

**OBSERVABILITY ESTIMATES FOR 1-D WAVE EQUATIONS  
WITH NON-LIPSCHITZ COEFFICIENTS**

*Francesco Fanelli*

BCAM – BASQUE CENTER FOR APPLIED MATHEMATICS  
Alameda de Mazarredo, 14 – 48009 Bilbao - Basque Country, SPAIN

ffanelli@bcamath.org

In this talk we will deal with the control problem for the 1-D wave equation

$$(1) \quad \omega(x) \partial_t^2 u - \partial_x^2 u = 0$$

on the interval  $[0, 1]$ , under minimal regularity assumptions over the coefficient  $\omega$ .

In a first time, we will show “classical” observability estimates for  $\omega$  satisfying an integral Zygmund condition. In particular, this result represents an improvement to the previous one for  $BV$  coefficients.

Then we will consider lower regularity hypothesis: for  $\omega$  log-Lipschitz or log-Zygmund, we will prove observability estimates “with a finite loss of derivatives”.

Finally, we will discuss the sharpness of our results.