Fast, Frugal & Focused

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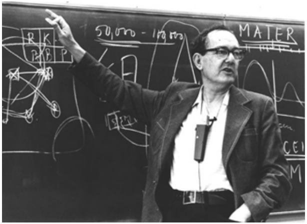
simon's question

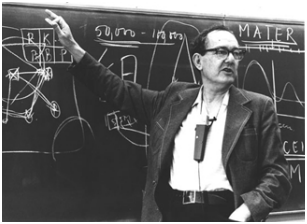
"How do human beings reason when the conditions for rationality postulated by the model of neoclassical economics are *not* met?"



Herbert Simon

bounded rationality

Theories of bounded rationality are more ambitious in trying to capture the actual process of decision as well as the substance of the final decision itself. A veridical theory of this kind can only be erected on the basis of empirical knowledge of the capabilities and limitations of the human mind; that is to say, on the basis of psychological research.



Herbert Simon

bounded rationality

Now if an organism is confronted with the problem of behaving approximately rational, or adaptively, in a particular environment, the kinds of simplifications [to its choice mechanisms] that are suitable may depend not only on the characteristics sensory, neural, and other of the organism, but equally upon the structure of the environment. Hence, we might hope to discover, by a careful examination of some of the fundamental structural characteristics of the environment, some further clues as to the nature of the approximating mechanisms used in decision making.

–Herbert Simon

H. A. Simon, "Rational Choice and the Structure of the Environment," *Psych Rev*, 63 (March 1956), p. 130.

bounded rationality

- *Psychological* – Cognitive Limitations
- *Ecological* – Environmental Constraints
- "Simon's scissors"

decision making task

Forced choice paired comparison task

Agent must decide which of two objects, A and B, has the larger value on some numerical criterion, C, based on their values on n binary cues $X_1, ..., X_n$.

heuristic structure and strategic biases

Take-the-Best

(Gigerenzer & Goldstein 1996)

Tallying $(1/N)$

(Dawes 1979)

heuristic structure and strategic biases

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Search Rule: Look up the cue with the highest validity

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heuristic structure and strategic biases

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Stopping Rule: If cue values differ $(+/-)$, stop search. If not, look up next cue.

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Stopping Rule: After m $(1 < m \leq M)$ cues, stop the search.

heuristic structure and strategic biases

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Decision Rule: Predict that the alternative with the positive cue value has the higher criterion value.

Bias: *ignore cues*

Tallying (1/N)

(Dawes 1979)

Search Rule: Look up cues in random order

Stopping Rule: After m ($1 < m \leq M$) cues, stop the search.

Decision Rule: Predict that the alternative with the higher number of positive cue values has the higher criterion value.

Bias: *ignore weights*

Evidence for "One good reason" strategies

"The results of [45 studies] firmly demonstrates that noncompensatory strategies (like TTB) were the dominant mode used by decision makers."

- Payne, Bettman & Johnson (1993), *The adaptive decision maker*.
- Gigerenzer et. al. (1999) *Simple heuristics that make us smart. Oxford Press.*
- Bergert and Nosofsky (2007) *J Exp Psych: LMC*; Reiskamp and Otto (2006) *J Exp Psych: General*.

Ford et al. (1989) *Organizational Behavior and Human Decision Processes*, p. 75

should people use heuristics?

Ideally, No.

- Heuristics are things people do because they cannot optimize;
- It is an imperfect approximation
- Characteristic of 'System I' thinking.
- Prone to error.

– Kahneman & Tversky School. (*Biases & Heuristics*)

Ideally, Yes.

- Can be as good as or even better than utility maximization
- Can be performed "intuitively" or "deliberatively".
- Are adaptive to the structure of the decision-making environment.

– Gigerenzer & The ABC Group (*Fast & Frugal Heuristics*)

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single variable decision rules

My Brunswikian Question

Under what *environmental conditions* do "single reason" rules perform well?

Egon Brunswik

single variable decision rules

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Under what environmental conditions do “single reason” rules perform well?

- Cues are highly intercorrelated (Hogarth & Karelaia 2005)
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- The single predictor cue is highly correlated with the other cues, but the remaining cues are uncorrelated with one another (Davis-Stober 2010).

epistemic coherence

is there a measure of epistemic coherence?

Is there a measure-of-coherence and measure-of-confirmation pair on which *more* coherence entails *higher* confirmation?

