A stability result for the Pólya-Szegő inequality

Giovanni Pisante

It is well known that the Dirichlet integral $\int_{\mathbb{R}^n} |\nabla u|^p$ decreases under the Steiner symmetrization about an hyperplane or the Schwarz spherical symmetrization about a point. This property is indeed the consequence of a Pólya-Szegő type inequality. Recently several studies has been devoted to the issue of characterizing the extremals in this type of inequalities and this led the research through the natural question of proving quantitative versions. The aim of the talk is to shed some light on this direction presenting a stability result for the Pólya-Szegő inequality. We will discuss in particular some natural geometric conditions needed to deal with the fact that, even when the Dirichlet integral of a function u and of its symmetral coincide, u can be very different from its symmetral. The talk is based on a jont work with M. Barchiesi, N. Fusco and G.M. Capriani