Confidence in Beliefs and Decision Making

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The business man himself not merely forms the best estimate he can of the outcome of his actions, but he is likely also to estimate the probability that his estimate is correct. (Knight, 1921, p226-227)

Belief: one dimension or two?

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. . . there is the degree to which one is confident in this endorsement.

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If the former is one's beliefs, the latter is one's **confidence in one's beliefs**. Together, they make up the agent's **doxastic state**.

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If the former is one's beliefs, the latter is one's **confidence in one's beliefs**. Together, they make up the agent's **doxastic state**.

Claim Confidence in beliefs is an important aspect of doxastic states.

Why confidence?

Where would a second dimension such as confidence make a difference?

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- In belief formation (or change)
- In choice

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One's confidence in a belief may depend upon the "amount" of information, that is, on aspects relevant to the modification and formation of belief.

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> *The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favorableness of the opinion itself. (Knight, 1921, p226-227)*

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- Ellsberg

Why confidence?

Reminder: Ellsberg

- Known urn: 100 balls, 50 red, 50 black.
- Unknown urn: 100 balls, each red or black.

	Known urn		Unknown urn	
	Red	Black	Red	Black
I	$ 100	$ 0	$ 0	$ 0
II	$ 0	$ 100	$ 0	$ 0
III	$ 0	$ 0	$ 100	$ 0
IV	$ 0	$ 0	$ 0	$ 100

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- Choices based on incomplete / controversial scientific evidence, where probabilities cannot necessary be given

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- In belief formation (or change)
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- Ellsberg
- Choices based on incomplete / controversial scientific evidence, where probabilities cannot necessary be given
- The problem of deferral – when should one defer?

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There appear to be many significant decisions where confidence in beliefs do, or should, play a role.

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There appear to be many significant decisions where confidence in beliefs do, or should, play a role.

But what role?

The role of confidence in choice

The role of confidence in choice

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?

The role of confidence in choice

- would we like decisions about climate change policy to be taken on the basis of “best hunch” estimates?
- and what about wagers between us?

The role of confidence in choice

- should we defer diagnosis to an expert when the situation is potentially critical?

The role of confidence in choice

- should we defer diagnosis to an expert when the situation is potentially critical?
- and what about wagering on the condition?

The role of confidence in choice

Maxim

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

The role of confidence in choice

Claim The role confidence should play in choice is subject to the following maxim:

Maxim

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Plan

1. develop a theory based on these claims
   - propose a model of confidence in beliefs
   - propose a family of decision rules which take confidence into account
2. defense and consequences of the theory:
   - conceptual and choice-theoretic properties
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Modelling confidence

Idea First attempt

- Represent beliefs by a set of probability measures (à la ...).

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- You are confident that the probability of C is greater than 0.3 if, for all p in $\mathcal{C}$, $p(C) \geqslant 0.3$
- If not, then you are unsure whether the probability of C is greater than 0.3.

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Problem

- confidence is represented as "binary": you are either fully confident about a probability judgement or completely unsure about it.
- in reality, confidence is not "binary": it comes in degrees.

Modelling confidence

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- represent beliefs by a **nested family of sets** of measures

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- represent beliefs by a **nested family of sets** of measures

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- you are **more confident** in $p(C) \geqslant 0.3$ than $p(B) \leqslant 0.2$ if the former holds for all probability measures in more sets than the latter.

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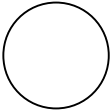
Modelling confidence

The representation in graphical form



$\Delta(\Sigma)$

Modelling confidence

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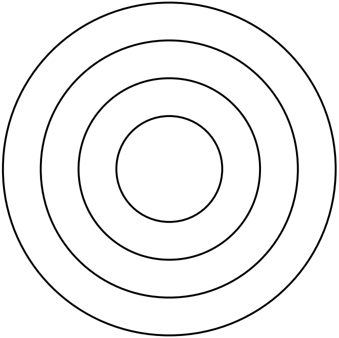

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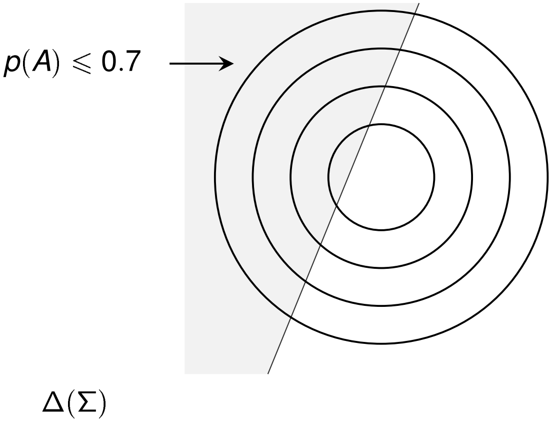