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Is there a measure-of-coherence and measure-of-confirmation pair on which *more* coherence entails *higher* confirmation?

- No (Erik Olsson)
- Qualified No (Luc Bovens and Stephan Hartmann)

incremental confirmation measures

$$inc_1(h, e_1, e_2) := P(h \mid e_1, e_2) - P(h \mid e_1)$$

$$inc_2(h, e_1, e_2) := \frac{P(h \mid e_1, e_2) - P(h \mid e_1)}{1 - P(h \mid e_1)}$$

$$ko(h, \mathbf{e}) := \frac{P(\mathbf{e} \mid h) - P(\mathbf{e} \mid \neg h)}{P(\mathbf{e} \mid h) + P(\mathbf{e} \mid \neg h)}, \text{ where } \mathbf{e} = \{e_1, e_2\}.$$

$$r_1(h, \mathbf{e}) := \log \frac{P(h \mid \mathbf{e})}{P(h)}$$

$$r_2(h, \mathbf{e}) := \log \frac{P(h \mid e_1, e_2)}{P(h \mid e_1)}$$

$$l(h, e) := \log \frac{P(\mathbf{e} \mid h)}{P(\mathbf{e} \mid \neg h)}$$

measures of coherence

- **Shogenji**: $S(x, y) := \dfrac{P(x, y)}{P(x)P(y)}$

- **Pearson's correlation coefficient**:

$$\rho_{X,Y} := \frac{P(X, Y) - P(X)P(Y)}{\sigma_X \sigma_Y} = \frac{P(X)[P(Y \mid X) - P(Y)]}{\sqrt{P(X)(1 - P(X))}\sqrt{P(Y)(1 - P(Y))}},$$

where variance of a binary variable, X, is $\sigma^2 = P(X)(1 - P(X))$.

- ◦ Several other proposals: (Bovens and Hartmann, Cross, Douven and Meijs, Glass, Fitelson, Heumer, Olsson).

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- Several other proposals: (Bovens and Hartmann, Cross, Douven and Meijs, Glass, Fitelson, Heumer, Olsson).
- We shall use divergence from mutual independence.

Focused Correlation (Myrvold 1996, Wheeler 2009)

$$For_h(e_1, \ldots, e_n) := \frac{S(e_1, \ldots, e_n \mid h)}{S(e_1, \ldots, e_n)} = \frac{\frac{P(e_1,\ldots,e_n \mid h)}{P(e_1 \mid h)\cdots P(e_n \mid h)}}{\frac{P(e_1,\ldots,e_n)}{P(e_1)\cdots P(e_n)}}$$

$$= \frac{P(h \mid e_1, \ldots, e_n) P(h)^{n-1}}{P(h \mid e_1) \cdots P(h \mid e_n)}.$$

$$For_h(\mathbf{e}) = \alpha \text{ is } \begin{cases} \text{inflationary} & \text{if } \alpha > 1 \\ \text{neutral} & \text{if } \alpha = 1 \\ \text{deflationary} & \text{if } \alpha < 1 \end{cases}$$

Wheeler (2009), 'Focused Correlation and Confirmation,' *British J Phil Sci*, 60(1): 79-100.

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- Yes!
- For the *class* of incremental confirmation measures and *focused correlation*:
  - Inflationary focused correlation $\Rightarrow$ positive incremental confirmation
  - More focused correlation $\Leftrightarrow$ more incremental confirmation

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constraints

(A0) *Regularity*: All events have positive measure

(A1) *Positive relevance*: $\forall e_i \in \mathbf{e}, P(h \mid e_i) > P(h)$

(A2) *Equal strength*: $\forall e_i, e_k \in \mathbf{e}, P(h \mid e_i) = P(h \mid e_k)$ and $P(h \mid \neg e_i) = P(h \mid \neg e_k)$

(A3) *Independent evidence*: $\forall E_j, E_k \; \mathrm{E}_j \perp\!\!\!\perp \mathrm{E}_k \mid \mathrm{H}.$

Ceteris Paribus Conditions

Witness Models	Positive Evidence	Equally Positive Evidence
A0* A1 A3	A0 A1	A0 A1 A2

Lemma 1 (Schlosshauer and Wheeler, 2011)

Let $\{e_1, e_2\}$ and $\{e_1, e_3\}$ satisfy (A0) and (A1) with respect to h. Then,

$$\frac{P(h \mid e_1, e_2)}{P(h \mid e_1, e_3)} = \frac{Foc_h(e_1, e_2)}{Foc_h(e_1, e_3)} \cdot \frac{P(h \mid e_2)}{P(h \mid e_3)}.$$

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- Observe:

$$\frac{inc_1(h, e_1, e_2)}{inc_1(h, e_1, e_3)} := \frac{P(h \mid e_1, e_2) - P(h \mid e_1)}{P(h \mid e_1, e_3) - P(h \mid e_1)}$$

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Observe: If (A2), then $P(h \mid e_2) = P(h \mid e_3)$.

