ENTANGLEMENT IN SOLIDS

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In Collaboration With…

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- *Beatrix Hiesmayr, Vienna*
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- *Acknowledements: T. C. Wei, S. Bose, A. Fisher…*

SUMMARY OF TALK

- Second Quantisation Entanglement.
- Simple Solid State Models.
- Spin and Space - Particle Statistics.
- Superconductivity.
- Macroscopic Witnesses.
- Experiments.
- Future directions.

Macroscopic Entanglement?

Ground state of some one-dimensional models can contain (two-spin) entanglement even in the thermodynamical limit of infinitely many spins!

Can we have high temperature (say 200K) macroscopic entanglement in nature as well?

Vedral, C. Eu. J. Phys (2003),
Vedral, N. J. Phys (2004).

"Physics is like sex. Sure, it may give some practical results, but that's not why we do it." - Feynman

Second Quantisation

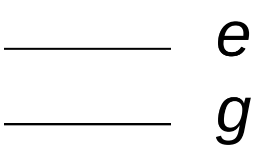
In order to study entanglement of identical particles need to talk about modes – second quantisation.

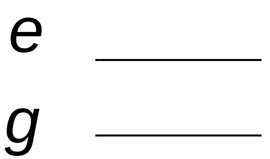
$$|0\rangle = |0\rangle_{k_1}|0\rangle_{k_2}|0\rangle_{k_3}|0\rangle|0\rangle|0\rangle\ldots \Rightarrow$$

$$(a^{\dagger}{}_{k_i} + a^{\dagger}{}_{k_j} + \ldots)|0\rangle$$

Different view: Ghirardi & Marinatto

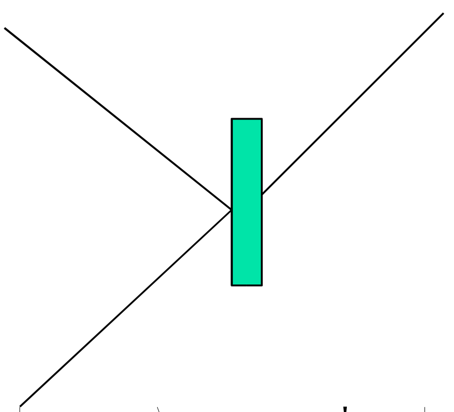
"Single particle" entanglement





$$|e\rangle_1 |g\rangle_2 + |g\rangle_1 |e\rangle_2$$

$$|\Psi\rangle = (b^\dagger{}_1 + b^\dagger{}_2)|0\rangle = |1\rangle_{b1}|0\rangle_{b2} + |0\rangle_{b1}|1\rangle_{b2}$$



$$|\Psi_{in}\rangle = (a^\dagger{}_1)|0\rangle$$

Computing Entanglement

Given the state $\rho_{123...N}$

Trace out all but "i" and "j" sites: $\rho_{ij} = tr_{\neq ij}\,\rho_{123...N}$

Entanglement:

$$E(\rho_{ij}) = \min_{sep} S(\rho_{ij} \,||\, \rho_{sep})$$

More difficult for more spins…

Example 1: Fermi Sea at T=0

$$|\Psi\rangle = \prod_{ps} a^{\dagger}{}_{s}(p)|0\rangle$$

$$[a^{\dagger}{}_{s}(p), a^{\dagger}{}_{t}(q)]_{+} = \delta_{st}\delta(p-q)$$

Density for two spins one at r and other at r':

$$\rho_{ss';tt'} = \langle\Psi|\psi^{\dagger}{}_{t'}(r')\psi^{\dagger}{}_{t}(r)\psi_{s'}(r')\psi_{s}(r)|\Psi\rangle =$$

$$= n^{2}(\delta_{st}\delta_{s't'} - \delta_{st'}\delta_{s't} f^{2}(r-r'))$$

Vedral, C. Eu. J. Phys (2003)

