Simulating entanglement without communication

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- *Simulating entanglement with a few bits of communication*
- *How strong can no-signaling correlation be?*
- *The elementary NonLocal machine*
- *Simulating entanglement with the NonLocal machine*
- *(is Nature sparing with resources? Experiments)*
- *Partially entangled states are more non-local than the singlet !*

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Simulating entanglement with a few bits of communication (+ shared randomness)

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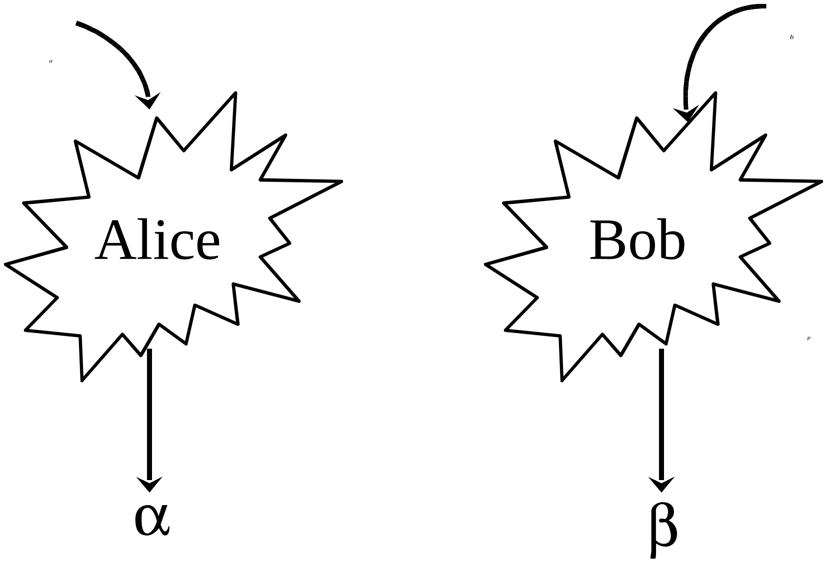
& define measurement bases.

The output α & β should reproduce the Q statistics:



<u>Case of singlet</u>: 8 bits, Brassard, Cleve, Tapp, PRL <u>83</u>, 1874 1999

2 bits, Steiner, Phys. Lett. A <u>270</u>, 239 2000,

Gisin s Phys. Lett. A <u>260</u>, 323, 1999

1 bit! Toner & Bacon, PRL <u>91</u>, 187904, 2003

0 bit: impossible (Bell inequality) ..but …

How strong can no-signaling correlation be?

CHSH-Bell inequality:

$E \quad a_1, b_1 \quad E \quad a_1, b_2 \quad E \quad a_2, b_1 \quad E \quad a_2, b_2 \quad 2$

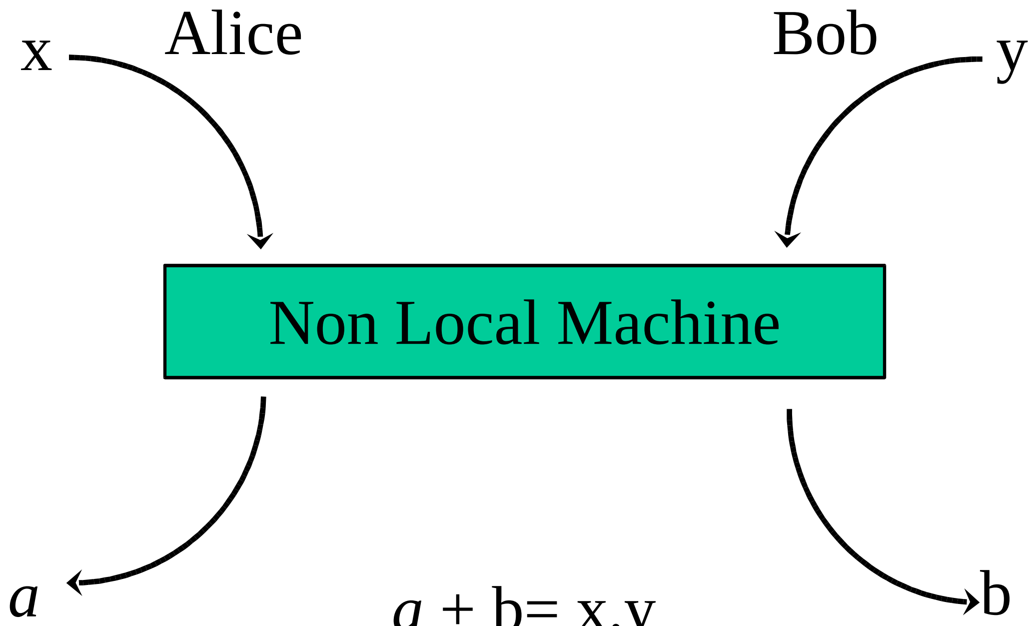
1) *Q correlation can violate Bell inequalities, but can't be used for signaling*
2) *Q correlation can't violate the CHSH-Bell inequality By more than a factor*

Are these two facts related? Could there be correlations that:

2) *Do not allow signaling, and*
3) *Do violate the CHSH inequality by more than ?*

Answer: yes! S. Popescu and D. Rohrlich, quant-ph/979026.

The NonLocal Machine

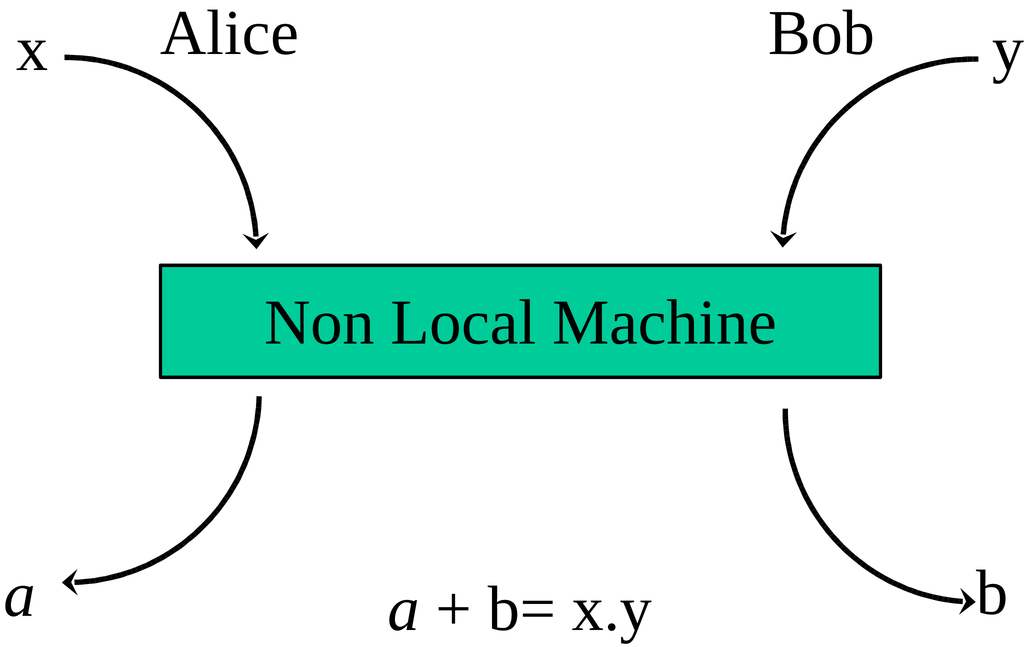


a + b= x.y

Prob(a=1|x,y) = ½, independent of y ⇒ no signaling

E(a,b|0,0) + E(a,b|0,1) + E(a,b|1,0) - E(a,b|1,1) = 4

The NonLocal Machine



$a + b = x.y$

A single bit of communication suffice to simulate the NL Machine (assuming shared randomness).
But the NL Machine does not allow any communication.

