DYNAMICS IN A QUANTUM COMPUTER WITH IMPERFECTIONS

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OUTLINE

- Entanglement evolution in a many-body quantum system with SI
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- Static VS dynamic imperfections
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- Generalized Entanglement in many body quantum systems with SI

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QCOMPUTER MODEL

$$H = H_0 + H_{ERR}(\ )$$

$$H_0 = \sum \Delta \ \sigma^z_i$$

$$H_{ERR} = \sum \ \delta_i(\ )\ \sigma^z_i + \sum \ J_{ij}(\ )\ \sigma^x_i \sigma^x_j$$

: Rate of change

Random and uniform

$$\delta_i \in [-\delta;\delta]$$

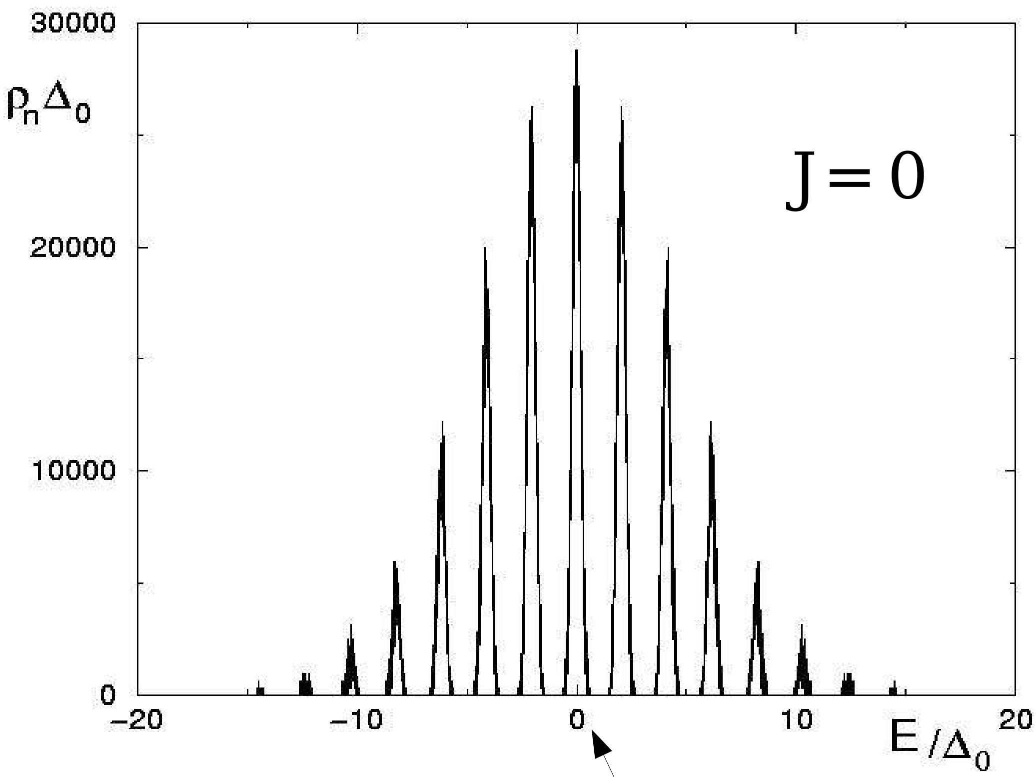
$$J_{ij} \in [-J;J]$$

Static Imperfections

$$1/\quad = 0$$

$$J, \delta << \Delta$$

STATIC IMPERFECTIONS



Central band

Gaussian shape

Quantum Chaos Border

$$J_c \approx \delta / n$$

Spacing level statistics,
Quantum entropy,
Fidelity decay
Local density of states...

