

Strategies and Interactive Beliefs in Dynamic Games

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- *Topic*: formal analysis of strategic thinking in dynamic games, backward and forward induction reasoning.
- *Tools*: epistemic (type) structures where states determine truth value of conditional statements.
- *Distinctive feature*: we use **only epistemic conditionals** of the form:
"if i learned h he would believe E with probability p ";
state=(actual actions,epistemic conditionals).
- *Motivation*: strategies cannot be (irreversibly) chosen, they are just beliefs on own contingent choices obtained from planning (objective vs subjective strategies).
- *Focus*: generic perfect information games.

- *Hierarchies of probabilistic beliefs*: Mertens & Zamir (IJGT 1985), Brandenburger & Dekel (JET 1993), Heifetz & Samet (JET 1998)
- *Rationalizability in games*: Bernheim (Ecma 1984), Pearce (Ecma 1984)
- *Dynamic interactive epistemology*: Ben Porath (REStud 1997), Battigalli & Siniscalchi (JET 1999,2002)

- 1 Preview on conditionals and Backward Induction (BI): semi-formal analysis of an example
- 2 Setup: perfect information (PI) games and epistemic structures
- 3 Strategies as epistemic constructs: some results
- 4 Conclusions

Preview: two kinds of type structures

Variants of Battigalli & Siniscalchi JET99: types are implicit representations of hierarchies of conditional beliefs. But, what are first-order beliefs about?

- \mathbf{H} =histories, \mathbf{Z} =paths (terminal nodes), $\mathbf{S}_i, \mathbf{S}_{-i}, \mathbf{S}$ =strategies
- "Traditional" \mathbf{S} -based structures: $(\bar{\mathbf{T}}_i, \bar{\beta}_i)_{i \in I}$, states in $\Omega = \mathbf{S} \times \mathbf{T}$, $(\mathbf{s}_i, \mathbf{t}_i)$ =state of i (\mathbf{s}_i is "objective"), \mathcal{H}_i =conditioning events,

$$\begin{aligned}\mathcal{H}_i &= \{\mathbf{S}_{-i}(\mathbf{h}) \times \bar{\mathbf{T}}_{-i} : \mathbf{h} \in \mathbf{H}\} \\ \bar{\beta}_i &: \bar{\mathbf{T}}_i \rightarrow \Delta^{\mathcal{H}_i}(\mathbf{S}_{-i} \times \bar{\mathbf{T}}_{-i}) \subset [\Delta(\mathbf{S}_{-i} \times \bar{\mathbf{T}}_{-i})]^{\mathcal{H}_i}\end{aligned}$$

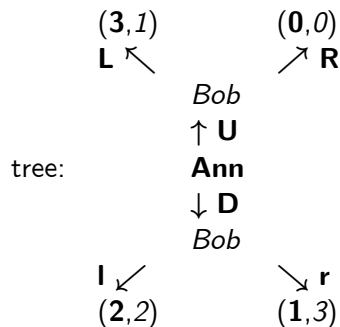
- Our \mathbf{Z} -based structures: $(\mathbf{T}_i, \beta_i)_{i \in I}$, states in $\Omega = \mathbf{Z} \times \mathbf{T}$, \mathbf{t}_i =epist.state of i (beliefs about others+plan),

$$\begin{aligned}\mathbf{H} &\cong \{\mathbf{Z}(\mathbf{h}) \times \mathbf{T}_{-i} : \mathbf{h} \in \mathbf{H}\} \\ \beta_i &: \mathbf{T}_i \rightarrow \Delta^{\mathbf{H}}(\mathbf{Z} \times \mathbf{T}_{-i}) \subset [\Delta(\mathbf{Z} \times \mathbf{T}_{-i})]^{\mathbf{H}}\end{aligned}$$