Density: Fermi Sea at T=0

$$\begin{matrix} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{matrix} \begin{pmatrix} 1-f^2 & 0 & 0 & 0 \\ 0 & 1 & -f^2 & 0 \\ 0 & -f^2 & 1 & 0 \\ 0 & 0 & 0 & 1-f^2 \end{pmatrix}$$

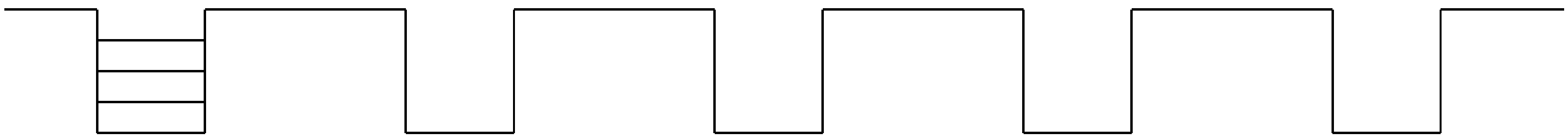
exchange

Entangled as long as the exchange term is larger than ½

Can do more electrons entanglement…

Vedral, C. Eu. J. Phys (2003), Lunkes, Brukner, Vedral, xxx (2004)

Example 2: Bosons at T=0



$$|\Psi\rangle = (a^{\dagger}{}_1 + a^{\dagger}{}_2 + \ldots a^{\dagger}{}_M)^N |0\rangle$$

Want:

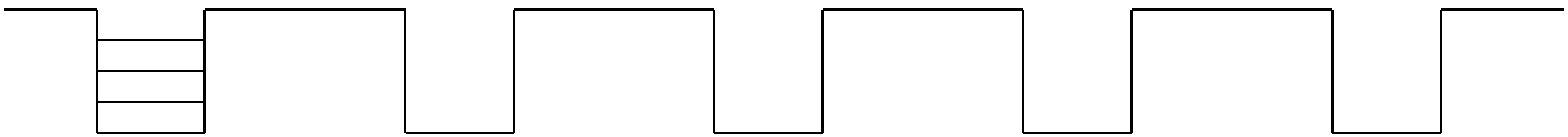
$$\rho(n_i, n_j; n'_i, n'_j) = \rho^{ij}$$

$$\rho^{ij} = \sum p_n \rho^{ij}{}_n$$

$$\rho^{ij}{}_n \propto \left(a^{\dagger}{}_i + a^{\dagger}{}_j\right)^n |0\rangle\langle 0| \left(a_i + a_j\right)^n$$

Infinite range entanglement!

Superconductivity



$$|\Psi\rangle = (a^{\dagger}{}_1 + a^{\dagger}{}_2 + \ldots a^{\dagger}{}_M)^N |0\rangle$$

$a^{\dagger}{}_i$ *Creates a Cooper pair of electrons in the spin singlet state at site "i".*

$e^- e^-$

$\uparrow\downarrow - \downarrow\uparrow$

Pauli exclusion:

$$|\Psi\rangle = |1100\rangle + |1010\rangle + \ldots |0011\rangle$$

ODLRO

$$\left|\Psi\right\rangle = (a^{\dagger}{}_1 + a^{\dagger}{}_2 + ...a^{\dagger}{}_n)^k \left|0\right\rangle$$

Two site density:

$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & c & c & 0 \\ 0 & c & c & 0 \\ 0 & 0 & 0 & b \end{pmatrix}$$

Off diagonal

$$a = \frac{k(k-1)}{n(n-1)}$$

$$b = \frac{(n-k)(n-k-1)}{n(n-1)}$$

$$c = \frac{k(n-k)}{n(n-1)}$$

Yang (1962):

$$k \to \infty, k/n \to const \Rightarrow c > 0$$

Computing Any Entanglement

$$|\varphi\rangle = \sqrt{\frac{k}{n}}\,|0\rangle + \sqrt{\frac{n-k}{n}}\,e^{i\varphi}\,|1\rangle$$

Closest disent. state

$$\rho = \frac{1}{2\pi}\int d\varphi\, \underbrace{|\varphi\rangle\langle\varphi| \otimes |\varphi\rangle\langle\varphi| \ldots |\varphi\rangle\langle\varphi|}_{n}$$

Rel. Ent.