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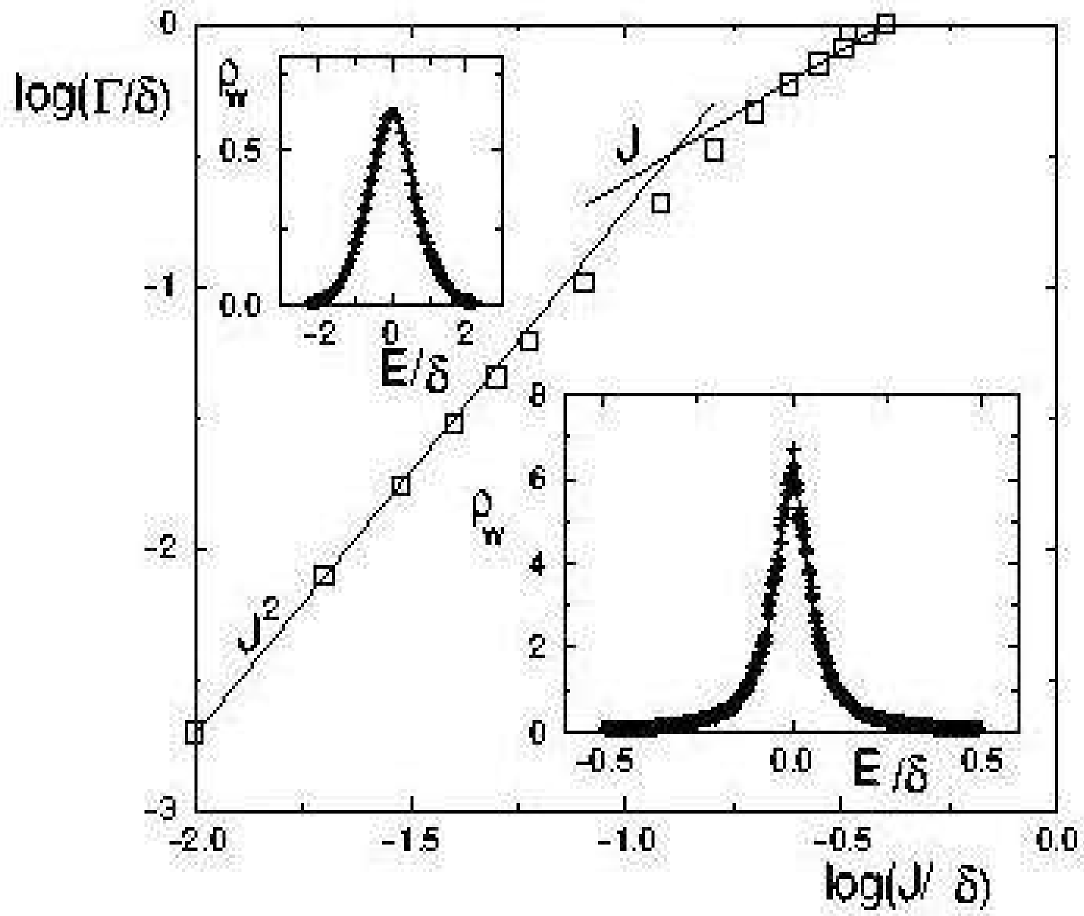
LOCAL DENSITY OF STATES



$$H\,|\Psi_{Imp}> = E\,|\Psi_{Imp}>$$

$$|\Psi_{Ex}> = |00..10..01>$$

$$\rho_W(E-E_i) = \sum_m W_{im}\,\delta(E-E_m)$$

$$W_{im} = |<\Psi_{Ex}|\Psi_{Imp}>|^2$$

Breit-Wigner

Gaussian

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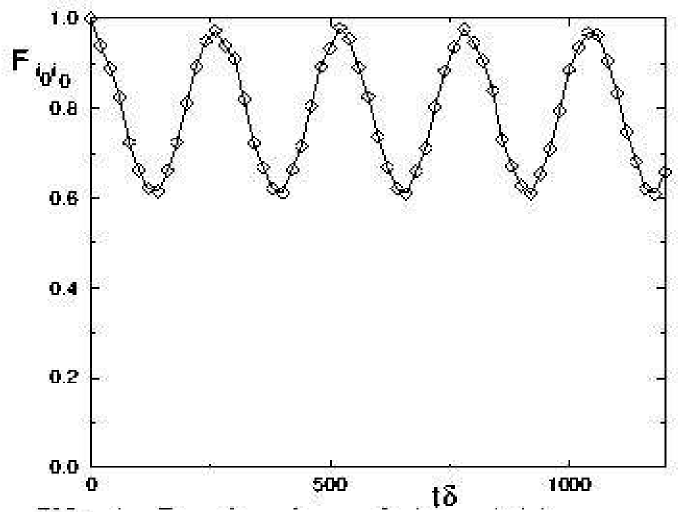
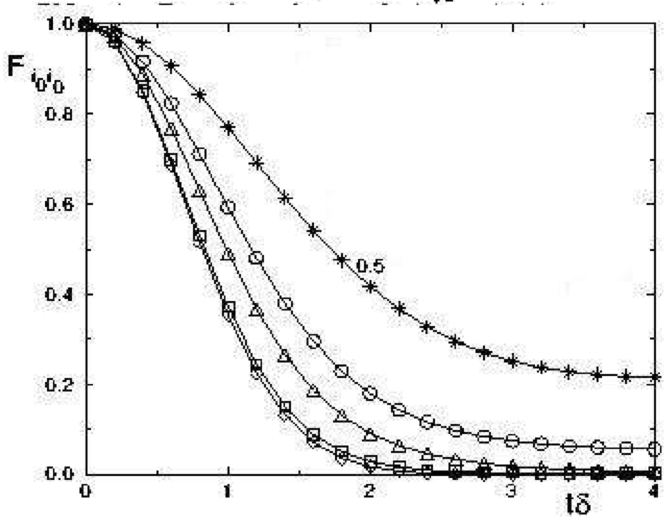
FIDELITY VS LDOS