Preview on conditionals and BI: an example

Stackelberg Minigame

outputs:	low (left)	high (right)
high (up)	3,1	0,0
low (down)	2,2	1,3



Preview on conditionals and BI: an example

Stackelberg Minigame

- *Ann believes "Bob would go Right given Up"*
=belief about a behavioral conditional
- **Given Up**, *Ann would believe "Bob goes Right"*
=conditional belief about behavior
- In **traditional analysis**: behavioral conditionals and beliefs about conditionals and opponents' beliefs
- In **our analysis**: actual actions (paths) and *conditional* beliefs about actions and beliefs

Preview on conditionals and BI: traditional analysis

States ω specify strategies $\mathbf{s}_i = \sigma_i(\omega)$ **objectively**

'Bob would go Right given Up' **false** at ω iff $\sigma_{\text{Bob}}(\omega) \in \{\mathbf{L.r}, \mathbf{L.l}\}$

$$\text{Rat}_{\text{Bob}} \subset [\mathbf{L.r}] = \{\omega : \sigma_i(\omega) = \mathbf{L.r}\} \Rightarrow \\ \mathbf{B}_{\text{Ann}}(\text{Rat}_{\text{Bob}}) \subset \mathbf{B}_{\text{Ann}}(\mathbf{L.r}) \text{ (uncond. belief)}$$



Bob
 \uparrow U
 Ann
 \downarrow D
 Bob

States also specify conditional beliefs

By built in independence:

$$\mathbf{B}_{\text{Ann}}(\mathbf{L.r}) \subset \mathbf{B}_{\text{Ann}}(\mathbf{L}|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\mathbf{r}|\mathbf{D}) = \\ = \mathbf{B}_{\text{Ann}}(\text{util} = 3|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\text{util} = 1|\mathbf{D})$$

$$\Rightarrow \text{Rat}_{\text{Ann}} \cap \text{Rat}_{\text{Bob}} \cap \mathbf{B}_{\text{Ann}}(\text{Rat}_{\text{Bob}}) \subset \\ \subset [\mathbf{U}, \mathbf{L.r}] \subset [\mathbf{U}, \mathbf{L}]$$



\Rightarrow **BI strategies and path obtain**

Preview on conditionals and BI: our analysis

“strategy” is an epistemic concept:

i 's cond. beliefs \Rightarrow (beliefs about others and) contingent plan of i

Rat. Planning: $\mathbf{RP}_{\text{Bob}} = \mathbf{B}_{\text{Bob}}(\mathbf{L}|\mathbf{U}) \cap \mathbf{B}_{\text{Bob}}(\mathbf{r}|\mathbf{D})$

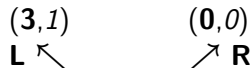
Material Consistency = path cons. with plan

Rationality = Material cons. + Rat. plan.

$\mathbf{R}_{\text{Bob}} \subset [\mathbf{U}, \mathbf{L}] \cup [\mathbf{D}, \mathbf{r}]$

But no event $[\mathbf{L}, \mathbf{r}]$. $\mathbf{B}_{\text{Ann}}(\mathbf{L}, \mathbf{r})$ not expressible!

$\mathbf{B}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}) \subset \mathbf{B}_{\text{Ann}}([\mathbf{U}, \mathbf{L}] \cup [\mathbf{D}, \mathbf{r}]) \not\subset$
 $\not\subset \mathbf{B}_{\text{Ann}}(\mathbf{L}|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\mathbf{r}|\mathbf{D})$ e.g. if $\mathbf{B}_{\text{Ann}}(\mathbf{D})$



Bob

\uparrow **U**

Ann

\downarrow **D**

Bob



Preview on conditionals and BI: an example

Our analysis

Recall: $R_{Bob} \subset [U, L] \cup [D, r]$, but

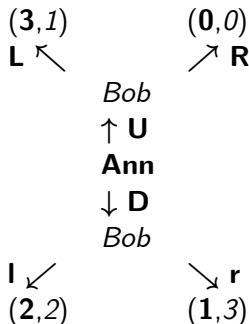
$B_{Ann}(R_{Bob}) \not\subset B_{Ann}(L|U) \cap B_{Ann}(r|D)$ e.g. if $B_{Ann}(D)$ (Ann plans **D**)