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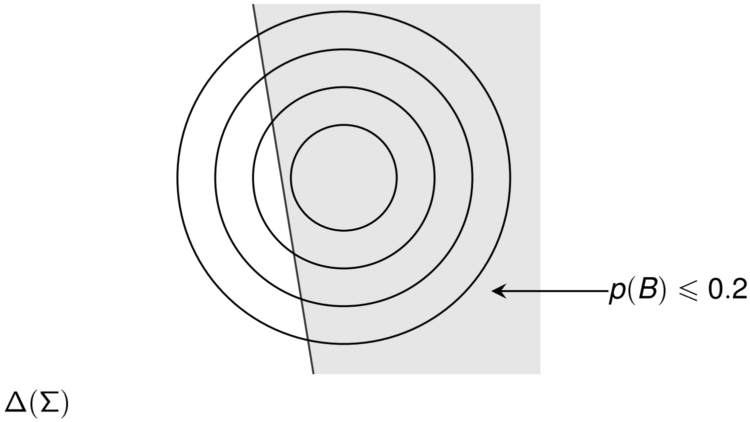

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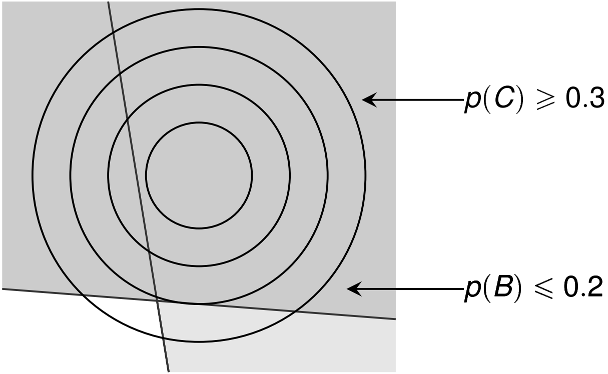

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Modelling confidence

Definition

A **confidence ranking** Ξ is a nested family of closed subsets of $\Delta(\Sigma)$.

A confidence ranking Ξ is **convex** if every $C \in \Xi$ is convex.
It is **continuous** if, for every $C \in \Xi$, $C = \overline{\bigcup_{C' \subsetneq C} C'} = \bigcap_{C' \supsetneq C} C'$.
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Remark

- this is equivalent to a weak order on the space of probability measures
- this is an ordinal structure.

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Decision

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    $f \geqq g$ implies $D(f) \supseteq D(g)$ (for stakes relation $\geqq$).

A family of decision theories

For each decision theory in the family:

Ingredients:

- utility function u
- confidence ranking Ξ
- cautiousness coefficient D

General form:

preferences concerning f are a function of $u(f(s))$ and $D(f)$ according to I

where:

1. I: decision rule
2. D respects the notion of stakes ($\geqq$)

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There are several ways of specifying the decision rule I and the notion of stakes relation.

Confidence, stakes and choice

Examples

Decision rules using sets of probabilities:

Confidence, stakes and choice

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- unanimity rule
  (an act is preferred to another if it has higher expected utility according to all probability measures in the set)

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  (evaluate an act by the lowest expected utility, calculated using all probability measures in the set)

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Decision rules using sets of probabilities:

- unanimity rule
  (an act is preferred to another if it has higher expected utility according to all probability measures in the set)
- maxmin expected utility
  (evaluate an act by the lowest expected utility, calculated using all probability measures in the set)
- Hurwicz or α-maxmin rule
- E-admissibility
- etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice of f:

Confidence, stakes and choice

Examples

Stakes involved in the choice of f:

- utility of worst consequence of f
- difference between utilities of best and worst possible consequences of f
- probability that f takes value below a certain threshold
- etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice between f and g:

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- difference between utilities of best and worst possible consequences of f
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- etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice between f and g:

- largest of utility of worst consequence of f and that of g
- largest of difference between utilities of best and worst possible consequences of f and that for g
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- etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice between f and g:

- largest of utility of worst consequence of f and that of g
- largest of difference between utilities of best and worst possible consequences of f and that for g
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- difference in utilities of worst consequences of f and g
- the largest utility difference in consequences of f and g, taken over all states
- etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice from a menu A:

- largest of utility of worst consequence of f and that of g
- largest of difference between utilities of best and worst possible consequences of f and that for g
- largest of probability that f takes value below a certain threshold and that for g
- difference in utilities of worst consequences of f and g
- the largest utility difference in consequences of f and g, taken over all states
- etc. etc.

