

A SCHRÖDINGER-MAXWELL SYSTEM

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In this talk Ω is a bounded open set in \mathbb{R}^N , $N \geq 2$, $M(x)$ is a symmetric measurable matrix such that, for $\alpha, \beta \in \mathbb{R}^+$,

$$\alpha|\xi|^2 \leq M(x)\xi\xi, \quad |M(x)| \leq \beta.$$

1. PAPER DEDICATED TO BENCI-FORTUNATO ON HIS "130TH" BIRTHDAY

Let $A, r > 0$ and $f(x) \in L^m(\Omega)$, $m \geq \frac{2N}{N+2}$ ("variational framework"), existence and "unexpected" summability properties of the solutions of

$$(1.1) \quad \begin{cases} 0 \leq u \in W_0^{1,2}(\Omega) : -\operatorname{div}(M(x)Du) + A\varphi u^r = f(x), \\ 0 \leq \varphi \in W_0^{1,2}(\Omega) : -\operatorname{div}(M(x)D\varphi) = u^{r+1}, \end{cases}$$

are studied in a recent paper. Moreover

$$\varphi u^r \in L^1(\Omega), \quad u^{r+1} \in L^1(\Omega).$$

If $A = r+1$ and M is symmetric, a method for the proof of the existence is the study of the critical points of

$$\frac{1}{2} \int_{\Omega} MDvDv - \frac{1}{2} \int_{\Omega} MD\psi D\psi + \int_{\Omega} v|\psi|^{r+1} - \int_{\Omega} f v$$

thanks to a theorem by Benci-Rabinowitz (work in progress with D. Arcoya and T. Leonori).

2. WORK IN PROGRESS WITH L. ORSINA

In a work in progress a "surprising" (existence of solutions in $W_0^{1,2}(\Omega)$ even outside of "variational framework") existence result is proved if $m = \frac{r+1}{r} < \frac{2N}{N+2}$, $r > \frac{N+2}{N-2}$.