# MINICOURSE TITLE: "HYPERBOLIC SAWTOOTH MAP" LECTURER: DMITRY DOLGOPYAT

The sawtooth map has the following form

$$x_{n+1} = \{x_n + y_n\}, \quad y_{n+1} = \{y_n + a x_{n+1} + b\}$$

where $\{\dots\}$ denotes the fractional part. If $a \notin (-4, 0)$ the sawtooth map has strong statistical properties. Namely it is ergodic (that is, it has no non-trivial invariant sets), mixing (that is, if $\phi, \psi \in L^2$ then the correlation function $\rho_{\phi,\psi}(n) = \int \phi(p_0)\psi(p_0)dp_0$ tends to 0), exponentially mixing (that is, $\rho_{\phi,\psi}(n)$ decays exponentially if $\phi$ and $\psi$ are Holder), and satisfies the Central Limit Theorem (that is, $\frac{\sum_{j=0}^{n-1}[\psi(p_j)-\int \psi(p)dp]}{\sqrt{n}}$ is asymptotically Gaussian for Holder $\psi$). Despite the simple form of the map, the proof of those results require some deep methods from smooth ergodic theory (Hopf chains, Growth Lemma for stable and unstable manifolds, coupling algorithm etc). We will use this map to illustrate some key ideas from hyperbolic dynamics.

If the time permits we will describe an application to piecewise linear Fermi-Ulam pingpong.

## Course outline.

- Dynamics for integer $a$. – Hopf argument – Existence of stable and unstable manifolds.
- Growth Lemma and local bounds on the sizes of invariant manifolds. – Hopf chains and ergodicity of sawtooth map.
- The square of the sawtooth map and weak mixing. – Couling algorithm and mixing. – Central Limit Theorem.

Our presentation will follow [1, 3, 4].

## References


[1] Chernov N., Markarian R. *Chaotic billiards,* Mathematical Surveys and Monographs **127** (2006) American Mathematical Society, Providence, RI.

[2] de Simoi J., Dolgopyat D. *Dynamics of some piecewise smooth Fermi-Ulam Models,* Chaos **22** (2012) paper 026124.

[3] Dolgopyat D. *Lectures on Bouncing Balls,* available at `http://www2.math.umd.edu/~dolgop/BBNotes.pdf`

[4] Liverani C., Wojtkowski M. P. *Ergodicity in Hamiltonian systems,* Dynamics reported **4** (1995) 130–202. Springer, Berlin.

[5] Ulam S. M. *On some statistical properties of dynamical systems,* In Proc. 4th Berkeley Sympos. Math. Statist., Prob., Vol. **III** (1961) 315–320.

[6] Zharnitsky V. *Instability in Fermi-Ulam "ping-pong" problem,* Nonlinearity **11** (1998) 1481–1487.