Hence, the NL Machine is a <u>strickly weaker ressource</u> than communication.

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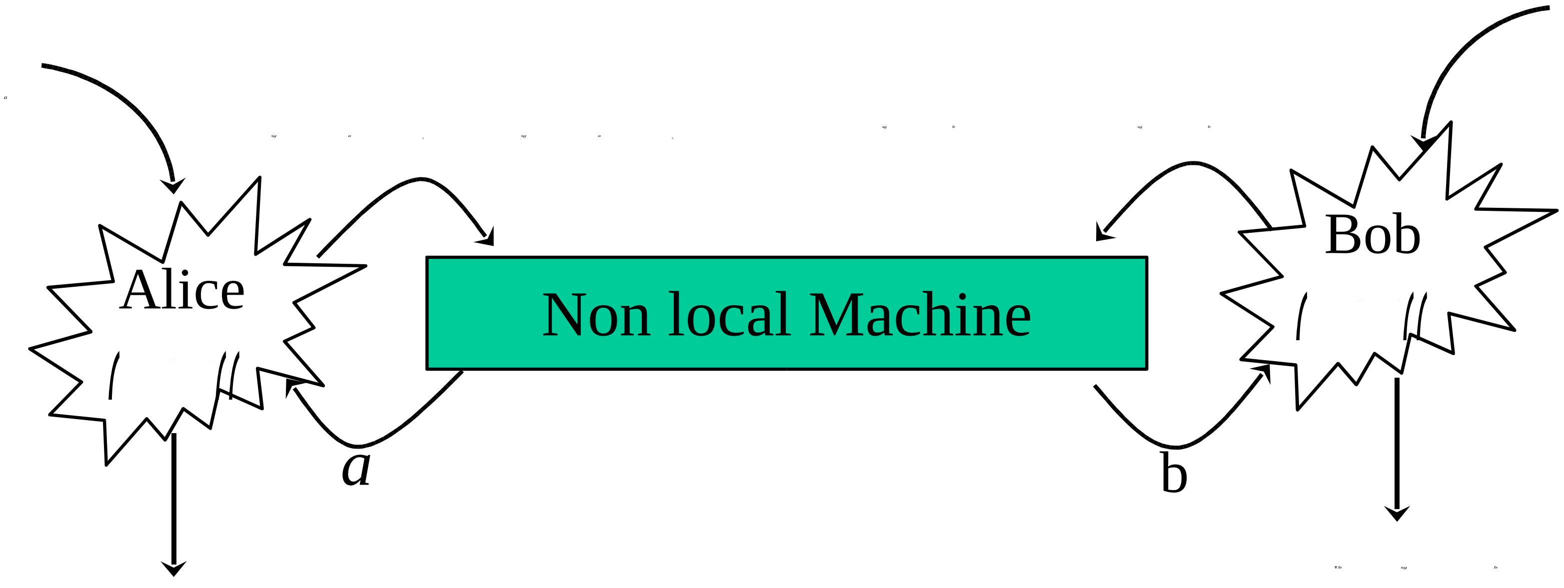
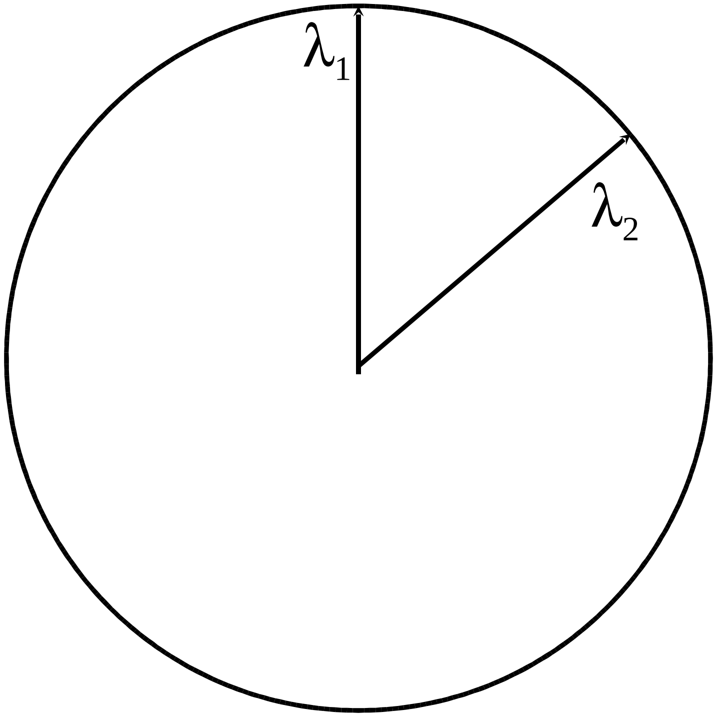
Simulating singlets with the NL Machine