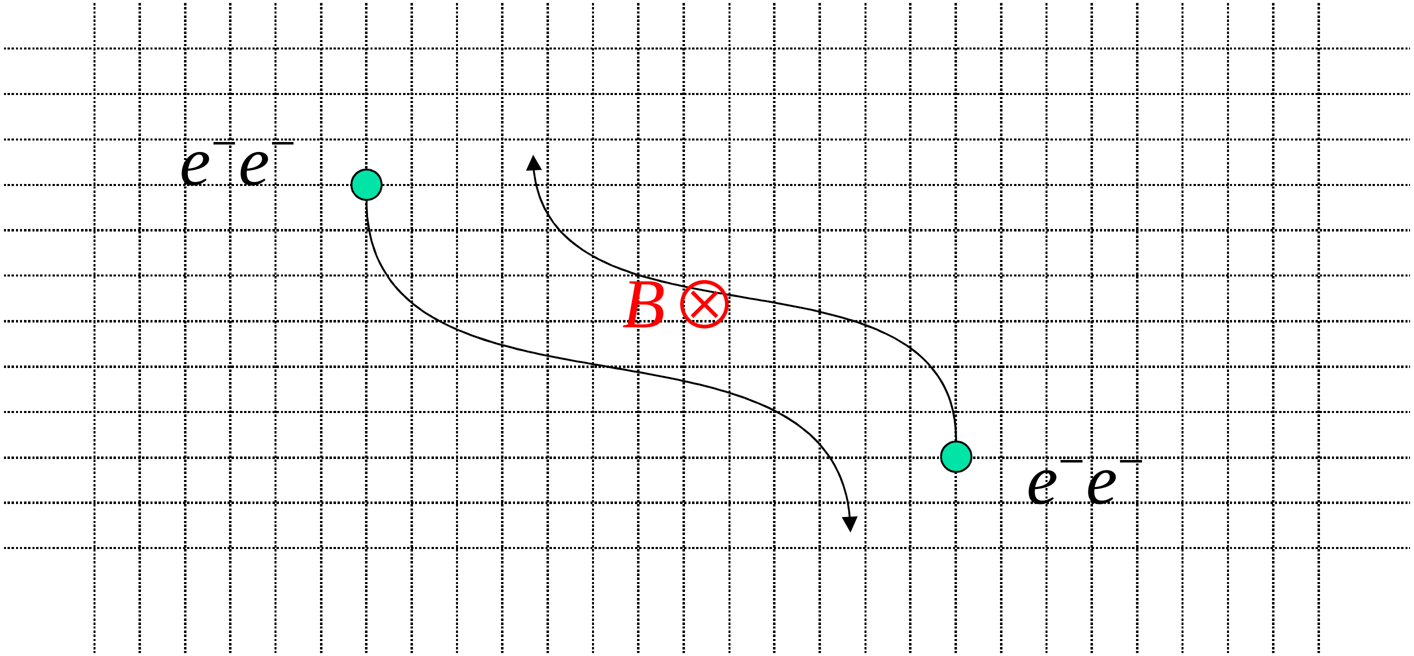
$$E(|\Psi\rangle) = S(|\Psi\rangle \,\|\, \rho) \approx \ln n$$

Wei et. al. (2004)

Works for any subsystem, Vedral, NJPhys 2004

High T entanglement!

