

Ellsberg's Paradox and the Value of Chances

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Ellsberg's paradox

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- Violates Savage's SEU theory

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- Typical responses:
 - 1 Beliefs not probabilities (e.g. capacities)
 - 2 New decision rules (e.g. Max Choquet EU)

Reframing the Problem

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- If Ellsberg is correct then problem needs reframing:
 - ① States are distributions of balls (as well as draws)
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- Suppose that the agent:
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 - ② Regards the two possible states as equally likely

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$$\textcircled{1} L_1 > L_2 \Leftrightarrow U\left(\frac{1}{3}\right) > 0.5U\left(\frac{2}{3}\right) + 0.5U(0)$$

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- So, given DMU, the Ellsberg preferences are consistent with SEU.

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Table: Risk Averse Preferences

- Intuitively someone is risk averse with respect to a divisible good G if they view losses and gain of quantities of G asymmetrically.

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- So chance risk aversion implies a preference for **hedging**.

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- So chance risk aversion implies **uncertainty aversion**.

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 - 2 PAA predicts uncertainty aversion here as well
- Evidence not unambiguous, but favours CRA

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- It is the latter: chances matter too.

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 - 3 I reject 2. In particular Linearity:

$$U(\text{Chance } x \text{ of } G) = x \cdot U(G)$$

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- Halevy (Econometrica 2007): Ambiguity aversion associated with failure to reduce compound lotteries.
- Krahermer and Stone (Econ. Theory 2011)
 - 1 Agents anticipate regret when learning that they picked a lottery with poor chances.
 - 2 Asymmetry of regret and joy explained by different reference points
- KMM (Econometrica 2005)
 - 1 Formally, the model proposed here is akin to KMM
 - 2 But interpretation is quite different ...

Formal Stuff

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- **Bradley:** For concave functions ϕ^1, \dots, ϕ^n :

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