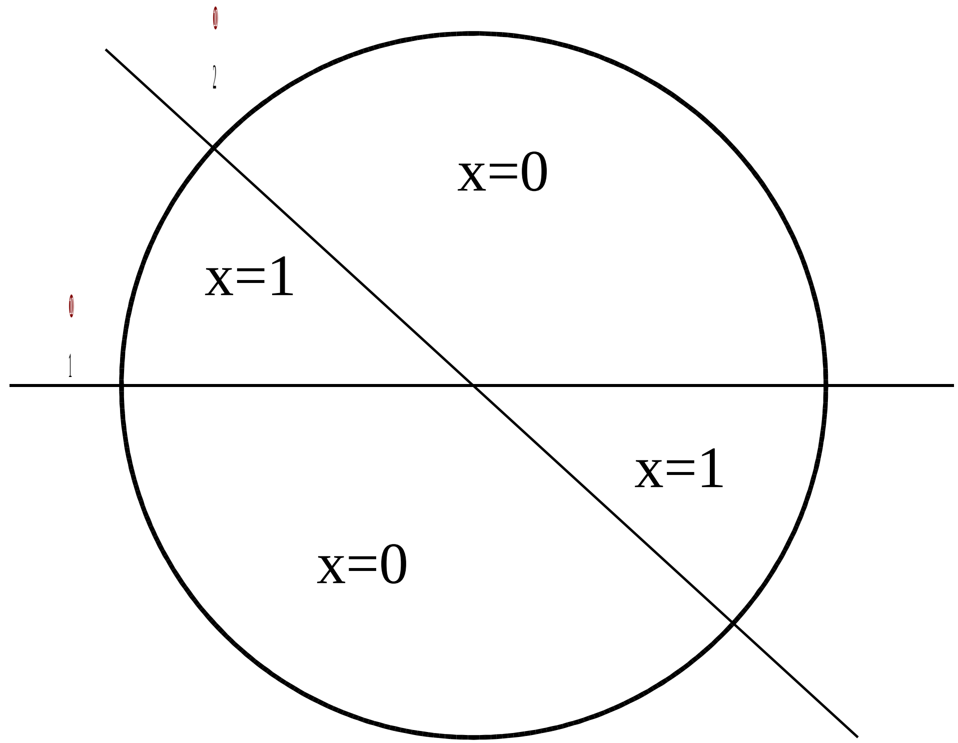


where ... and ... are distributed uniformly on S

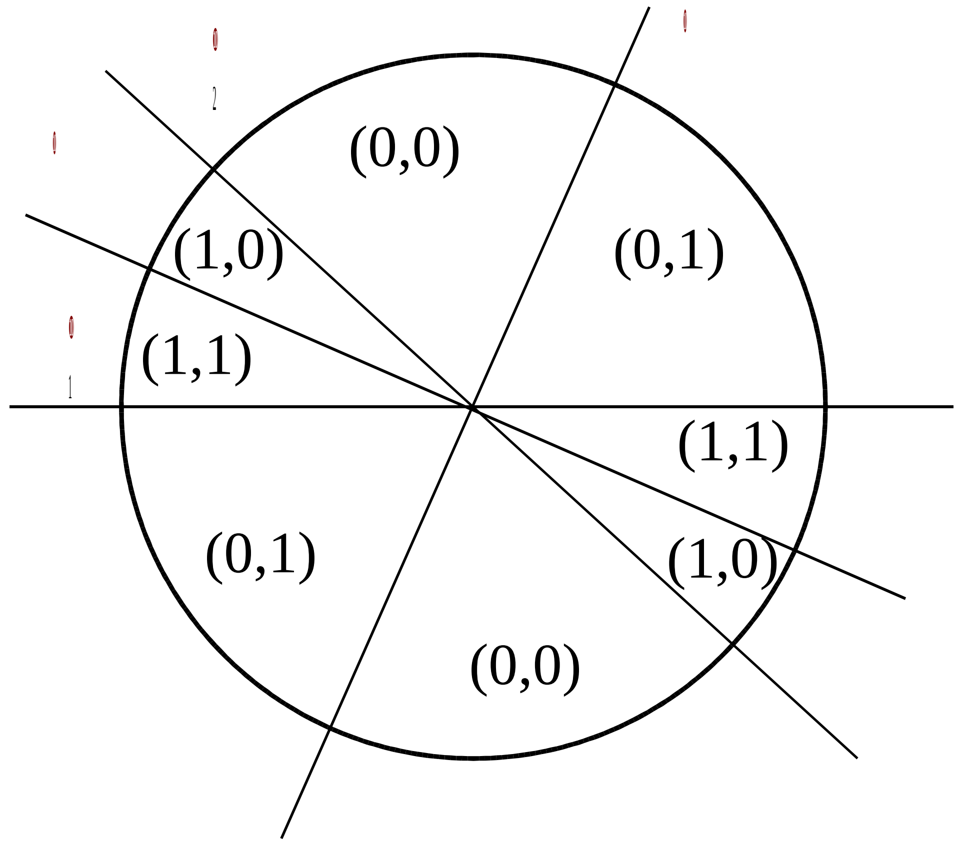
Given & , the statistics of α & β is that of the singlet state:

hint for the proof:




x=0
x=1
x=1
x=0

(x,y)



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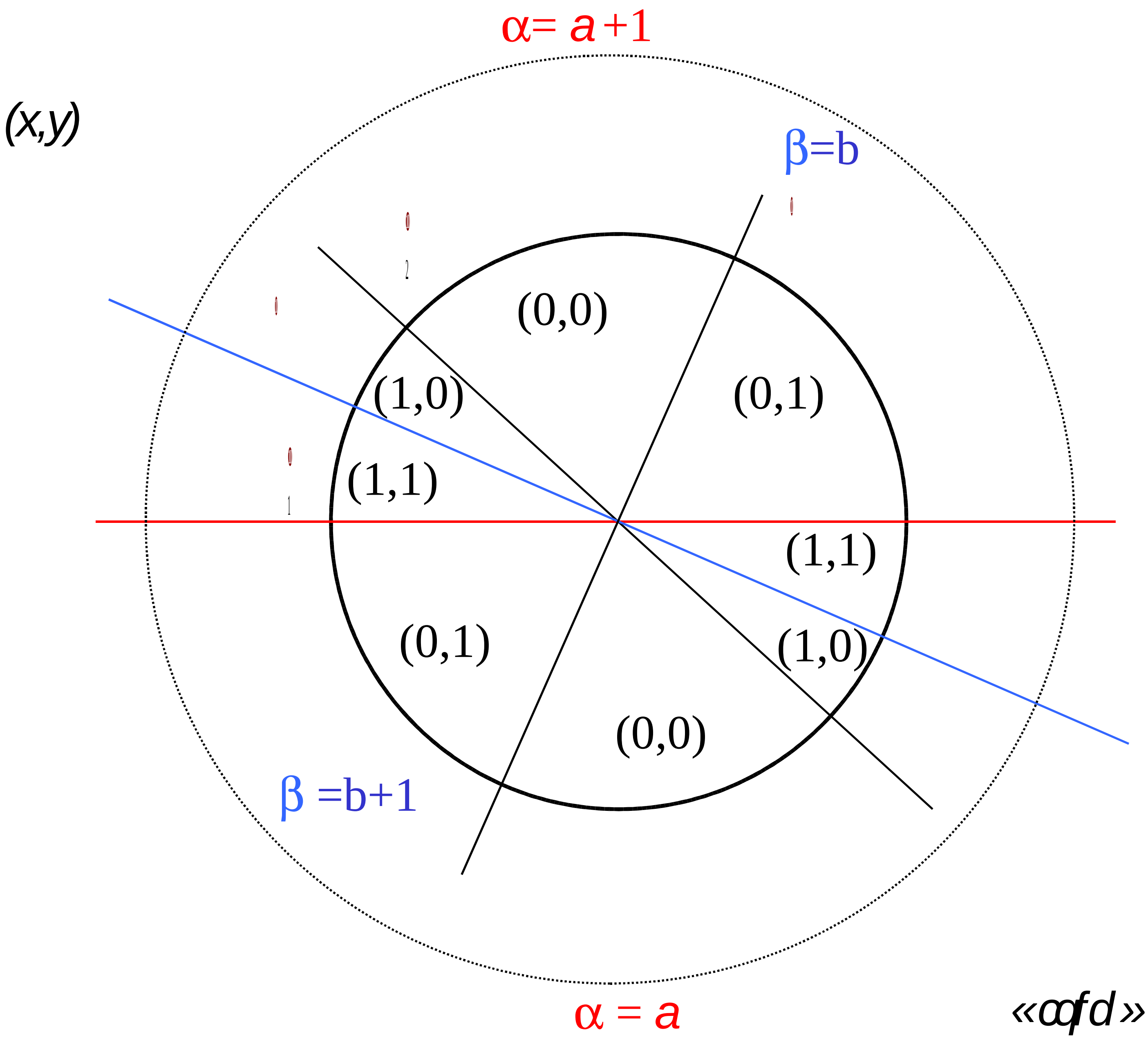
α= a+1
(x,y)
β=b
(0,0)
(1,0)
(0,1)
(1,1)
(1,1)
(0,1)
(1,0)
(0,0)
β =b+1
α = a

(

physical speculations…

How come the Correlations?