Meisner Effect



Sewell, (1995).

$$|01\rangle + |10\rangle \rightarrow |01\rangle + e^{iBS}|10\rangle \Rightarrow b \rightarrow e^{iBS} b$$

$$\therefore e^{iBS} = 1 \Rightarrow BS = 0 \Rightarrow B = 0$$

Higgs and Mass

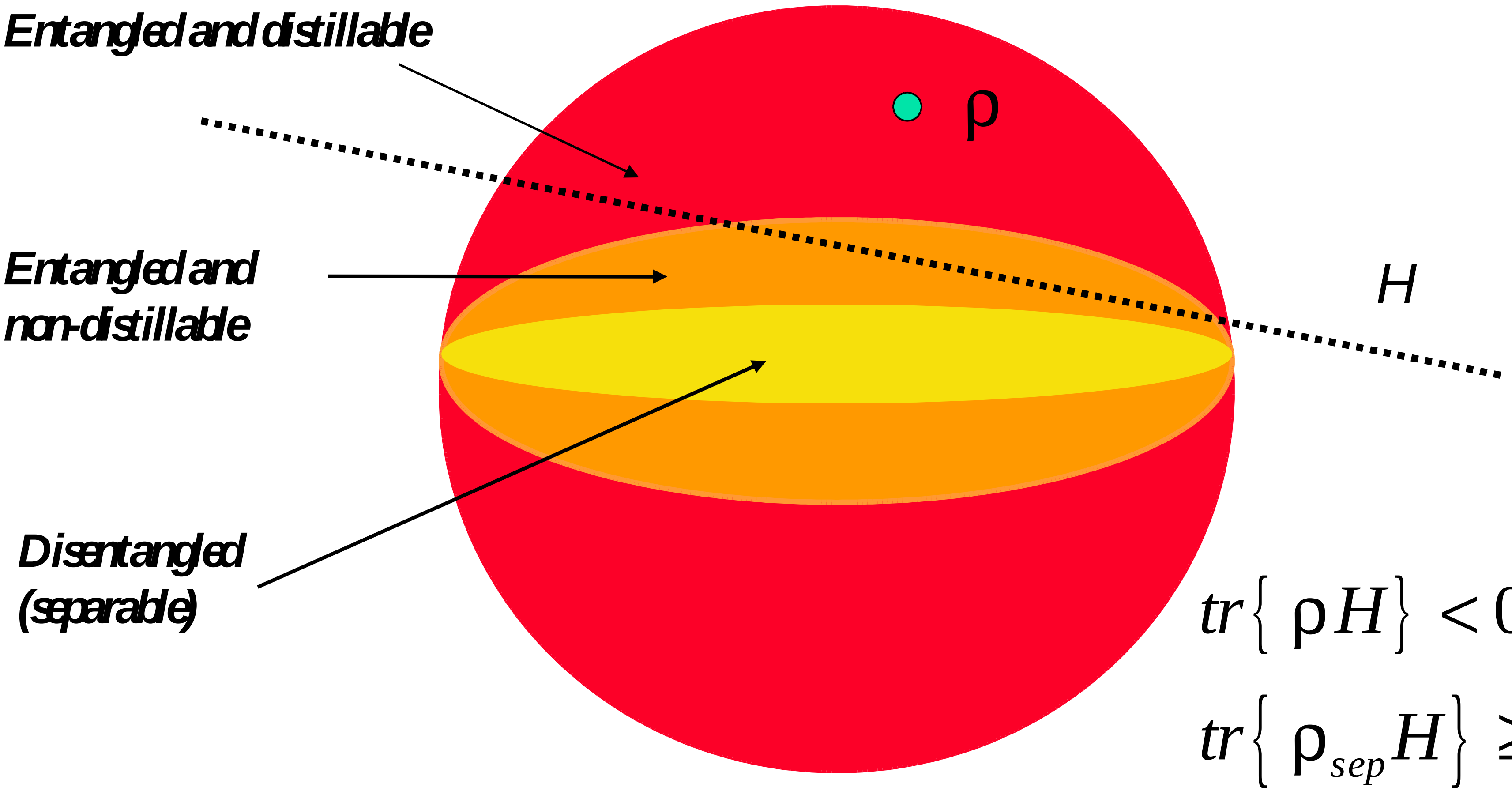
Boson mediators derived from local gauge invariance are necessarily massless (long range).

$$\Psi(r,t) = e^{i\Theta(r,t)}\Psi(r,t)$$

Explain mass by postulating an all-present field which is condensed and interacts with other fields, making thier bosons massive (short range) through a mechanism equivalent to Meissner (Higgs).