Since there is no event $[L.r]$,
 $B_{Ann}(L.r)$ not expressible

If $B_{Ann}(D)$ (Ann plans **D**) the following is possible

$B_{Ann}([U, L] \cup [D, r]) \cap \neg B_{Ann}(L|U)$

$\exists \omega \in [D, r] \cap R_{Ann} \cap R_{Bob} \cap B_{Ann}(R_{Bob})$



\Rightarrow **Imperfect Nash equilibrium may obtain**

Preview on conditionals and BI: an example

How to recover elementary BI, route 1: Strong Belief in rationality

- Ann *strongly believes* \mathbf{E} if she believes \mathbf{E} given each \mathbf{C} with $\mathbf{C} \cap \mathbf{E} \neq \emptyset$
- $[\mathbf{U}] \cap \mathbf{R}_{\text{Bob}} \neq \emptyset, [\mathbf{D}] \cap \mathbf{R}_{\text{Bob}} \neq \emptyset$
- hence $\mathbf{SB}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}) \subset \mathbf{B}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}|\mathbf{D})$
- but $\mathbf{R}_{\text{Bob}} \subset [\mathbf{U}, \mathbf{L}] \cup [\mathbf{D}, \mathbf{r}]$
- hence $\mathbf{SB}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}) \subset \mathbf{B}_{\text{Ann}}(\mathbf{L}|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\mathbf{r}|\mathbf{D})$ (BI cond. beliefs)
- thus $\mathbf{R}_{\text{Ann}} \cap \mathbf{R}_{\text{Bob}} \cap \mathbf{SB}_{\text{Ann}}(\mathbf{R}_{\text{Bob}}) \subset [\mathbf{U}, \mathbf{L}]$ (BI path)

Preview on conditionals and BI: an example

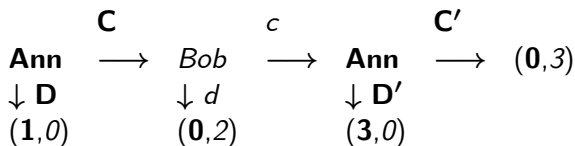
How to recover elementary BI, route 2: Own-Action Independence

- Suppose
 - (i) Ann's conditional beliefs about Bob's beliefs (hence his plan) are *independent* of her actions (**Ind**),
 - (ii) Ann strongly believes that Bob is materially consistent:
- $\mathbf{Ind} \cap \mathbf{B}_{\text{Ann}}(\mathbf{RP}_{\text{Bob}}) \cap \mathbf{SB}_{\text{Ann}}(\mathbf{MC}_{\text{Bob}}) \subset \mathbf{B}_{\text{Ann}}(\mathbf{L}|\mathbf{U}) \cap \mathbf{B}_{\text{Ann}}(\mathbf{r}|\mathbf{D})$
- Thus
 - $\mathbf{R}_{\text{Ann}} \cap \mathbf{R}_{\text{Bob}} \cap \mathbf{Ind} \cap \mathbf{B}_{\text{Ann}}(\mathbf{RP}_{\text{Bob}}) \cap \mathbf{SB}_{\text{Ann}}(\mathbf{MC}_{\text{Bob}}) \subset [\mathbf{U}, \mathbf{L}]$
- (Note: such independence and belief in consistency are implicit in traditional analysis)

- Route 1 (SB, with higher-level epistemic assumptions) leads to the BI-path in generic PI games
- Route 2 (OAI) leads to the BI-path in generic two-stage PI games, not in longer games such as the Centipede (even with higher-level epistemic assumptions)