Is there a theory which explains how the correlations come (i.e. in which the correlations are derived from more primitive concepts) and in which there is no signaling?

Is Nature sparing with resources?

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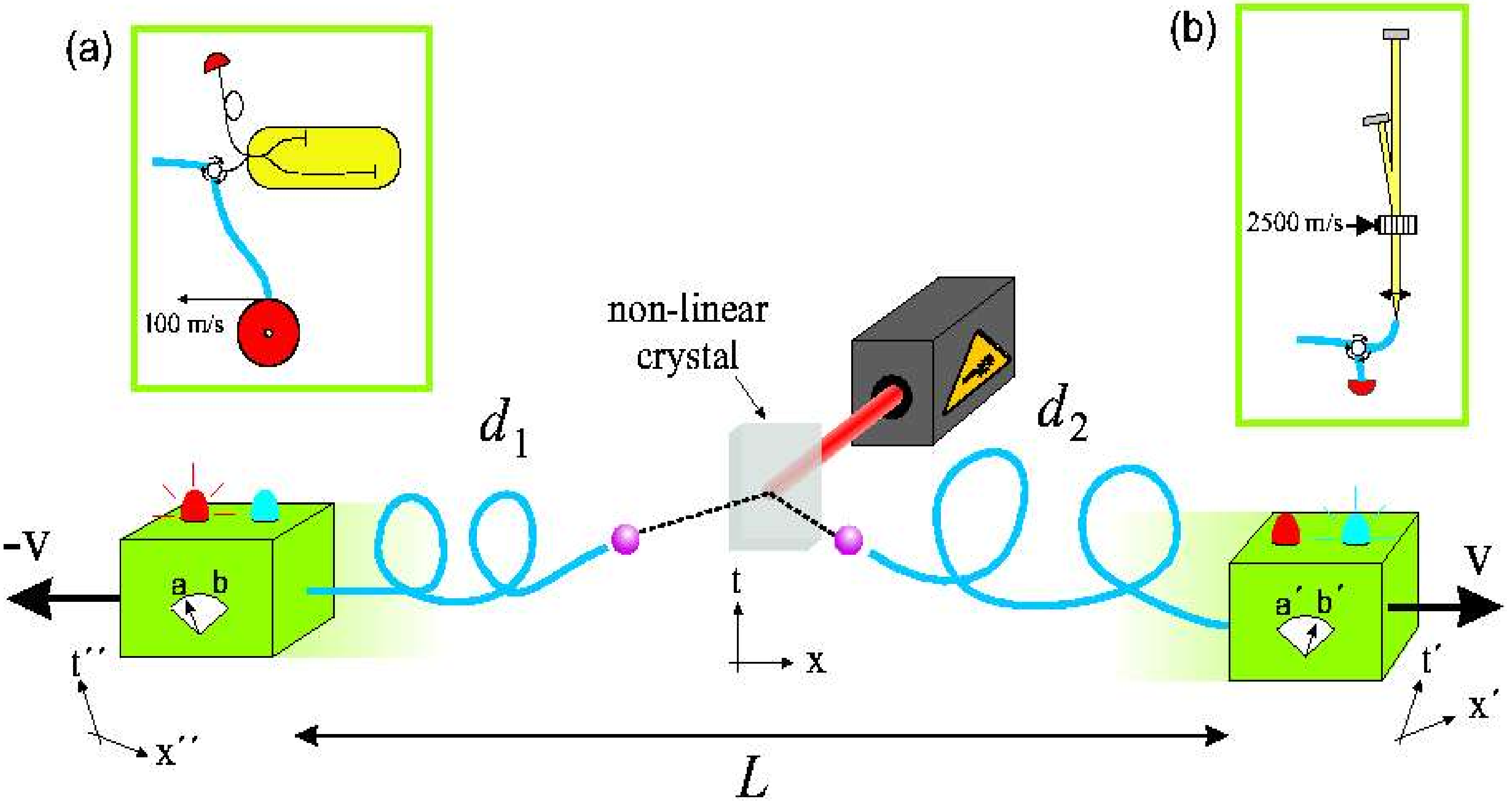
Experiments



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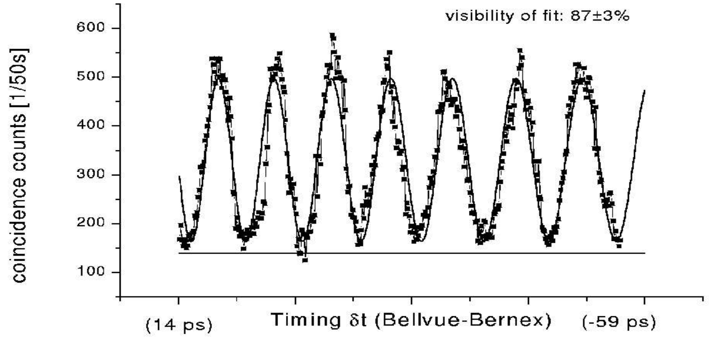
Experimental results

Simultaneous measurement in the reference frame determined by the swiss Alps:

$$_{QI} \qquad \frac{10\ km}{1\ mm}c \qquad 10^{7}c$$



Simultaneous measurement in the reference frame determined by the cosmic background radiation:

$$\upsilon_{QI} > 2.10^{4}\, c$$

The measurement devices are in relative motion such that each one, on its own inertial reference frame, detects its photon before the other:
2-photon interference fringes are still observed!

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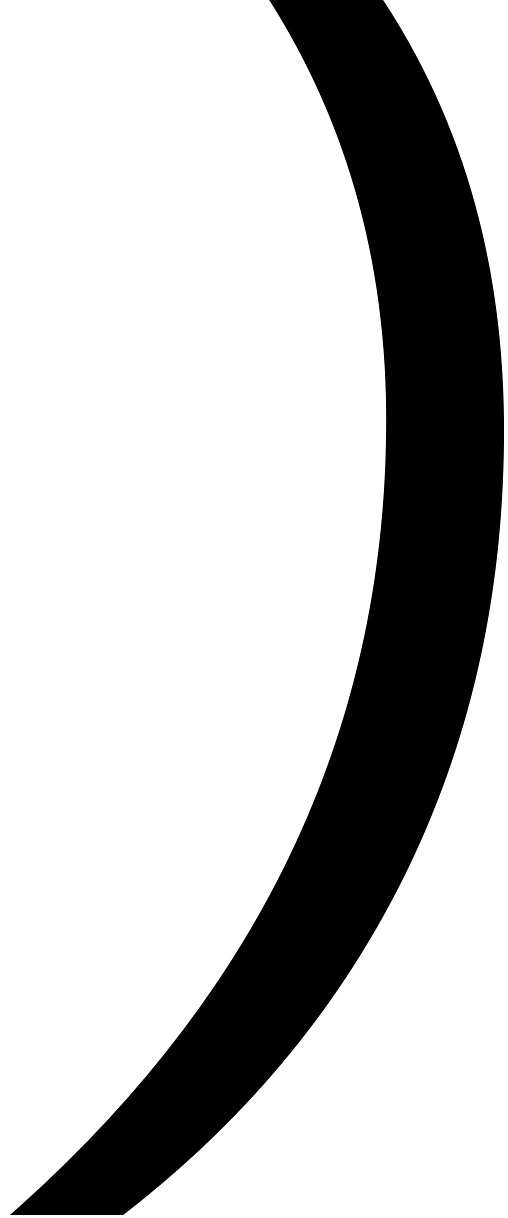
Correlation cry out for explanation!

a derivation from a more primitive concept

unlikely to be « hidden communication »

The NL-machine defines an elementary unit of nonlocality, a nl-bit.



back to solid stuff …

Simulating partial entanglement

Partially entangled states seem more nonlocal than the max entangled ones !