V. Vedral, *Meisner and Mass as Entanglement Witnesses*, xxx, 2004.

Mixed State Entanglement



$$tr\{\rho H\} < 0$$

$$tr\{\rho_{sep} H\} \geq 0$$

Energy as the Witness

Heisenberg model:

$$H = -\lambda \vec{\sigma}\vec{\sigma} \Rightarrow$$

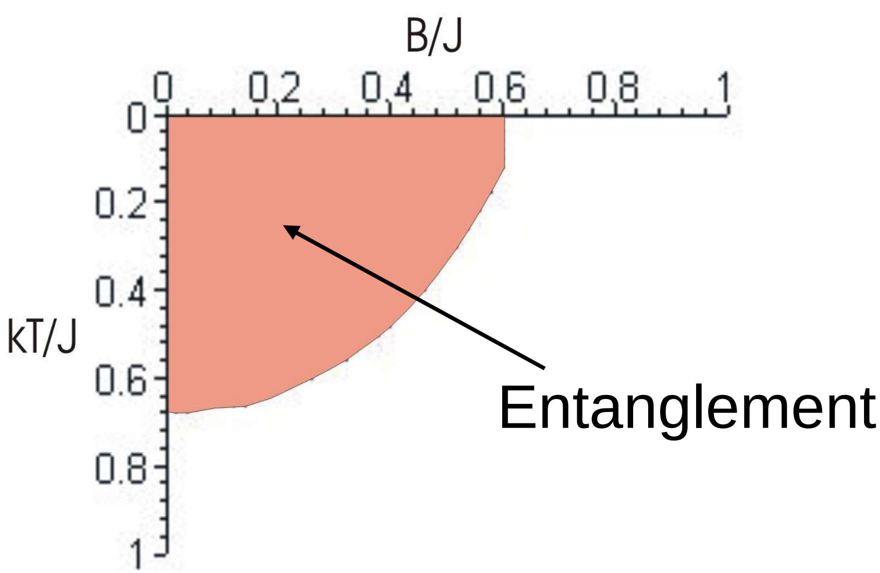
$$|\, tr\{\rho_{sep} H\}\,| \leq \lambda \;\&\; tr\{\rho_{\sin} H\} = -3\lambda$$

Toth, xxx (2003).