"Centipede" example: strong belief analysis

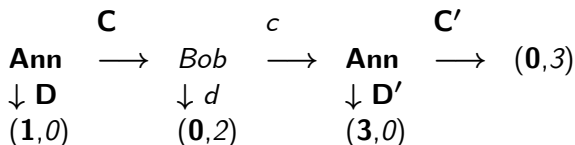
Forward induction reasoning yields the BI outcome



- $R_{Ann} \subset \neg[\mathbf{C}, \mathbf{c}, \mathbf{C}']$
- $R_{Bob} \cap SB_{Bob}(R_{Ann}) \subset [\mathbf{D}] \cup [\mathbf{C}, d]$
- $R_{Ann} \cap SB_{Ann}(R_{Bob} \cap SB_{Bob}(R_{Ann})) \subset [\mathbf{D}]$

"Centipede" example: initial common belief analysis

Not enough to get the BI outcome



- $R_{Ann} \subset \neg[C, c, C']$
- But, **without** $SB_{Bob}(R_{Ann})$, R_{Bob} has no behavioral implication!
Thus
- $R_{Ann} \cap R_{Bob} \cap Ind \cap B_{Ann}(R_{Bob}) \cap SB_{Ann}(MC_{Bob}) \subset \neg[C, c, C']$
- \Rightarrow initial common belief in $R \cap Ind \cap SB(MC)$ only buys $\neg[C, c, C']$

(We can show this by example and as corollary of a general theorem)

Setup

Perfect Information (PI) games

- $i \in I$, *players*
- $h \in H$, *histories/nodes* (H_i , owned by i), H **finite**
- $z \in Z \subset H$, *terminal histories/paths*; $Z(h) = \{z : h \preceq z\}$
- $a \in A(h)$, *actions* at $h \in H \setminus Z$
- $u_i : Z \rightarrow \mathbb{R}$, *utility/payoff* s.t. **no relevant ties**

Setup

Epistemic structures for PI games: general

- $\omega \in \Omega = \mathbf{X} \times \mathbf{T} = \mathbf{X} \times \prod_{i \in I} \mathbf{T}_i$, *states of the world*
- $\mathbf{x} \in \mathbf{X}$, *eXternal* (non-epistemic) *states*, **finite** (e.g. $\mathbf{X} = \mathbf{S}$ or $\mathbf{X} = \mathbf{Z}$)
- (implicitly understood *path function* $\pi : \mathbf{X} \rightarrow \mathbf{Z}$)
- for $\mathbf{Y} \subset \mathbf{X}$, $[\mathbf{Y}] = \mathbf{Y} \times \mathbf{T}$ *eXternal events*
- $\beta_i : \mathbf{T}_i \rightarrow \mathbf{B}_i$, $\beta_i(\mathbf{t}_i)$ *epistemic state of i at ω* ($\mathbf{B}_i = \mathbf{Beliefs}_i$ to be specified)

- Conditioning events (hypotheses) correspond to histories:

$$[\mathbf{h}] = \{(\mathbf{x}, \mathbf{t}) \in \mathbf{X} \times \mathbf{T}_{-i} : \pi(\mathbf{x}) \in \mathbf{Z}(\mathbf{h})\}$$

- Probability measures concentrated on conditioning events $\mathbf{C} = [\mathbf{h}]$, $\mathbf{D} = [\mathbf{h}']$... related by *chain rule*:

$$\mathbf{E} \subset \mathbf{C} \subset \mathbf{D} \Rightarrow \mu_i(\mathbf{E}|\mathbf{D}) = \mu_i(\mathbf{E}|\mathbf{C})\mu_i(\mathbf{C}|\mathbf{D}) \quad (\text{ch.r})$$

- Write

$$\mu_{i,h}(\cdot) = \mu_i(\cdot|[\mathbf{h}]), \mu_i = (\mu_{i,h}(\cdot))_{h \in \mathbf{H}} \in [\Delta(\mathbf{X} \times \mathbf{T})]^{\mathbf{H}}$$