Partially entangled states are more robust against the detection loophole

Bell inequalities are more violated by partially entangled states than by max entangled ones (for dim > 2 & all known cases).

When testing Bell inequality, the use of a partially entangled state provides more information per experimental run than the use of max entangled states.
(work under progress by T. Acin, R. Gilles & N. Gisin)

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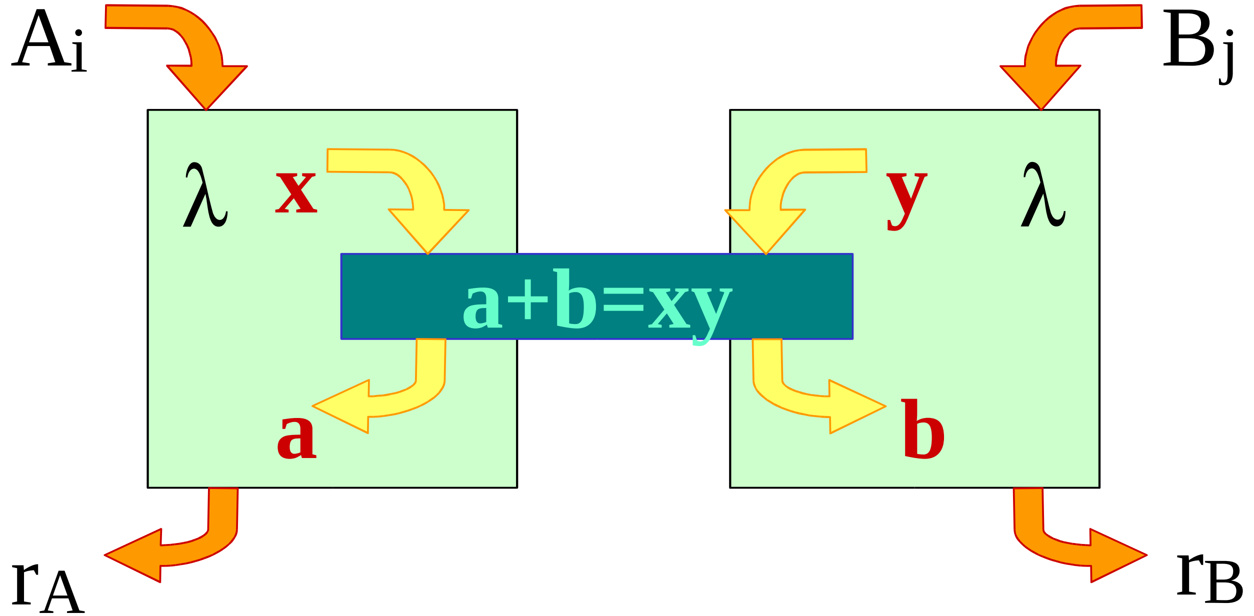
How to prove that some correlation can't be simulated with a single use of the nonlocal machine?

$\Rightarrow$ *same idea as Bell inequality, i.e.*

1. List all possible strategies
2. Notice that they constitute a convex set
3. Notice that this convex set has a finite number of extremal points (vertices), i.e. it's a polytope
4. Find the polytope's facets
5. Express the facets as inequalities



For given A_i *and* λ, *there are 6 extremal local strategies:*

1. $\mathbf{r_A}=0$ *3.* $X=0$ *and* $\mathbf{r_A=a}$ *5.* $X=0$ *and* $\mathbf{r_A=a+1}$

2. $\mathbf{r_A=1}$ *4.* $X=1$ *and* $\mathbf{r_A=a}$ *6.* $X=1$ *and* $\mathbf{r_A=a+1}$

For 2 settings per side, there are 6^4 *strategies defining 264 different vertices.*

The polytope is the same as the "no-signaling polytope" studied by J. Barrett et al in quant-ph/0404097

Consequently, no quantum state can violate such a 2-settings inequality

The 1 nl-bit inequality

For 3 settings per side:

- *there are 6^6 strategies defining 3880 different vertices.*
- *There is a unique new inequality:*

y \ x	-2	0	0
-2	+1	+1	+1
-1	+1	+1	-1
0	+1	-1	0

≤ 0

y \ x	$P(\mathbf{r}_A = 0 \mid x)$
$P(\mathbf{r}_B = 0 \mid y)$	$P(\mathbf{r}_A = \mathbf{r}_B = 0 \mid x,y)$

Recall: for standard Bell inequalities (i.e. with no nonlocal machines) and 3 settings per side, there is also a unique new inequality:

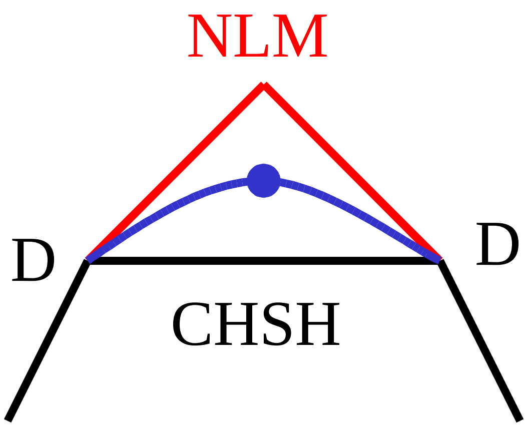
I3322 =

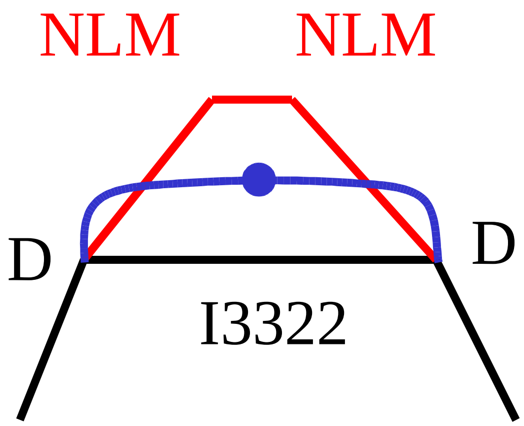
y \ x	-1	0	0
-2	+1	+1	+1
-1	+1	+1	-1
0	+1	-1	0