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$$F(t) = |\langle \phi_{imp}(t) | \phi_{ext}(t) \rangle|^2$$

$$F(t) = \mathscr{F}\left[\rho_W(E-E_i)\right]$$

Breit-Wigner → Exponential

↓

Gaussian → Gaussian

FIDELITY REGIMES

PERTURBATIVE

$F(t) = 1 - (J/\delta)^2 n\, t^2$

perturative theory

VALID FOR

$J\, t \ll 1$

EXPONENTIAL

$F(t) = \exp(-\Gamma_f\, t)$

$\Gamma_f = |V|^2 \rho_f = J^2 \rho_f$

FERMI GOLDEN RULE

$\rho_f = n/\delta$

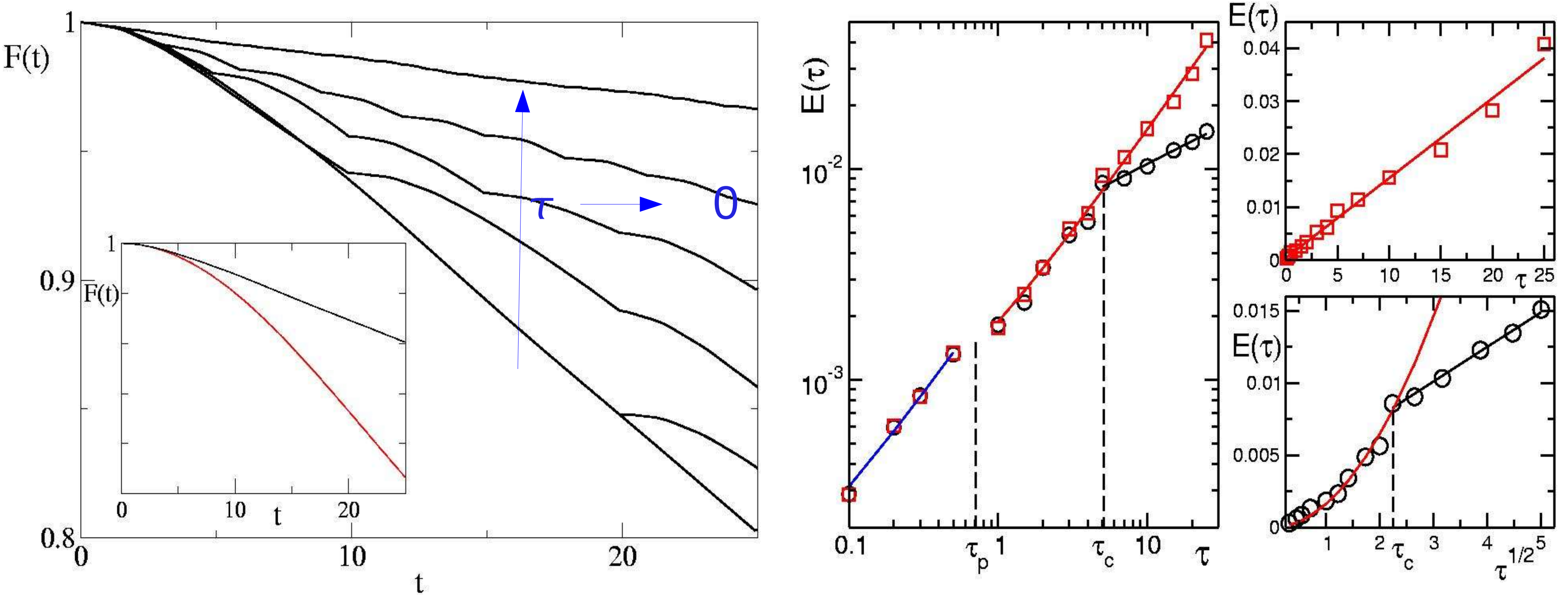
GAUSSIAN

$F(t) = \exp(-J^2 n t^2)$

product of oscillations

ERGODIC

DYNAMICAL IMPERFECTIONS