Conditional probability systems (CPS's) on $(\mathbf{X} \times \mathbf{T}, \mathbf{H})$:

$$\Delta^{\mathbf{H}}(\mathbf{X} \times \mathbf{T}_{-i}) = \{\mu_i : \mu_{i,h}([\mathbf{h}]) = \mathbf{1} \text{ and (ch.r) holds}\}$$

We focus on epistemic **type structures** where eXternal states are complete sequences of actions: $\mathbf{X} = \mathbf{Z}$

$$\Omega = \mathbf{Z} \times \prod_{i \in I} \mathbf{T}_i, \quad \beta_i : \mathbf{T}_i \rightarrow \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i}), \quad (i \in I)$$

meaning:

- **no knowledge** about \mathbf{z} at the outset
- **information** about moves **cannot** directly **disclose** anything about types/**beliefs**

$$\Omega = \mathbf{Z} \times \prod_i \mathbf{T}_i, \quad \beta_i : \mathbf{T}_i \rightarrow \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i}),$$

Technical assumptions (in a sense, w.l.o.g.): for each $i \in \mathbf{I}$

- \mathbf{T}_i **compact metrizable**, hence $\Delta^H(\mathbf{Z} \times \mathbf{T}_{-i})$ compact metrizable (B&S JET99)
- β_i **continuous**

Definition

A type structure is *complete* if β_i is **onto** for every $i \in \mathbf{I}$.

Main example: the canonical structure of hierarchies of beliefs.

Canonical structure: types as hierarchies of beliefs

- First-order beliefs: $\mathbf{T}_i^1 = \Delta^H(\mathbf{Z})$
- Second-order beliefs:

$$\mathbf{T}_i^2 = \left\{ (\mu_i^1, \mu_i^2) \in \mathbf{T}_i^1 \times \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i}^1) : \forall h \in \mathbf{H}, \text{marg}_{\mathbf{Z}} \mu_{i,h}^2(\cdot) = \mu_i^1 \right\}$$

- ... recursive def. of $\mathbf{T}_i^n \subset \mathbf{T}_i^{n-1} \times \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i}^{n-1})$...
- $\mathbf{T}_i =$ projective limit of $(\mathbf{T}_i^n)_{n \in \mathbb{N}}$, set of *infinite hierarchies of beliefs* satisfying common full belief in coherence
- homeomorphism $\beta_i : \mathbf{T}_i \rightarrow \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i})$ by a generalization of Kolmogorov's extension theorem

Standard (monotonic) belief operators:

[for $\mathbf{E} \subset \mathbf{Z} \times \mathbf{T}$, let $\mathbf{E}_{t_i} = \{(z, t_{-i}) : (z, t_i, t_{-i}) \in \mathbf{E}\}$ section at t_i]

- conditional belief $\mathbf{B}_i(\mathbf{E}|\mathbf{h}) = \{(z, t_i, t_{-i}) : \beta_i(t_i)(\mathbf{E}_{t_i}|\mathbf{h}) = 1\}$,
- initial belief $\mathbf{B}_i(\mathbf{E}) = \mathbf{B}_i(\mathbf{E}|\mathbf{h}^0)$ (\mathbf{h}^0 =empty hist., root)

Nonmonotonic belief operator

- strong belief $\mathbf{SB}_i(\mathbf{E}) = \bigcap_{\mathbf{h}:\mathbf{E} \cap [\mathbf{h}] \neq \emptyset} \mathbf{B}_i(\mathbf{E}|\mathbf{h})$

Setup

Dynamic programming

Given $\mu_i \in \Delta^H(\mathbf{Z} \times \mathbf{T}_{-i})$ derive

$$\mu_i(\mathbf{a}|\mathbf{h}) = \mu_i([\mathbf{h}, \mathbf{a}]|[\mathbf{h}])$$

for all $\mathbf{h} \in \mathbf{H} \setminus \mathbf{Z}$, $\mathbf{a} \in \mathbf{A}(\mathbf{h})$.