Confidence, stakes and choice

Examples

Stakes involved in the choice from a menu A in a context γ:

- largest of utility of worst consequence of f and that of g
- largest of difference between utilities of best and worst possible consequences of f and that for g
- largest of probability that f takes value below a certain threshold and that for g
- difference in utilities of worst consequences of f and g
- the largest utility difference in consequences of f and g, taken over all states
- etc. etc. etc.

Plan

1. develop a theory based on these claims
   - propose a model of confidence in beliefs
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Challenges and desiderata

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- it corresponds to a reasonable (pre-technical) intuition
- it has acceptable choice-theoretical consequences
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Lets see how this family does, by considering two members.

Conceptual properties

Incomplete preferences

- “Unanimity” decision rule
- any notion of stakes on pairs of acts (assumed to satisfy some basic properties)

Conceptual properties

Incomplete preferences

$f \preceq g$ if and only if:

$$\sum_{s \in S} u(f(s)).p(s) \leqslant \sum_{s \in S} u(g(s)).p(s)$$

for all $p \in D((f, g))$

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Interpretation:

- no preference between f and g: defer the choice between them.

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Interpretation:

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Under such a rule:

- choices made at low stakes may be suspended (but not reversed) at higher stakes.
- for higher stakes, one is decisive only if one is confident enough in appropriate beliefs.

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Conclusion This yields the following advice for deferral:

Defer when the confidence in relevant beliefs is not sufficient to match the importance of the decision.

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Comparison Few “incomplete preference” rules defended by invoking plausible maxims of this sort.

Comparison This rule is not as extreme as the unanimity rule.

Conceptual properties

Careful preferences

- “Maxmin EU” decision rule
- any notion of stakes on acts (assumed to satisfy some basic properties)

Conceptual properties

Careful preferences

$f \preceq g$ if and only if:

$$\min_{p \in D(f)} \sum_{s \in S} u(f(s)).p(s) \leqslant \min_{p \in D(g)} \sum_{s \in S} u(g(s)).p(s)$$

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Under such a rule:

- for higher stakes, one is effectively only relying on beliefs in which one has sufficient confidence.
- behaviour is as “pessimistic” as one’s confidence: the more confident in appropriate beliefs or the lower the stakes, the less pessimistic.

Conceptual properties

Careful preferences

$f \preceq g$ if and only if:

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Conclusion This gives the following advice for high-stakes decisions:

Choose boldly if one has sufficient confidence; choose cautiously if not.

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Comparison Few "non-EU" rules correspond so closely to plausible maxims of this sort.

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What would you want from a decision rule?

- ✓ it corresponds to a reasonable (pre-technical) intuition
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- it is conceptually clear about the roles of different mental attitudes

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Preliminaries

The Anscombe-Aumann framework

- S non-empty finite set of states
- $\Delta(\Sigma)$ set of probability measures on S
- X nonempty set of outcomes
- $\Delta(X)$ set of consequences
- $\mathcal{A}$ set of acts (functions $S \to \Delta(X)$)
- $\preceq$ preference relation on $\mathcal{A}$

Notation:

- $u(f(s)) = \sum_{x \in supp(f(s))} f(s)(x)u(x)$.
- $f_\alpha g$: shorthand for $\alpha f + (1-\alpha)g$.

Incomplete preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and reflexivity $\preceq$ is non-trivial and reflexive.

Completeness $f \preceq g$ or $f \succeq g$.

Transitivity if $f \preceq g$ and $g \preceq h$, then $f \preceq h$.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity the sets $\{\alpha \in [0,1] | \ f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] | \ f_\alpha h \succeq g\}$ are closed in $[0,1]$.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

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Standard unanimity model (Bewley):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$:

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Consistency when the stakes decrease, one cannot suspend (determinate) preferences.

Incomplete preferences

Representation theorem

Theorem

$\preceq$ *satisfies axioms above*

$\Leftrightarrow$ *there exists* $u : X \to \Re$, Ξ *and* $D : \Re \to \Xi$ *such that, for all* $f, g \in \mathcal{A}$, $f \preceq g$ *iff*

$$\sum_{s \in S} u(f(s)).p(s) \leqslant \sum_{s \in S} u(g(s)).p(s) \quad \forall p \in D((f, g))$$

Furthermore u *is unique up to positive affine transformation, and* Ξ *and* D *are unique.*

Incomplete preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

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Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

Conclusion

- As reasonable as "unanimity" incomplete preference model.
- Does not fall prey to Dynamic Consistency arguments

Careful preferences

Axioms

Expected utility (Anscombe and Aumann):

For all $f, g, h \in \mathcal{A}$, $\alpha \in (0,1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

Independence $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Continuity the sets $\{\alpha \in [0,1] |\ f_\alpha h \preceq g\}$ and $\{\alpha \in [0,1] |\ f_\alpha h \succeq g\}$ are closed in $[0,1]$.

Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Careful preferences

Axioms

Standard maxmin EU model (Gilboa-Schmeidler):

For all $f, g, h \in \mathcal{A}$, $c \in \Delta(X)$, $\alpha \in (0, 1)$:

Non triviality and weak order $\preceq$ is non-trivial, reflexive, transitive and complete.

C-Independence $f \preceq g$ iff $f_\alpha c \preceq g_\alpha c$.

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Monotonicity if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

Uncertainty Aversion For all $f, g \in \mathcal{A}$, $\alpha \in (0, 1)$, if $f \sim g$ then $f_\alpha g \succeq f$.

Careful preferences

Axioms

Current careful preference model:

For all $f, g, h \in \mathcal{A}$, $c \in \Delta(X)$, $\alpha \in (0,1)$:

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S-Independence (i) if $f_\alpha d$ involves lower stakes than f, then $f \succeq c$ implies $f_\alpha d \succeq c_\alpha d$
(ii) if $f_\alpha d$ involves higher stakes than f, then $f \preceq c$ implies $f_\alpha d \preceq c_\alpha d$

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Monotonicity applies to acts of the same stakes

Uncertainty Aversion applies to acts of the same stakes

Careful preferences

Representation theorem

Theorem

$\preceq$ satisfies axioms above

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Furthermore u is unique up to positive affine transformation, and Ξ and D are unique.

Conclusion

- There is a mild weakening of the independence axiom with respect to the maxmin EU model: it simply requires taking the stakes into account.

Choice-theoretic properties

Summary

Incomplete preferences

- Difference from “unanimity” incomplete preference model: allow indeterminacy when the stakes increase
- Independence holds: model does not fall prey to Dynamic Consistency arguments

Careful preferences

- Difference from “maxmin EU” model: allow one to exhibit more caution when the stakes increase

No more

Choice theoretic properties

Dutch Books

Consider the bet, with stakes S, yielding €S if A and €0 if not A.

Betting quotient $q(A)$: value such that you are indifferent between buying and selling the bet at stakes S for €$q(A)S$.

The argument (approximately):

No Dutch Book can be made against you	$\Leftrightarrow$	your betting quotients are probabilities

Choice theoretic properties

Dutch Books

Consider the bet, with stakes S, yielding €S if A and €0 if not A.

Assumptions:

- €$q(A)S$ is the price at which you are indifferent between buying and selling the bet
- €$q(A)S$ is the buying / selling price for all stakes S
- if you are willing to enter into some transactions separately, you are willing to enter into the set taken together

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Choice theoretic properties

Dutch Books

Consider the bet, with stakes S, yielding €S if A and €0 if not A.

Assumptions:

- you have a buying price € $\underline{q_S}(A)S$ and a selling price €$\overline{q_S}(A)S$
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- if you are willing to enter into some transactions separately, you are willing to enter into the set taken together at stakes not higher than the stakes in the initial transactions.

Choice theoretic properties

Dutch Books

Consider the bet, with stakes S, yielding €S if A and €0 if not A.

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Choice theoretic properties

Dutch Books

Consider the bet, with stakes S, yielding €S if A and €0 if not A.

In this case:

No Dutch Book can be made against you	$\Leftrightarrow$	buying / selling prices are minimal / maximal probabilities of a confidence ranking

Challenges and desiderata

What would you want from a decision rule?

- ✓ it corresponds to a reasonable (pre-technical) intuition
- ✓ it has acceptable choice-theoretical consequences
- it is conceptually clear about the roles of different mental attitudes

Lets see how this family does, by considering two members.

Separation of beliefs and desires

The model contains three elements:

- Utility function
- Confidence ranking
- Cautiousness coefficient

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There is a natural comparison of decisiveness

- DM 1 is more decisive than DM 2 if he has the same preferences as DM 2 whenever DM 2's preferences are determinate.

Separation of beliefs and desires

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There is a natural comparison of decisiveness

that corresponds precisely to differences in the cautiousness coefficient

For two decision makers with the same u and Ξ

1 is less decisive

$\Leftrightarrow$ $D_1((f,g)) \supseteq D_2((f,g))$ for all pairs f and g.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
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And there is a natural comparison of attitude to uncertainty

- DM 1 is more averse to uncertainty than DM 2 if, whenever 1 prefers f to non-ambiguous c, then so does 2.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
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And there is a natural comparison of attitude to uncertainty

that corresponds precisely to differences in the cautiousness coefficient.

For two decision makers with the same u and Ξ

- 1 is more averse to uncertainty

$\Leftrightarrow$ $D_1(f) \supseteq D_2(f)$ for all acts f.