Critical T, B

Brukner & Vedral quant-ph/xxx2004

XX Heisenberg Interaction

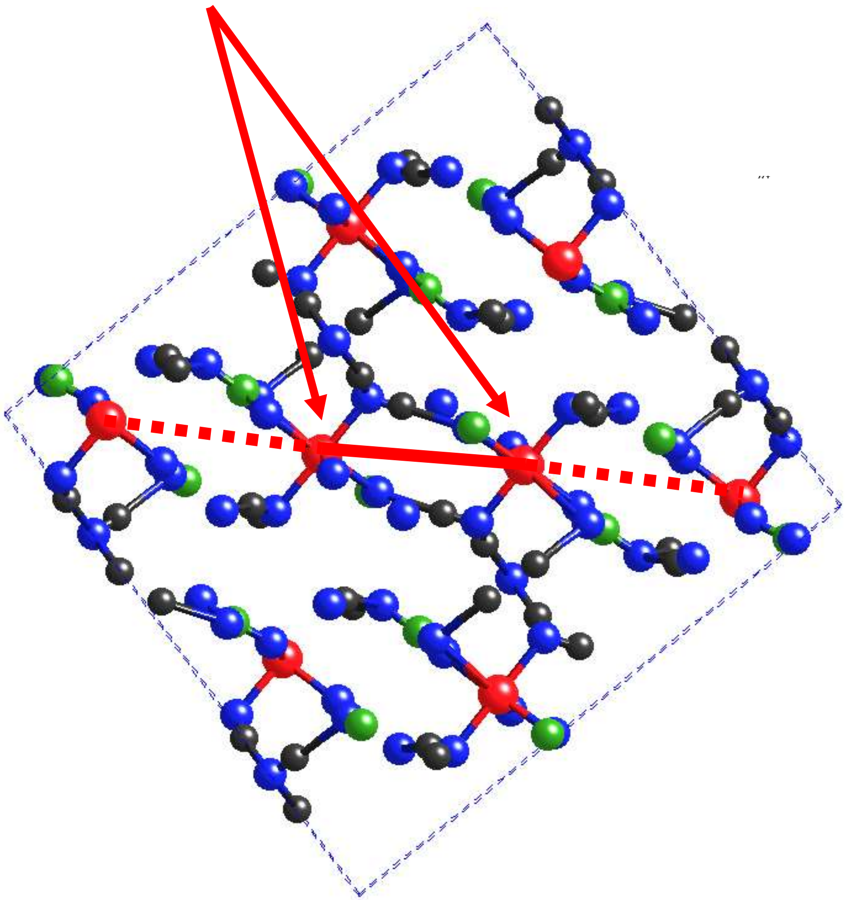


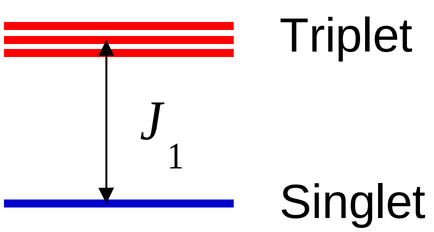
Katsura (1962)
Exactly Solvable: *U*, *M*, …