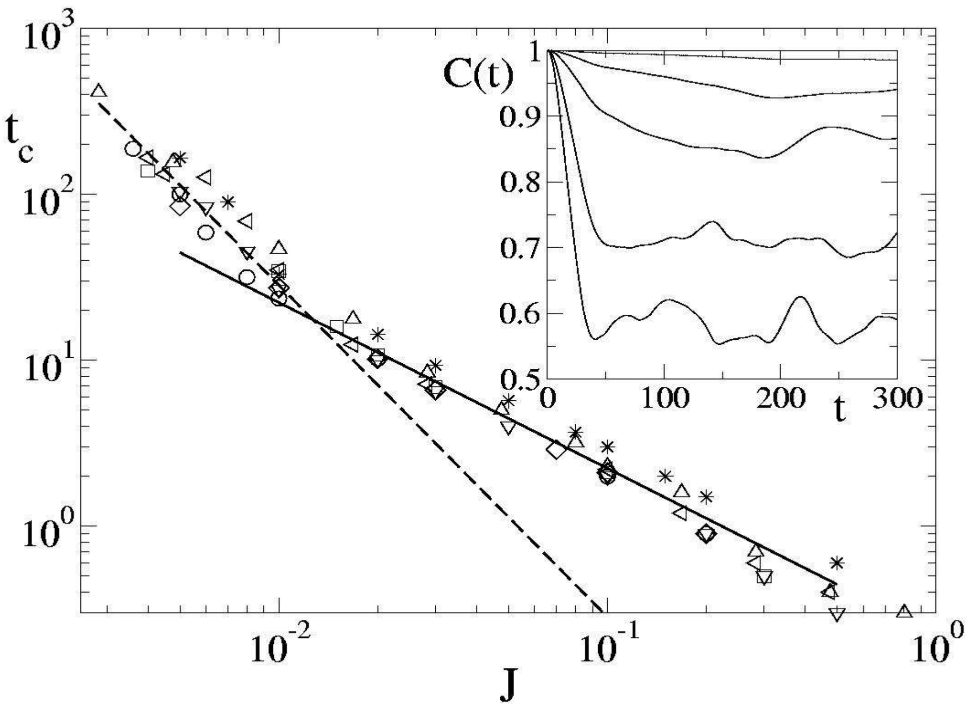
- If $\tau \neq 0$, the system feels an effective imperfections strength given by $J_{eff} = J\,\tau^{1/2}$ $\delta_{eff} = \delta\,\tau^{1/2}$
- For $\tau \ll 1$ the system is always in perturbative regime
- Quasi-static regime: the fidelity decay follows the static regime decay

ENTANGLEMENT

- Study of entanglement dynamics
- Concurrence (non-trivial function of LDOS)
- Initial state: bell and completely separable with zero magnetization in the central band
- Product of bell states, GHZ, W state
- Concurrence and Purity decay in presence of static imperfections