Separation of beliefs and desires

The model contains three elements:

- Utility function = Desires over outcomes
- Confidence ranking = Beliefs and confidence in beliefs
- Cautiousness coefficient = Attitude to choosing in the absence of confidence

Conclusion There is a clean separation between beliefs and desires (attitudes to outcomes and to choosing in the absence of confidence).

Comparison The unanimity model, as well as most other “incomplete preference” rules, do not exhibit such a separation.

Comparison Maxmin EU, as well as many other “non-EU” models of decision making, do not exhibit such a separation.

In summary

On examination of two members of the proposed family:

- They embody plausible maxims for choice
- They both involve a neat separation of the decision maker's doxastic and conative attitudes
- They do not have particularly unreasonable consequences for choice

There is no reason to suspect that these properties do not extend to other reasonable members of the family.

Plan

1. develop a theory based on these claims
   - propose a model of confidence in beliefs
   - propose a family of decision rules which take confidence into account
2. defense and consequences of the theory:
   - conceptual and choice-theoretic properties
   - (briefly) consequences for decision making

Plan

1. develop a theory based on these claims
   - propose a model of confidence in beliefs
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2. defense and consequences of the theory:
   - conceptual and choice-theoretic properties
   - (briefly) consequences for decision making

Some consequences . . .

Were a decision maker to wish to use any of these rules, he would need to fix:

- his utility function
- his beliefs and confidence in beliefs
- his attitude to choosing in the absence of confidence

What would that mean for, for example, (public) decision making?

Some consequences . . .

- The decision maker's cautiousness coefficient . . .

  . . . reflects a value judgement on the extent one can rely on beliefs of limited confidence in important decisions.

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Hence:

The beliefs used in a decision may depend on the stakes involved . . .

Some consequences . . .

- The decision maker's cautiousness coefficient . . .

  . . . reflects a value judgement on the extent one can rely on beliefs of limited confidence in important decisions.

Hence:

The beliefs used in a decision may depend on the stakes involved . . .

. . . but the beliefs themselves don't.

Some consequences . . .

Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation (1992 Rio declaration).

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Some interpretations:

"Prescriptive" A decision rule: threat + uncertainty ⇒ precautionary action

"Argumentative" A rule of dialogue: lack of evidence cannot be used as an argument

"Epistemic" Rules for beliefs: what you believe depends on the purposes or stakes

Some consequences . . .

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  . . . but the beliefs themselves don't.
- The decision maker's cautiousness coefficient . . .
  . . . reflects a value judgement on the extent one can rely on beliefs of limited confidence in important decisions.

Conclusion

The higher the stakes involved in a decision, the more confidence is needed in a belief for it to play a role in the decision.

We have:

- a maxim concerning the role of confidence in choice
- a formal model of confidence in beliefs and a family of decision rules embodying the maxim
- these rules have attractive conceptual and choice-theoretic properties: intuitiveness, separation of beliefs and desires, reasonable consequences for choice.
- they may have interesting consequences for high-stakes decision making.

Thank you.

Confidence in Beliefs and Decision Making

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Games and Decisions
8-10 July 2013

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Why confidence?

Public decision making

The governor must decide whether to allow a factory project

- fumes from the factory could affect district farming area.
- probabilities controversial, but he retains estimate of 10^{-5}.
- with this probability, project retained.

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- consequences are larger by a factor of a thousand, in governor's opinion.

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- with this probability, project retained.

The governor must decide whether to allow a GM crops project

- probability of infecting non-GM area same as probability of fumes arriving there.
- consequences are larger by a factor of a thousand, in governor's opinion.
- Yet it is not *prima facie* unreasonable to turn down the project!

back Intro back Axioms

Why confidence?

Public decision making

Stereotyped version

Urn with 10^6 balls; at least 990000 blue and at least 1 red.
Advisers' estimate: at most 10 are red.

	Colour of ball drawn from urn	
	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
p_0	0	0

f: factory; g: GM crops.

Preferences: $f \succ p_0$ and $g \prec p_0$.

back Intro back Axioms

Preliminaries

The Anscombe-Aumann framework

S	non-empty finite set of states
$\Delta(\Sigma)$	set of probability measures on S
X	nonempty set of outcomes
$\Delta(X)$	set of consequences
$\mathcal{A}$	set of acts (functions $S \to \Delta(X)$)
$\preceq$	preference relation on $\mathcal{A}$

Notation:

- $u(f(s)) = \sum_{x \in supp(f(s))} f(s)(x)u(x)$.
- $f_\alpha g$: shorthand for $\alpha f + (1-\alpha)g$.
- $\leqq$: the stakes relation.
- $f \asymp g$: $f \leq g$ or $f \geq g$.