Consider only the conditional prob. of i 's opponents' actions:

$$\mu_i(\mathbf{a}|\mathbf{h}), \mathbf{h} \in \mathbf{H}_{-i}, \mathbf{a} \in \mathbf{A}(\mathbf{h})$$

\Rightarrow subjective decision tree $\Gamma_i(\mu_i)$ for i

μ_i is consistent with dynamic programming on $\Gamma_i(\mu_i)$ iff (OSD)

$$\forall \mathbf{h} \in \mathbf{H}_i, \mu_i \left(\arg \max_{\mathbf{a} \in \mathbf{A}(\mathbf{h})} \mathbf{V}_i((\mathbf{h}, \mathbf{a}), \mu_i) | \mathbf{h} \right) = 1,$$

$$\text{with } \mathbf{V}_i((\mathbf{h}, \mathbf{a}), \mu_i) = \sum_{\mathbf{z} \in \mathbf{Z}(\mathbf{h}, \mathbf{a})} \mathbf{u}_i(\mathbf{z}) \mu_i(\mathbf{z} | \mathbf{h}, \mathbf{a})$$

Strategies as beliefs

Rational planning

The plan of i at (z, t_i, t_{-i}) (plan of t_i) is derived from $\beta_i(t_i)$

Definition

Pl. i plans rationally at (z, t_i, t_{-i}) if

$$\forall h \in H_i, \beta_i(t_i) \left(\arg \max_{a \in A(h)} V_i((h, a), \mu_i) | h \right) = 1.$$

Event: RP_i

- RP_i is just a property of i 's beliefs/types: i expects to take locally maximizing actions conditional on each $h \in H_i$.
- **Interpretation:** i has beliefs about others and computes his plan (beliefs about himself) by dynamic programming ("folding back") on the corresponding subjective decision tree.

Strategies as beliefs

Material Consistency and Material Rationality

Connect beliefs to behavior:

Definition

Pl. i is *materially consistent* at (z, t_i, t_{-i}) if he does not violate his plan on path z

$$\forall h \in H_i, \forall a \in A(h), \\ (h, a) \preceq z \Rightarrow \beta_i(t_i)(a|h) > 0.$$

Event: MC_i

Definition

Pl. i is *materially rational* at (z, t_i, t_{-i}) if he plans rationally and does not violate his plan at (z, t_i, t_{-i}) . Event: $R_i = MC_i \cap RP_i$.

Strategies as beliefs

Common belief in R and Nash equilibrium

- Recall: results apply to **finite PI games with NRT**, in such games the **BI strategy is unique** and (mixed) **Nash equilibria yield a unique path** with prob. 1.
- Similar to traditional analysis: initial common belief in material rationality does not yield BI or Nash paths in games of depth **$d > 2$** (e.g. Centipede).

- *Unlike traditional analysis:* in games of **depth 2**, correct belief in rationality does **not** yield **BI**, only a **Nash path** (cf. initial example).

Proposition

In a game Γ of depth 2, $\forall z \in \mathbf{Z}$, z is a Nash path if and only if

$$(z, \mathbf{t}) \in \bigcap_i \mathbf{R}_i \cap \mathbf{B}_i(\mathbf{R}_{-i})$$

for some \mathbf{t} , in some type structure $(\mathbf{T}_i, \beta_i)_{i \in I}$ for Γ .

Strategies as beliefs

Common Strong Belief in Material Rationality

- One way to obtain elementary BI (games of depth 2) is to assume that the first mover strongly believes in the material rationality of the co-player.
- We can go further and replicate "traditional" results on common strong belief in rationality and Nash eq. (Battigalli-Friedenberg, 2010), or EFR and BI in complete structures (Battigalli-Siniscalchi, 2002).

