Beliefs and Strategies in the Absentminded Driver Problem

Alexandru Marcoci

LSE

Pisa
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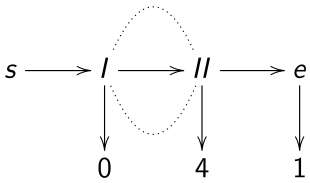
- Philosophical view: focus on beliefs.
- I will offer a new justification for the solution to the Absentminded Driver problem.
    - What should the driver believe?
    - Can we solve the problem while keeping the driver's beliefs fixed? What would that entail?

The Absentminded Driver

Having one last beer in the bar a driver is contemplating the road home. From the bar he needs to take the highway and then exit at the second intersection in order to sleep in his own bed. However, the way home is treacherous: if he exits at the first intersection he ends up in a very bad neighbourhood risking his life and if he continues on the highway after the second intersection he will need to sleep in a motel. Knowing he had one too many he realizes that he will not be able to remember passing by an intersection and hence he will not be able to acknowledge reaching the second. Therefore he asks himself what he should do.

The Absentminded Driver



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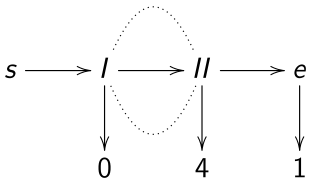
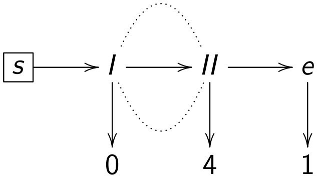


The root represents the bar, nodes I and II represent the first and the second exit, respectively, and the last node stands for the motel. The pay-offs we will be working with are 0 for ending up in the bad neighbourhood, 4 for getting home, and 1 for sleeping in a motel.

Ex ante



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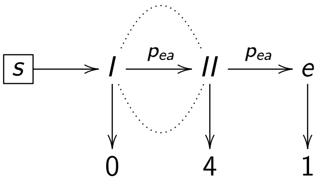

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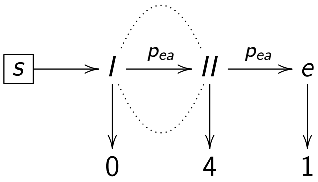

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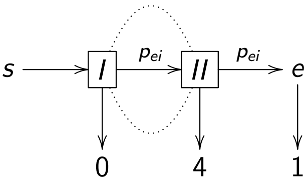
Ex interim



Problem 1 $EU_{ei}(mixing)$

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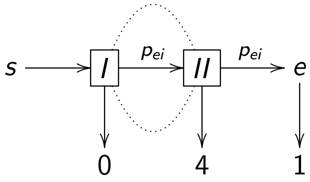

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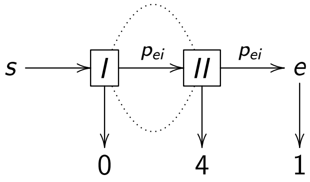

$$EU_{ei}^{(p_{ei},a)}(mixing) = a \times ((1-p_{ei}) \times 0 + p_{ei}(1-p_{ei}) \times 4 + p_{ei}^2 \times 1) \\ + (1-a)((1-p_{ei}) \times 4 + p_{ei} \times 1)$$

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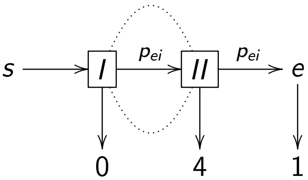

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$$p_{ea} = \frac{2}{3}$$
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Problem 2 If $a \neq 1$, then $p_{ea} \neq p_{ei}$ and the driver has an incentive to switch strategies upon reaching an intersection.

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- Normalizing, $a = \frac{1}{1+p_{ei}}$, while $1 - a = \frac{p_{ei}}{1+p_{ei}}$.
- The optimal strategy feeds back into your belief.

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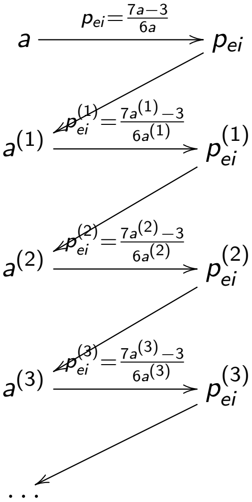
4. But then the optimal strategy must change as well ...

From Strategies back to Beliefs



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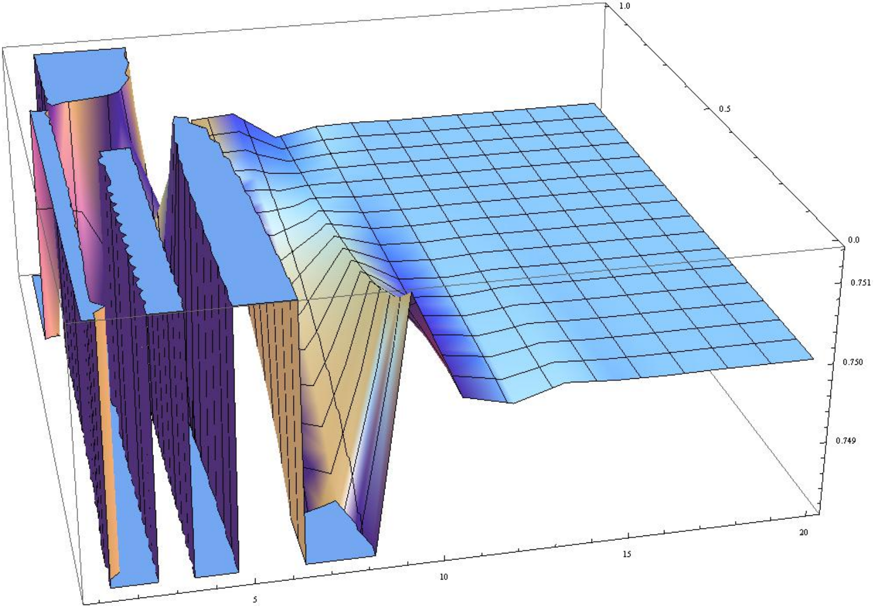
By the feedback mechanism, the equilibrium between your beliefs and your strategies is reached when the driver believes he is at the first intersection with probability $\frac{9}{13}$ and he continues with probability $\frac{4}{9}$.


1.0
0.5
0.0
0.751
0.750
0.749
5
10
15
20

Rational belief again

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- Why should Rationality demand Stability? Because otherwise you never get to action.

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- This differs from the usual way of presenting which simply says that *a* should not be 1. I offer a justification for this: *a* has to be Rational (in the above sense). 1 is not.
- However, this does not solve Problem 2: $p_{ei} = \frac{4}{9}$ and $p_{ea} = \frac{2}{3}$.

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- Then, the optimal ex interim strategy is to continue with $\frac{2}{3}$, and hence $p_{ea} = p_{ei}$.
- Consistency and Stability come apart. By updating according to Consistency the driver does not always reach a Stable belief.

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- The justification for continuing with $\frac{2}{3}$ comes from the fact that ex interim, the only rational belief (Consistent and Stable) is $\frac{3}{5}$.