Incomplete Preference

Main axioms

C-completeness For all $c, d \in \Delta(X)$, $c \asymp d$.

S-Transitivity For all $f, g, h, e, e' \in \mathcal{A}$, $\alpha, \beta \in (0,1]$ such that $(f,h) \leqq (f_\alpha e, g_\alpha e)$ or $f(s) \sim g(s)$ for all $s \in S$, and $(f,h) \leqq (g_\beta e', h_\beta e')$ or $g(s) \sim h(s)$ for all $s \in S$, if $f_\alpha e \preceq g_\alpha e$ and $g_\beta e' \preceq h_\beta e'$, then $f \preceq h$.

Independence For all $f, g, h \in \mathcal{A}$ and for all $\alpha \in (0,1)$ such that $f \asymp g$ and $f_\alpha h \asymp g_\alpha h$, $f \preceq g$ if and only if $f_\alpha h \preceq g_\alpha h$.

Consistency For all $f, g, h \in \mathcal{A}$ and $\alpha \in (0,1)$ such that $(f_\alpha h, g_\alpha h) \leqq (f, g)$, if $f \asymp g$, then $f_\alpha h \asymp g_\alpha h$.

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Choice-theoretic properties

Crucial axioms

EU and unanimity preferences (Bewley, 2002):

Transitivity If $f \preceq g$ and $g \preceq h$, then $f \preceq h$

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[a] $f \asymp g$: $f \preceq g$ or $f \succeq g$.

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(i) if $f \preceq g$ and $g \preceq h$, then $f \asymp h$[a]

(ii) moreover, in such a case, $f \preceq h$

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Current proposal:

(i) No: can have $f \preceq g$ and $g \preceq h$ but $f \not\asymp h$

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Choice-theoretic properties

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(ii) Yes: can never have $f \preceq g$, $g \preceq h$ and $f \succ h$.

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Choice-theoretic properties

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Current proposal:

S-transitivity If $f \preceq g$ and $g \preceq h$ when the stakes are higher than for (f, h), then $f \preceq h$

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(ii) Yes: can never have $f \preceq g$, $g \preceq h$ and $f \succ h$.

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Choice-theoretic properties

EU and unanimity preferences:

Independence For all $f, g, h \in \mathcal{A}$ and $\alpha \in (0,1)$, $f \preceq g$ iff $f_\alpha h \preceq g_\alpha h$.

Choice-theoretic properties

Crucial axioms

EU and unanimity preferences and current proposal:

Independence For all $f, g, h \in \mathcal{A}$ and $\alpha \in (0,1)$ such that $f \asymp g$ and $f_\alpha h \asymp g_\alpha h$, $f \leq g$ iff $f_\alpha h \leq g_\alpha h$.

Choice-theoretic properties

EU and unanimity preferences and current proposal:

Independence The standard condition holds whenever the preferences in question are determinate.

Choice-theoretic properties

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Current proposal:

Consistency When the stakes decrease, one cannot suspend (determinate) preferences.

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Choice-theoretic properties

Crucial axioms

EU and unanimity preferences and current proposal:

Independence The standard condition holds whenever the preferences in question are determinate.

Current proposal:

Consistency When the stakes decrease, one cannot suspend (determinate) preferences.

Conclusion

- Independence is not violated by this member of the proposed family of decision rules.
- Only a mild weakening of transitivity, and a consistency axiom needed to take account of the effect of stakes on determinacy of preference.

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Careful preferences

Main axioms

S-independence For all $f \in \mathcal{A}$, $c, d \in \Delta(X)$ and $\alpha \in (0,1)$,

(i) if $d \succeq \hat{f}$, then $f \succeq c$ implies $f_\alpha d \succeq c_\alpha d$

(ii) if $d \preceq \hat{f}$, then $f \preceq c$ implies $f_\alpha d \preceq c_\alpha d$

S-Monotonicity For all $f, g \in \mathcal{A}$, $c, d \in \Delta(X)$ and $\alpha \in (0,1]$ with $\hat{f} \sim \widehat{g_\alpha d}$ and $g_\alpha d \sim c_\alpha d$, if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq c$, and if $f(s) \succeq g(s)$ for all $s \in S$, then $f \succeq c$.

S-Uncertainty Aversion For all $f, g \in \mathcal{A}$, $c, d \in \Delta(X)$ and $\alpha, \beta \in (0,1)$ with $\hat{f} \sim \hat{g} \sim \widehat{(f_\alpha g)_\beta d}$, if $f \sim g \sim c$ then $(f_\alpha g)_\beta d \succeq c_\beta d$.