Recall:

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(A2*) Variable evidence: $\forall e_i, e_j \in \mathbf{e}, \frac{For_h(e_1,e_3)}{For_h(e_1,e_2)} < \frac{P(h|e_2)}{P(h|e_3)} \leq 1.$

idea: Evidence varies within a safe range if: the relative boost to h from combining e_2 to e_1 versus combining e_3 to e_1 is **greater than** the relative boost to h (if any) from e_3 alone versus e_2 alone.

relaxing (A2)

Thm 2: If $\mathbf{e} = \{e_1, e_2\}$ and $\mathbf{e}' = \{e_1, e_3\}$, and $(\mathbf{e} \cup \mathbf{e}', h)$ satisfies (A0, A1, A2*), then all of the following inequalities hold:

- $For_h(\mathbf{e}) > For_h(\mathbf{e}')$
- $r_1(h, \mathbf{e}) > r_1(h, \mathbf{e}')$
- $r_2(h, \mathbf{e}) > r_2(h, \mathbf{e}')$
- $l(h, \mathbf{e}) > l(h, \mathbf{e}')$
- $ko(h, \mathbf{e}) > ko(h, \mathbf{e}')$
- $inc_1(h, \mathbf{e}) > inc_1(h, \mathbf{e}')$
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single variable decision rules

Question

Under what environmental conditions do "single reason" rules perform well?

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Theorem (Wheeler & Katsikopoulos, underway)

Let $v_i = P(c \mid e_i)$ and $P(c) = \frac{1}{2}$, for a criterion c and cues e_i.

$$P(c \mid e_1) = \frac{P(c \mid e_1, \ldots, e_n)}{2v_2 2v_3 \cdots 2v_n} \times \frac{\dfrac{P(e_1, \ldots, e_n)}{P(e_1) \ldots P(e_n)}}{\dfrac{P(e_1, \ldots, e_n \mid c)}{P(e_1 \mid c) \cdots P(e_n \mid c)}}$$

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and

$$\frac{P(c \mid e_1)}{P(c \mid e_1, \ldots, e_n)} = \prod_{i=2}^{n} P(c \mid x_i) \times 2^{n-1} \times \frac{\frac{P(e_1, \ldots, e_n)}{P(e_1) \ldots P(e_n)}}{\frac{P(e_1, \ldots, e_n \mid c)}{P(e_1 \mid c) \cdots P(e_n \mid c)}}$$

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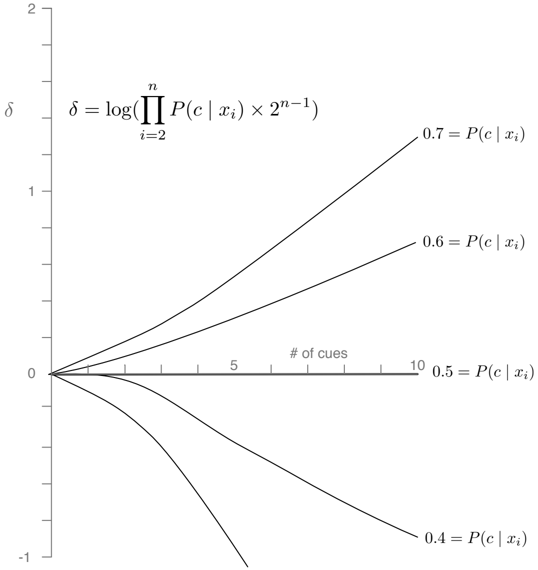

2
1
0
-1
δ
$\delta = \log(\prod_{i=2}^{n} P(c \mid x_i) \times 2^{n-1})$
$0.7 = P(c \mid x_i)$
$0.6 = P(c \mid x_i)$
# of cues
5
10
$0.5 = P(c \mid x_i)$
$0.4 = P(c \mid x_i)$