Drawback: U not directly observable

Alternating Spin Chains

$Cu(NO_3)_2 \cdot 2.5D_2O$: 1D dimerized spin-1/2 system

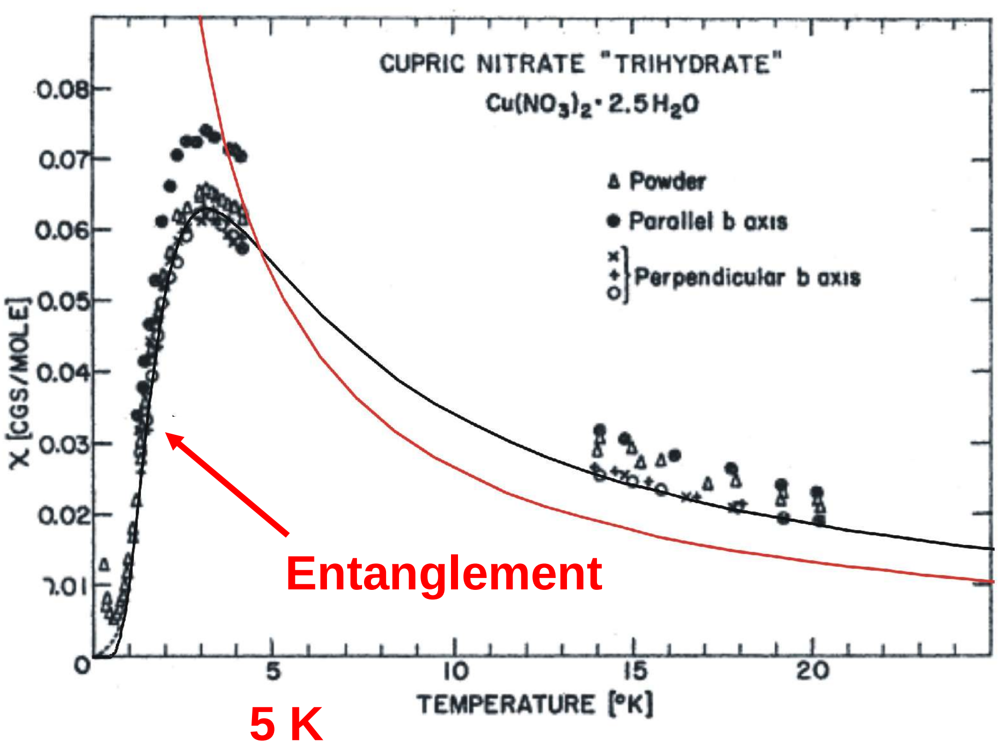




Arnesen, Bose, Vedral, PRL 2000

Magnetic Susceptibility of CN

Berger *et al.*, Phys. Rev. 1963



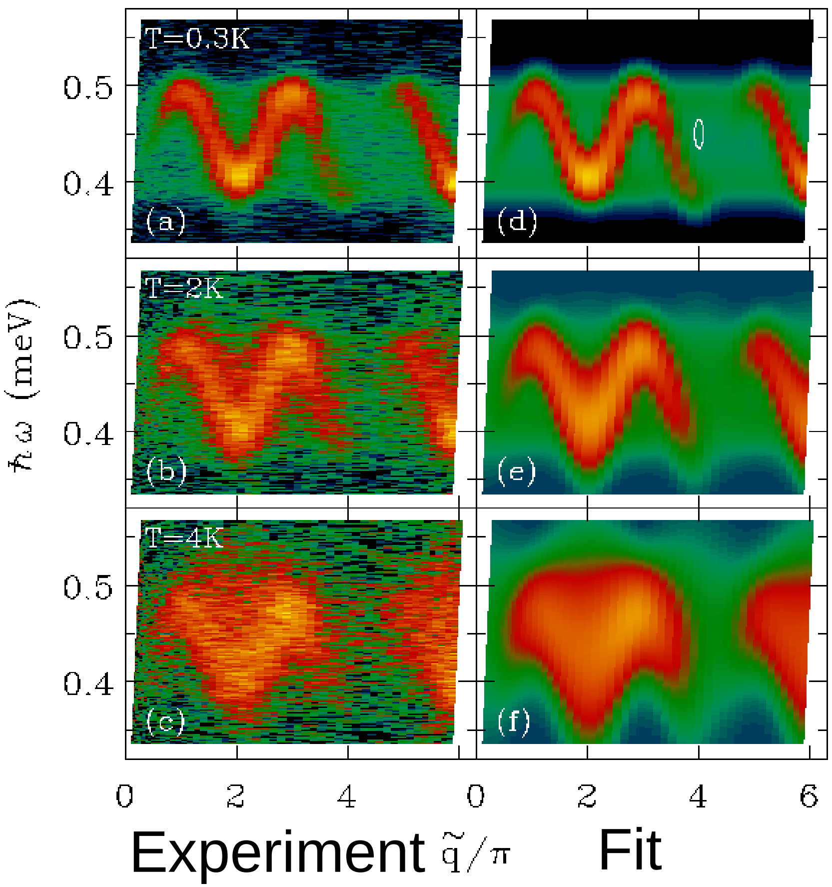
Isotropy (!)

Entanglement Witness

Brukner,Vedral & Zeilinger quant-ph xxx2004

Neutron Magnetic Diffraction of CN

Xu *et al*., PRL (2000)



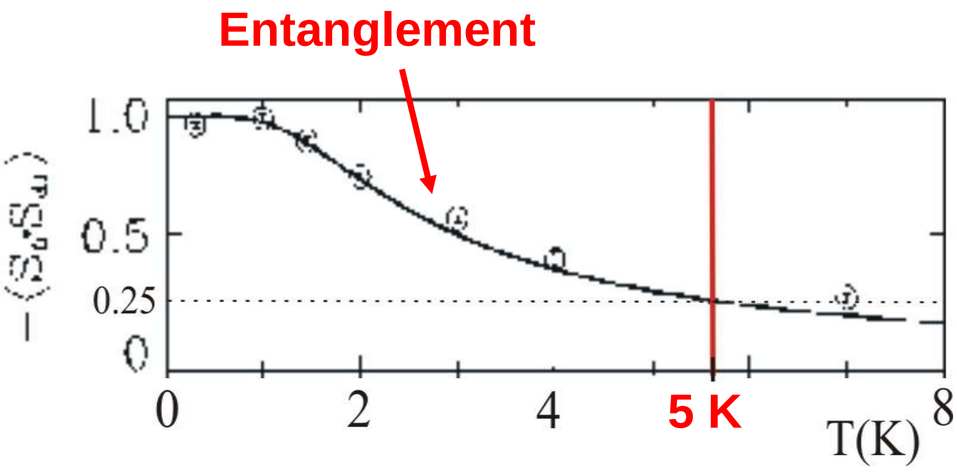
(macroscopic) intensity of neutron scattering