Choice theoretic properties

S-independence

C-independence $f \preceq g$ iff $f_\alpha c \preceq g_\alpha c$

S-independence (i) if $f_\alpha d$ involves lower stakes than f, then $f \succeq c$ implies $f_\alpha d \succeq c_\alpha d$

(ii) if $f_\alpha d$ involves higher stakes than f, then $f \preceq c$ implies $f_\alpha d \preceq c_\alpha d$

	Colour of ball drawn from urn	
	Blue	Red
f	10 000	-1 M
g	10 M	-1 000 M
p_0	0	0

C-independence $f \succeq p_0 \Leftrightarrow g \succeq p_0$.

S-independence $g \succeq p_0 \Rightarrow f \succeq p_0$, but not vice versa.

Choice-theoretic propertites

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Monotonicity For all $f, g \in \mathcal{A}$, if $f(s) \preceq g(s)$ for all $s \in S$, then $f \preceq g$.

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- c is the 'certainty equivalent' of g when evaluated at stakes corresponding to f
- So the axiom says:
  - if g dominates f, then it is preferred to f when it is evaluated at the stakes level of f
  - if g is dominated by f, then f is preferred to it when it is evaluated at the stakes level of f.

Choice-theoretic propertites

S-Uncertainty Aversion

Uncertainty Aversion For all $f, g \in \mathcal{A}$, $\alpha \in (0,1)$, if $f \sim g$ then $f_\alpha g \succeq f$.

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S-Uncertainty Aversion	Uncertainty Aversion
$(A, -100; 400) \sim (A, -25; 50)$	$(A, -100; 400) \sim (A, -25; 50)$
$(A, -81.25; 312.5) \succeq$ both	$(A, -81.25; 312.5) \succeq$ both

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Confidence equivalence

Definition

Let $\preceq^1$ and $\preceq^2$ be preference relations satisfying the axioms. $\preceq^1$ and $\preceq^2$ are *confidence equivalent* if (i) for all $c, d \in \Delta(X)$, $c \preceq^1 d$ iff $c \preceq^2 d$ and (ii) for all $d \in \Delta(X)$, there exists $d' \in \Delta(X)$ and $\alpha \in (0,1)$ such that, for all $f \in \mathcal{A}$ and $c \in \Delta(X)$ with $\hat{f} \sim d$, $f \succeq^1 c$ iff $f_\alpha d' \succeq^2 c_\alpha d'$.

Proposition

Let $\preceq^1$ and $\preceq^2$ be preference relations satisfying the axioms and represented by utility functions, confidence rankings and cautiousness coefficients (u_1, Ξ_1, D_1) and (u_2, Ξ_2, D_2) respectively. $\preceq^1$ and $\preceq^2$ are confidence equivalent if and only if u_1 is a positive affine transformation of u_2 and $\Xi_1 = \Xi_2$.

back

Attitudes to confidence

Aversion to choosing in the absence of confidence

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In a nutshell

1 is more averse to choosing in the absence of confidence than 2 if he ceases to prefer f over c at lower stakes.

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1 is more averse to choosing in the absence of confidence than 2 if he ceases to prefer f over c at lower stakes.

Definition

Suppose $\preceq^1$ and $\preceq^2$ satisfy the axioms and are represented by the same u and Ξ. Then $\preceq^1$ is **more averse to choosing in the absence of confidence** than $\preceq^2$ if, for all $f \in \mathcal{A}$, $c, d, e \in \Delta(X)$ and $\alpha \in (0,1]$, if $f_\alpha d \succeq^1 c_\alpha d$ whenever $\widehat{f_\alpha d} \succeq^1 e$, then $f_\alpha d \succeq^2 c_\alpha d$ whenever $\widehat{f_\alpha d} \succeq^2 e$.

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Attitudes to confidence

Ambiguity Aversion

Attitudes to confidence

Ambiguity Aversion

Definition

Suppose $\preceq^1$ and $\preceq^2$ satisfy the axioms and are represented by the same u and Ξ. Then $\preceq^1$ is **more ambiguity averse** than $\preceq^2$ if, for any $f \in \mathcal{A}$ and $c \in \Delta(X)$, if $f \succeq^1 c$ then $f \succeq^2 c$.

Attitudes to confidence

Ambiguity Aversion

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- Standard (Ghirardato and Marinacci, 2002; Klibanoff et al., 2005).