≤ 0

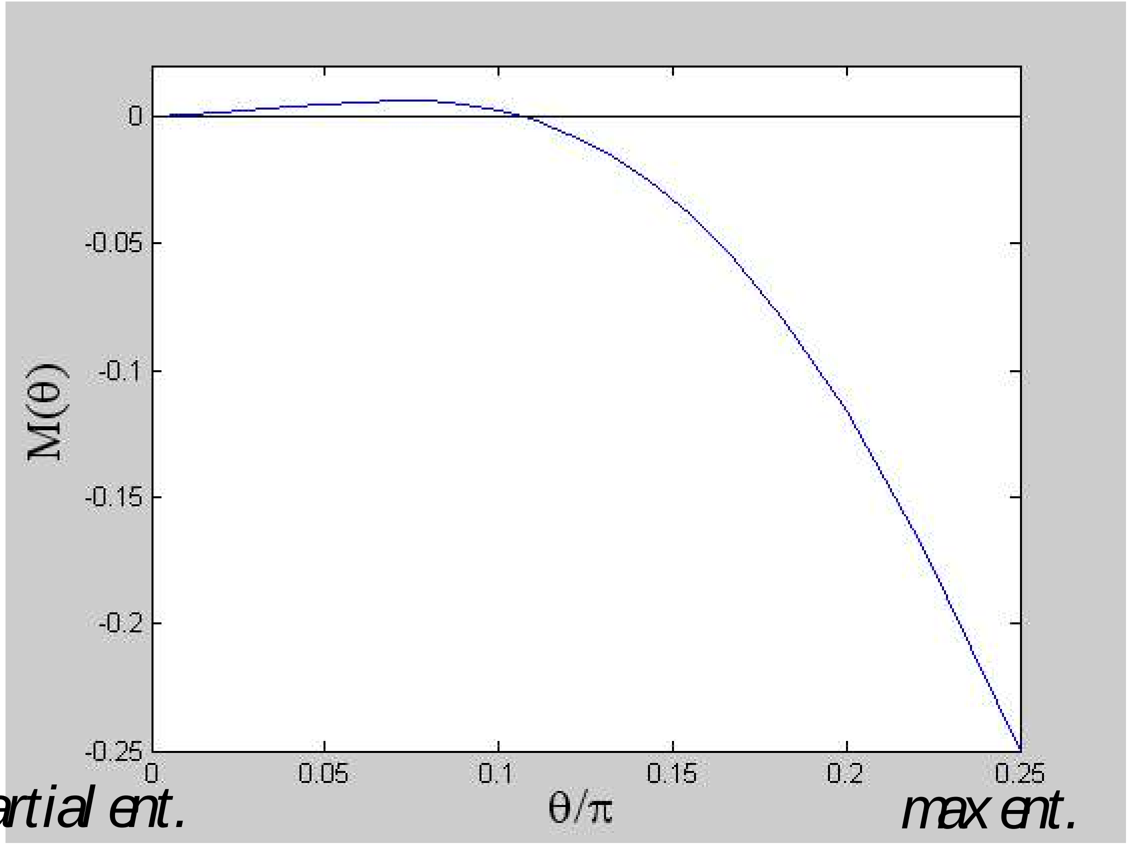
Collins & Gisin, J.Phys.A 37, 1775, 2004

Geometric intuition





Very partially entangled states do violate the 1-nl bit-Bell inequality :



partial ent. *max ent.*

- ***Very partially entangled states can't be simulated with only 1 nl-bit***
- ***Partially entangled states are <u>more nonlocal</u> than the singlet !***

Conclusions

- *One can simulate the singlet Q correlations with qualitatively less than a bit of communication.*
- *Causal (ie no signaling) correlation can be stronger than the Q correlation.*
- *The NL Machine inspired by the CHSH-Bell inequality suffice to simulate singlet correlation, quant-ph/0410027*
  (one instance of the NL Machine per simulated outcome).
- *Is Nature sparing with resources?*

Nonlocality ≠ entanglement

quant-ph 0412109

Simulating partial entanglement

Partially entangled states seem are nonlocal than the max entangled ones !

Nonlocality $\neq$ entanglement

The CHSH-Bell inequality is monogamous

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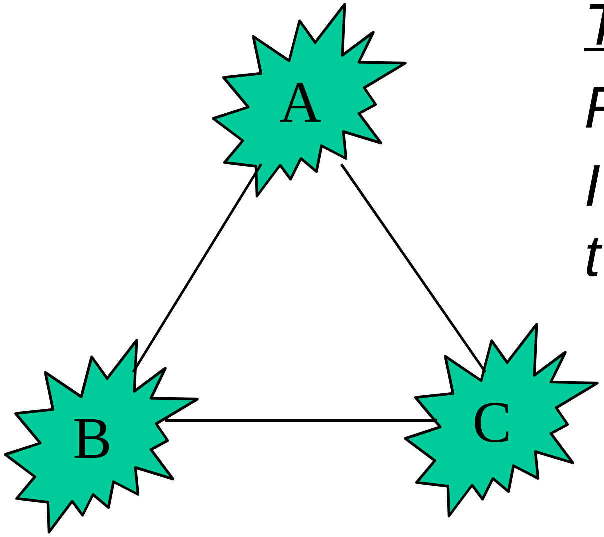


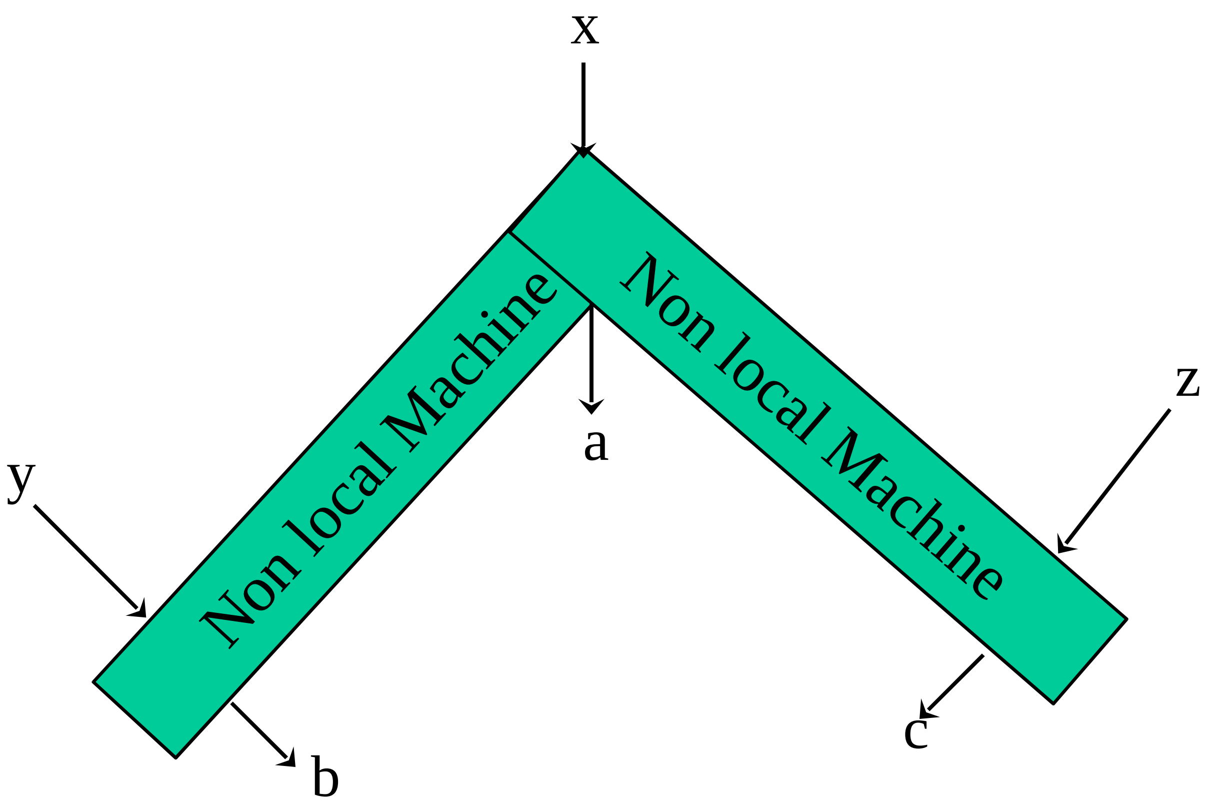
Theorem:

For all 3-qubit state Ψ_{ABC}*:*

If A-B violates the CHSH-Bell inequality then neither A-C nor B-C violates it.