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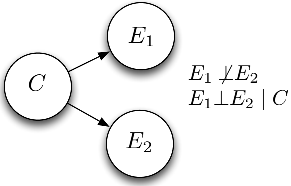
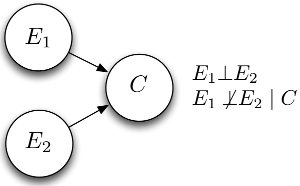


Katsikopoulos & Martignon



Baucells, Carrasco & Hogarth

collaborators

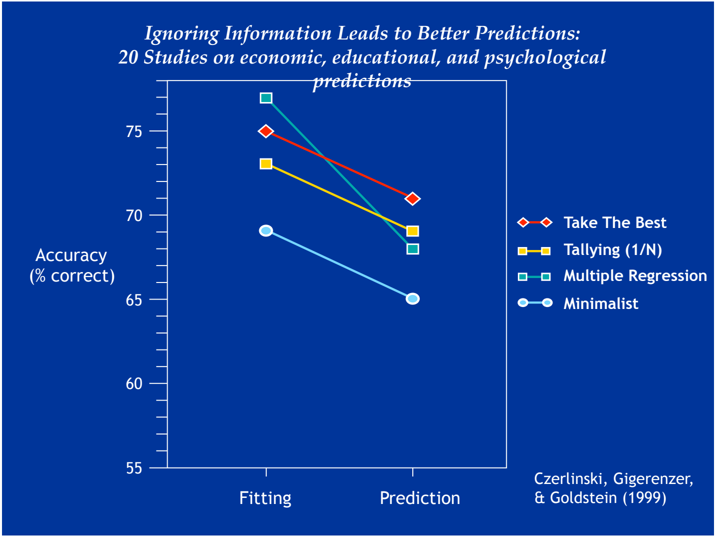
CCC

Richard Scheines, Carnegie Mellon University

Maximilian Schlosshauer, Niels Bohr Institute

CCC & Heuristics

Konstantinos Katsikopoulos, Max Planck Institute


Ignoring Information Leads to Better Predictions:
20 Studies on economic, educational, and psychological predictions
Accuracy
(% correct)
75
70
65
60
55
Fitting
Prediction
Take The Best
Tallying (1/N)
Multiple Regression
Minimalist
Czerlinski, Gigerenzer, & Goldstein (1999)

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If you accept *Focused Correlation* as a basis for measures of coherence, then

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Next Question

Where does this impossibility business come in?

focused correlation within sfcc models

$$For_H(E_1, E_2) := \frac{S(E_1, E_2 \mid H)}{S(E_1, E_2)} = \frac{\frac{P(E_1 \cap E_2 | H)}{P(E_1|H)P(E_2|H)}}{\frac{P(E_1 \cap E_2)}{P(E_1)P(E_2)}}$$

$$= \frac{1}{\frac{P(E_1 \cap E_2)}{P(E_1)P(E_2)}}$$

NB: Since $For_H(E_1, E_2) < 1$, the precondition for Prop 1 is not satisfied.

focused correlation within sfcc models

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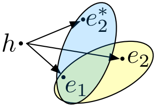
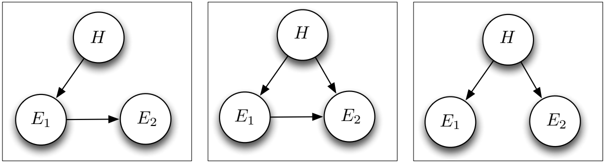
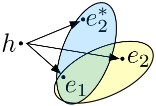
resolving a discontinuity

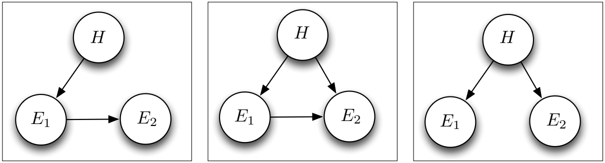




Wheeler (2012) 'Explaining the Limits of Olsson's Impossibility Result,' *The Southern Journal of Philosophy*, 50(1): 136-50.

resolving a discontinuity





$$P(E_2) = P(E_2^*)$$

or

$$\text{(A2)} \left[\begin{array}{l} P(H|E_1) = P(H|E_2) \\ P(H|E_1) = P(H|E_2^*) \end{array}\right.$$

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key coherence references

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