary conditions become also sufficient;
3. show that all the transference plans are concentrated on $\cup_\alpha B_\alpha$.

Explicit procedures are provided in the three cases above, the principal one being that the problem has an hidden structure of linear preorder with universally measurable graph.