microscopic structure correlation function

Quantum Correlations in CN

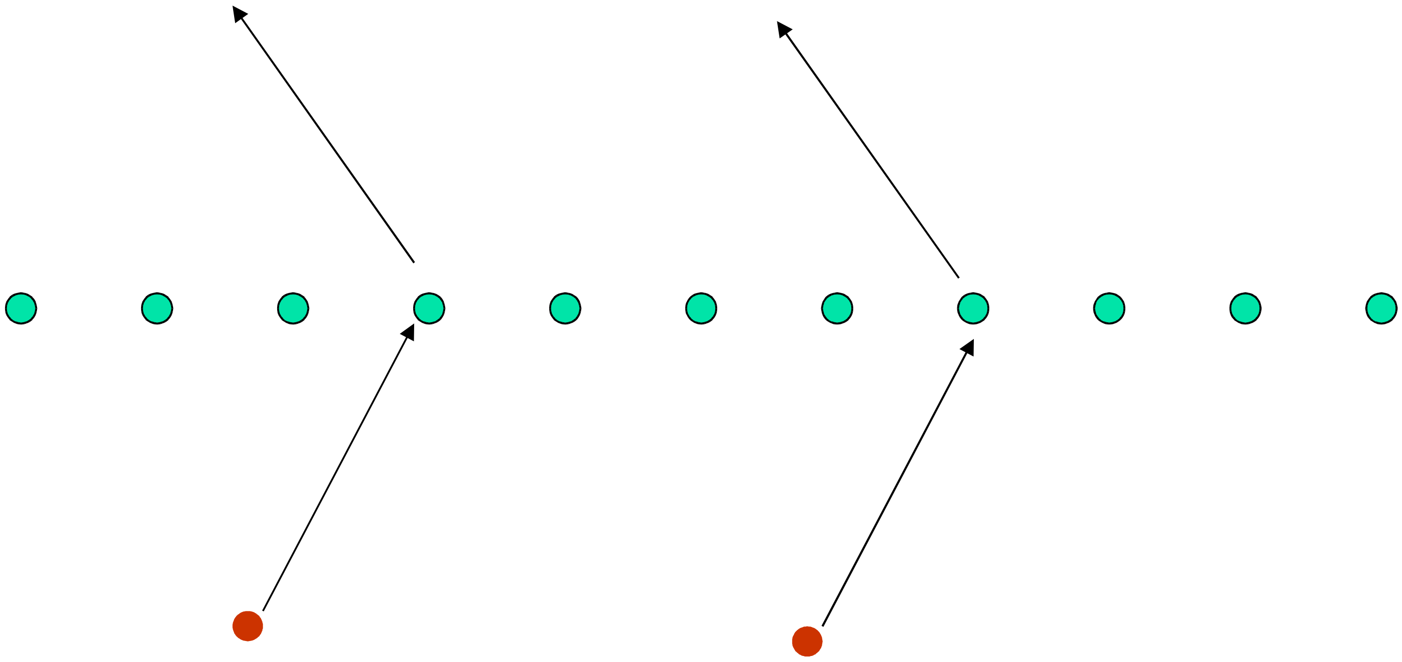
State Entangled

Fit with more than 1000 data points per parameter



Concurrence:

Extraction



Neutron scattering: should be able to extract as much as we like!

c.f. Reznik 2000,2004

FUTURE DIRECTIONS

- Quantunify multipartite entanglement?
- Macroscopic entanglement: high T?
- Extracting and measuring entanglement.
- Using this entanglement.