Strategies as beliefs

Common Strong Belief in Material Rationality

- $R_i^0 = R_i$, $R_i^{k+1} = R_i^k \cap SB_i(R_{-i}^k)$
- $CSBR = \bigcap_{k,i} R_i^k$, correct Common Strong Belief in R
- $\pi : S \rightarrow Z$ strategy-path function

Proposition

(i) In any type structure

$$\text{proj}_Z CSBR \subset \{\text{Nash-paths}\}$$

(ii) In a **complete** (or otherwise "sufficiently rich") type structure

$$\text{proj}_Z CSBR = \pi(\text{EFR}) = \{\text{BI-path}\}.$$

Strategies as beliefs

Own Action Independence

- Reason for non-BI result: Ann may initially believe in \mathbf{R}_{Bob} but she need not believe in \mathbf{R}_{Bob} conditional on taking an unplanned action.
- This cannot happen if Ann's beliefs about Bob's type *conditional on her own actions* do not depend on the conditioning action, and she strongly (hence always) believes in \mathbf{MC}_{Bob} .
- *Own-action independence* = \mathbf{i} 's conditional beliefs about \mathbf{t}_{-i} do not depend on \mathbf{i} 's actions (event \mathbf{Ind}_i)

Proposition

In every "rich" (e.g., complete) type structure for a game of depth 2

$$\text{proj}_{\mathbf{Z}} \left(\bigcap_i \mathbf{R}_i \cap \mathbf{Ind}_i \cap \mathbf{B}_i(\mathbf{R}_{-i}) \cap \mathbf{SB}_i(\mathbf{MC}_{-i}) \right) = \{\mathbf{BI-path}\}.$$

What about longer games (e.g. Centipede)?

Strategies as beliefs

Independence

- $\mathbf{RInd}_i^0 = \mathbf{R}_i \cap \mathbf{Ind}_i \cap \mathbf{SB}_i(\mathbf{MC}_{-i})$,
- $\mathbf{RInd}_i^{k+1} = \mathbf{RInd}_i^k \cap \mathbf{B}_i(\mathbf{RInd}_{-i}^k)$
- $\mathbf{CBRInd} = \bigcap_{i,k} \mathbf{RInd}_i^k$

All the paths consistent with the "Dekel-Fudenberg procedure", $\pi(\mathbf{S}^\infty \mathbf{W})$, are also consistent with **CBRInd**:

Proposition

There are type structures (including the canonical one) such that $\forall \mathbf{s} \in \mathbf{S}^\infty \mathbf{W}$, $\exists (\mathbf{z}, \mathbf{t}) \in \mathbf{CBRInd}$ such that $\mathbf{z} = \pi(\mathbf{s})$.

Corollary

*There are non-BI paths consistent with **CBRInd** in Centipede (of depth $\mathbf{d} > 2$).*

Conclusions






- Many results of the “traditional analysis” with behavioral conditionals in the state of the world make a lot of sense. They are built on often implicit assumptions of consistency of plans with actual behavior, strong belief in consistency (perceived intentionality) and self/opponents independence.
- We rule out behavioral conditionals, but allow for epistemic ones. This forces an interpretation of strategies as epistemic constructs and imposes discipline: in this setup strategies cannot be chosen, they can only be planned.
- Consistency and own-action independence have to be assumed explicitly. This seems fitting for a formal analysis of strategic reasoning.






Conclusions





Imperfect information (with perfect recall)

- $\mathbf{h}_i \in \mathbf{H}_i$ information sets (personal histories)
- Conditions/hypotheses for \mathbf{i} : $[\mathbf{h}_i]$ and $[\mathbf{h}_i, \mathbf{a}_i]$ ($\mathbf{a}_i \in \mathbf{A}_i(\mathbf{h}_i)$)
- Value of action \mathbf{a}_i at \mathbf{h}_i : $\mathbb{E}_{\beta_i(t_i)}[\mathbf{u}_i | \mathbf{h}_i, \mathbf{a}_i]$
- Potential conflict between our analysis and standard decision theory.
- *Own-action independence* and *strong belief in material consistency* allow to reconcile our analysis with traditional theory.

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