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Attitudes to confidence

Characterisation

Theorem

Suppose that $\preceq^1$ and $\preceq^2$ satisfy axioms and are represented by the same u and Ξ. The following are equivalent:

(i) $\preceq^1$ *is more averse to choosing in the absence of confidence than* $\preceq^2$

(ii) $\preceq^1$ *is more ambiguity averse than* $\preceq^2$

(iii) $s_1(f) \geqslant s_2(f)$ *for all* $f \in \mathcal{A}$

(iv) $D_1(r) \supseteq D_2(r)$ *for all* $r \in \Re$.

back

Attitudes to confidence

Characterisation

Conclusion:

- the cautiousness coefficient D fully captures the agent's attitude to choosing in the absence of confidence
- there is separation of beliefs and tastes (Ξ plays no role)
- in this model, attitude to choosing in the absence of confidence is equivalent to the "standard" notion of ambiguity attitude

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Decisiveness and attitudes to confidence

Definition

Let $\preceq$ satisfy the axioms. $(f,g) \preccurlyeq (f',g')$ if

$$f_\alpha h \succeq g_\alpha h \Rightarrow f'_{\alpha'} h' \succeq g'_{\alpha'} h'$$

whenever $(f_\alpha h, g_\alpha h) \equiv (f'_{\alpha'} h', g'_{\alpha'} h')$.

$\preceq^1$ and $\preceq^2$ are *confidence equivalent* if $\preccurlyeq^1 = \preccurlyeq^2$.

back

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whenever $(f_\alpha h, g_\alpha h) \equiv (f'_{\alpha'} h', g'_{\alpha'} h')$.

$\preceq^1$ and $\preceq^2$ are *confidence equivalent* if $\preccurlyeq^1 = \preccurlyeq^2$.

Proposition

$\preceq^1$ and $\preceq^2$ are confidence equivalent iff u_2 is a positive affine transformation of u_1, and $\Xi_1 = \Xi_2$.

▸ back

Decisiveness and attitudes to confidence

Definition

$\preceq^1$ is **less decisive** than $\preceq^2$ if, for all $f, g \in \mathcal{A}$, $f \preceq^1 g \Rightarrow f \preceq^2 g$.

Proposition

Suppose that $\preceq^1$ *and* $\preceq^2$ *satisfy the axiomsand are confidence equivalent. The following are equivalent:*

(i) $\preceq^1$ *is less decisive than* $\preceq^2$

(ii) $D_2((f,g)) \subseteq D_1((f,g))$ *for all* $(f,g) \in \mathcal{A}^2$.

back

Deferral and forced choice

$\preceq^d$ (deferral present) satisfies axioms above

back

back2

Deferral and forced choice

$\preceq^d$ (deferral present) satisfies axioms above

$\preceq^n$ (deferral absent) is complete

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$\preceq^d$ (deferral present) satisfies axioms above

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and

Benchmark on certainty

For all $f, g \in \mathcal{A}$, if there is no $c \in \Delta(X)$ such that $f_\alpha h \succeq^d c_\alpha h$ but $g_{\alpha'} h' \not\succeq^d c_{\alpha'} h'$ for some $h, h' \in \mathcal{A}$ and $\alpha, \alpha' \in (0, 1]$ with $\sigma(f_\alpha h, c_\alpha h) = \sigma(g_{\alpha'} h', c_{\alpha'} h') = \sigma(f, g)$, then $g \not\prec^n f$.

back back2

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if and only if, for all $f, g \in \mathcal{A}$, $f \preceq^n g$ iff

$$\min_{p \in D(\sigma(f,g))} \sum_{s \in S} u(f(s)).p(s) \leqslant \min_{p \in D(\sigma(f,g))} \sum_{s \in S} u(g(s)).p(s)$$

back back2

Properties of $\leqq$

(Weak Order) $\leqq$ is reflexive, transitive and complete.

(Symmetry) for all $f, g \in \mathcal{A}$, $(f, g) \equiv (g, f)$.

(Extensionality) for all $f, f', g, g' \in \mathcal{A}$, if $f(s) \sim f'(s)$ and $g(s) \sim g'(s)$ for all $s \in S$, then $(f, g) \equiv (f', g')$.

(Continuity) For all $f, f', g, g', h \in \mathcal{A}$, the sets $\{(\alpha, \beta) \in [0,1]^2 |\ (f_\alpha h, g_\beta h) \geqq (f', g')\}$ and $\{(\alpha, \beta) \in [0,1]^2 |\ (f_\alpha h, g_\beta h) \leqq (f', g')\}$ are closed in $[0,1]^2$.

(Richness) For all $f, f', g, g' \in \mathcal{A}$ such that $f(s) \nsim g(s)$ for some $s \in S$ and $f'(s) \nsim g'(s)$ for some $s \in S$, there exists $h, h' \in \mathcal{A}$ and $\alpha, \alpha' \in (0,1]$ such that $(f_\alpha h, g_\alpha h) \leqq (f', g') \leqq (f_{\alpha'} h', g_{\alpha'} h')$.

back