Optimal transport in the quadratic case with a convex constraint: an application of the Champion-De Pascale-Juutinen method

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The following problem arises naturally in [5]:

$$(P) \quad \inf\{\int c(x,y)\ d\gamma(x,y) :\ \gamma \in \Pi(f_0, f_1)\},$$

where $f_0 << dx$ and f_1 are probability measures on ^{d}, $\Pi(f_0, f_1)$ are the probability measures on ${}^d \times {}^d$ with fixed marginals f_0 and f_1 and $c(\cdot,\cdot)$ is given by

$$c(x,y) = \begin{cases} |y-x|^2 & \text{if } y-x \in C \\ +\infty & \text{otherwise,} \end{cases}$$

the subset C of ^{d} being convex. The question is, if there exists $\gamma \in \Pi(f_0, f_1)$ such that $\int c(x,y)\ d\gamma(x,y) < +\infty$, can we find a solution for (P) which does not divide masses? As shown by L. Caravenna in [1], when the cost take $+\infty$ as a value, the dual problem may not have a solution. The classical methods for strictly convex cost need this solution. We develop an alternative method adapted from the works of T.Champion, L.De Pascale and P.Juutinen (see [2], [3] and [4]). Joint work with F. Santambrogio.

References

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[4] T. Champion, L. De Pascale, P. Juutinen, *The ∞-Wasserstein distance: local solutions and existence of optimal transport maps.* SIAM J. Math. Anal. 40 (2008), no. 1, 1–20.

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