(see V. Scarani & NG, PRL 87, 117901,2001, see also B. Therhal et al., PRL 90,157903,2003)

Causal NonLocal machines are monogamous



If a+b=x.y, a+c=x.z then b+c=x(y+z), and Alice can signal to B-C

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The new inequality for qubits with 3 settings

This is the only new inequality for 3 inputs and binary outputs.

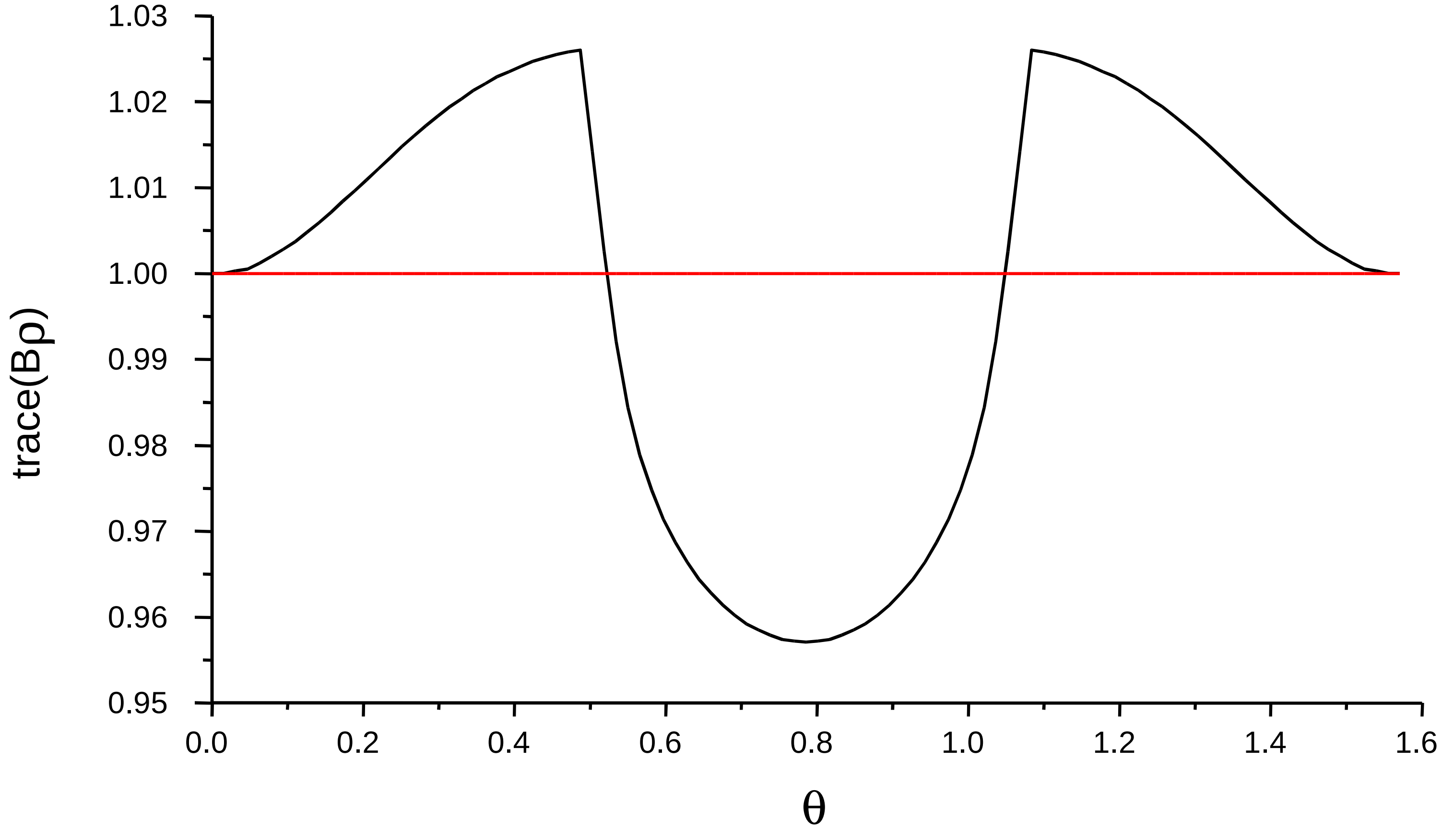
D. Collins et al., J.Phys. A 37, 1775-1787, 2004

$$
\begin{aligned}
I3322 = {} & P(a=0,b=0|x=0,y=0)+P(0,0|0,1)+P(0,0|0,2) \\
& + P(0,0|1,0)+P(0,0|1,1)-P(0,0|1,2) \\
& + P(0,0|2,0)-P(0,0|2,1) \\
& - P(a=0|x=0) - 2P(b=0|y=0) - P(b=0|y=1) \\
& \leq 0
\end{aligned}
$$

For each θ, let λ_{CHSH} be the critical weight such that

$$\rho(\theta) = \lambda_{CHSH} P_{\cos(\theta)|00\rangle+\sin(\theta)|11\rangle} + (1-\lambda_{CHSH}) P_{|01\rangle}$$

is at the limit of violating the CHSH inequality



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The I3322-Bell inequality is not monogamous

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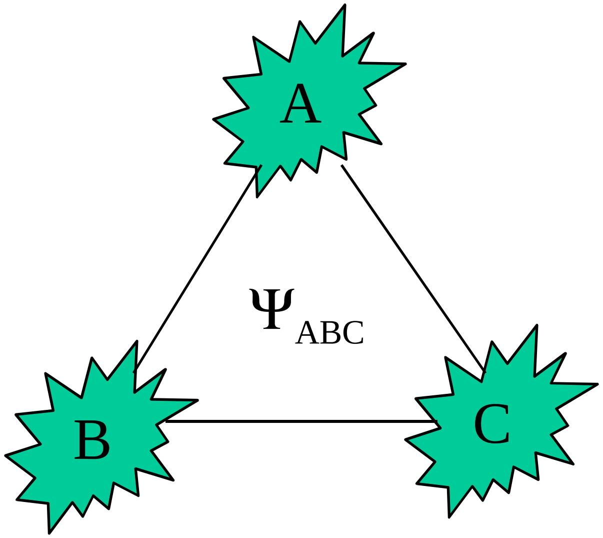
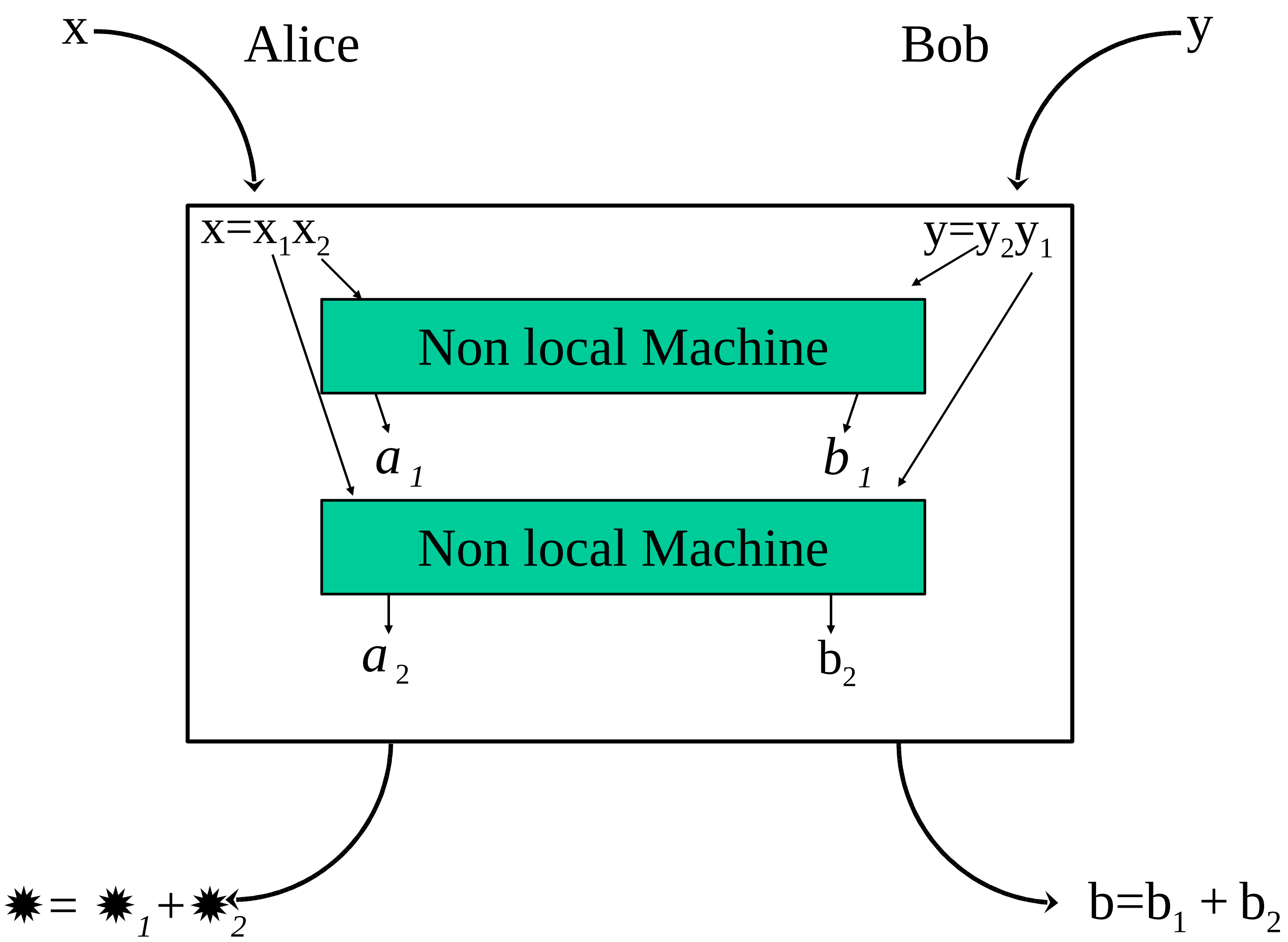


There exists a 3-qubit state Ψ_{ABC}, such that A-B violates the I3322-Bell inequality and A-C violates it also.

(see D. Collins et al., J.Phys. A 37, 1775-1787, 2004)

The NonLocal machine optimal for the I3322 Bell inequality



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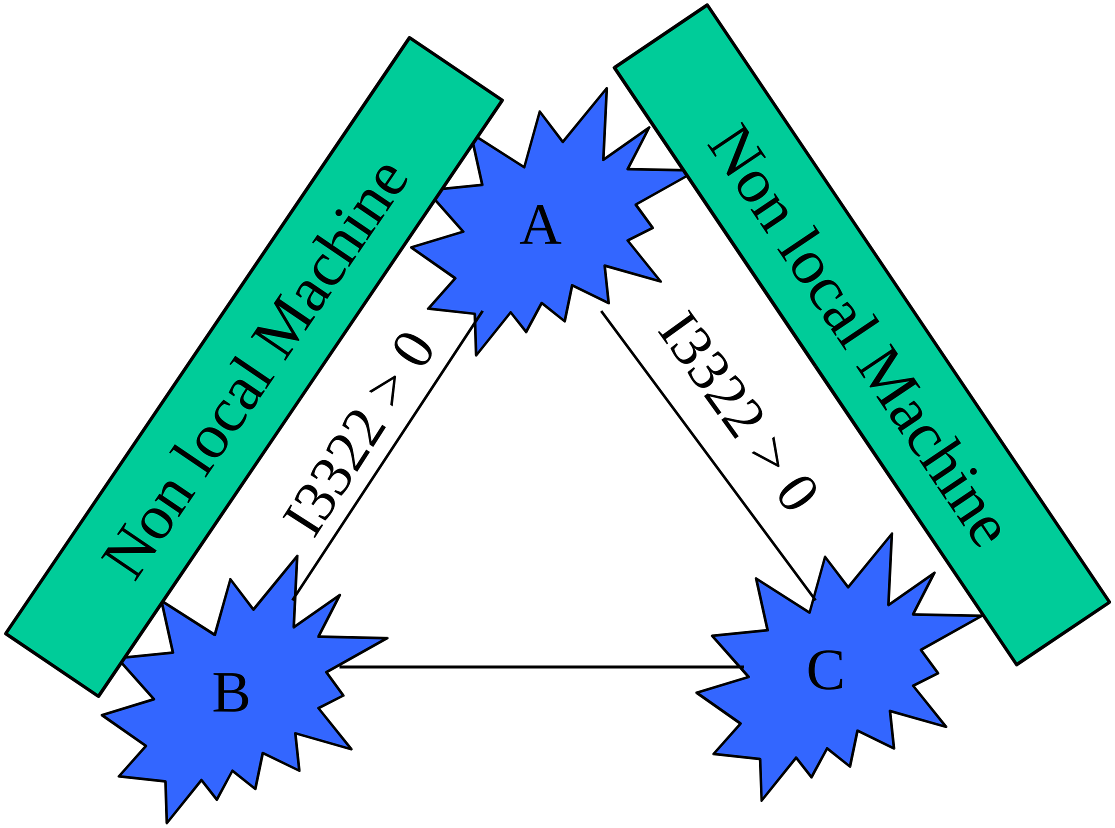
The NL Machine I3322 is not monogamous



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