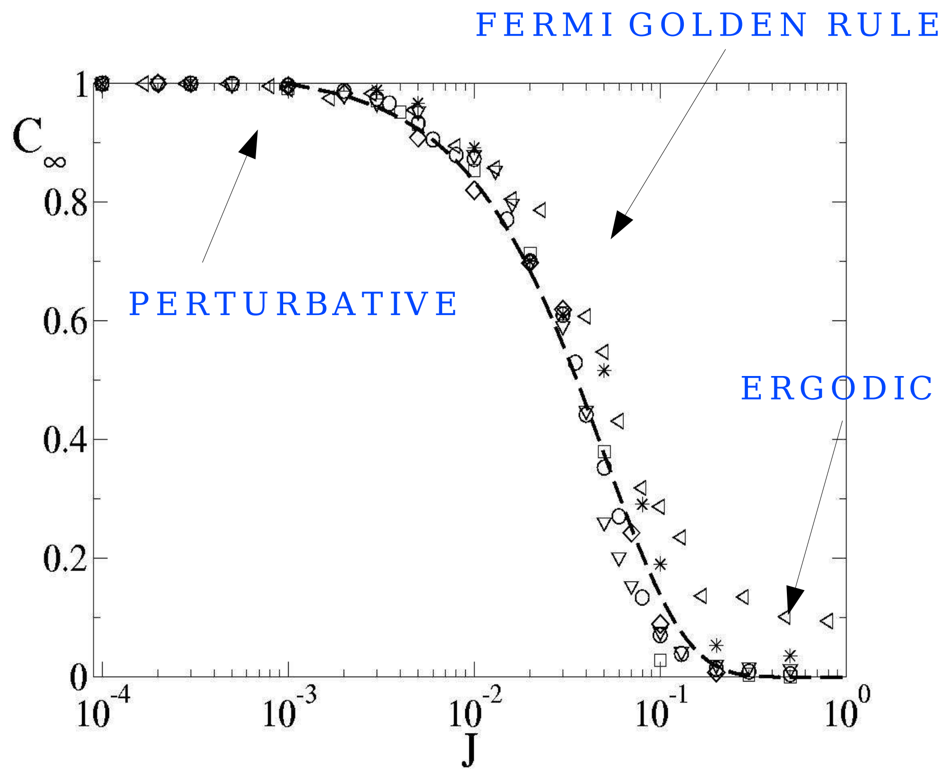
ENTANGLEMENT
FOLLOWS FIDELITY DECAY

CONCURRENCE



$$C(t_c) = 0.9$$

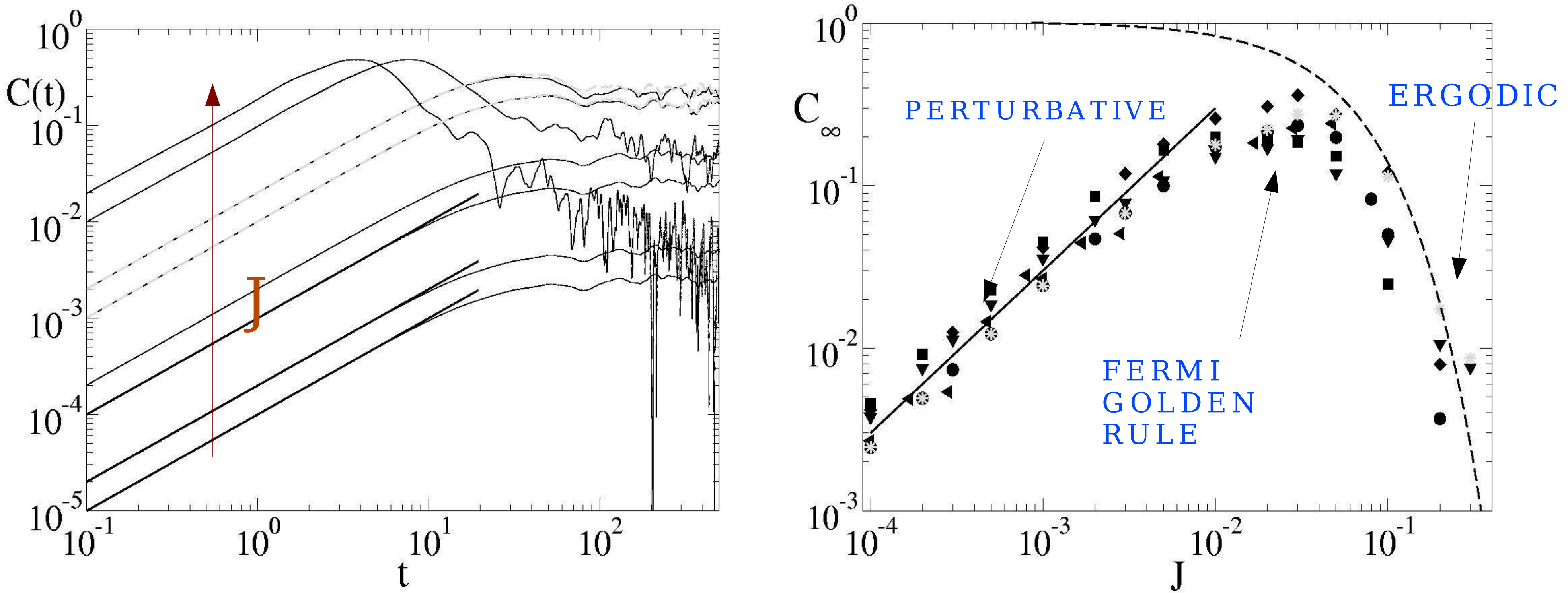
$$t_c^E = 1/J \qquad t_c^F = 1/J^2$$



A $t >> t_c$

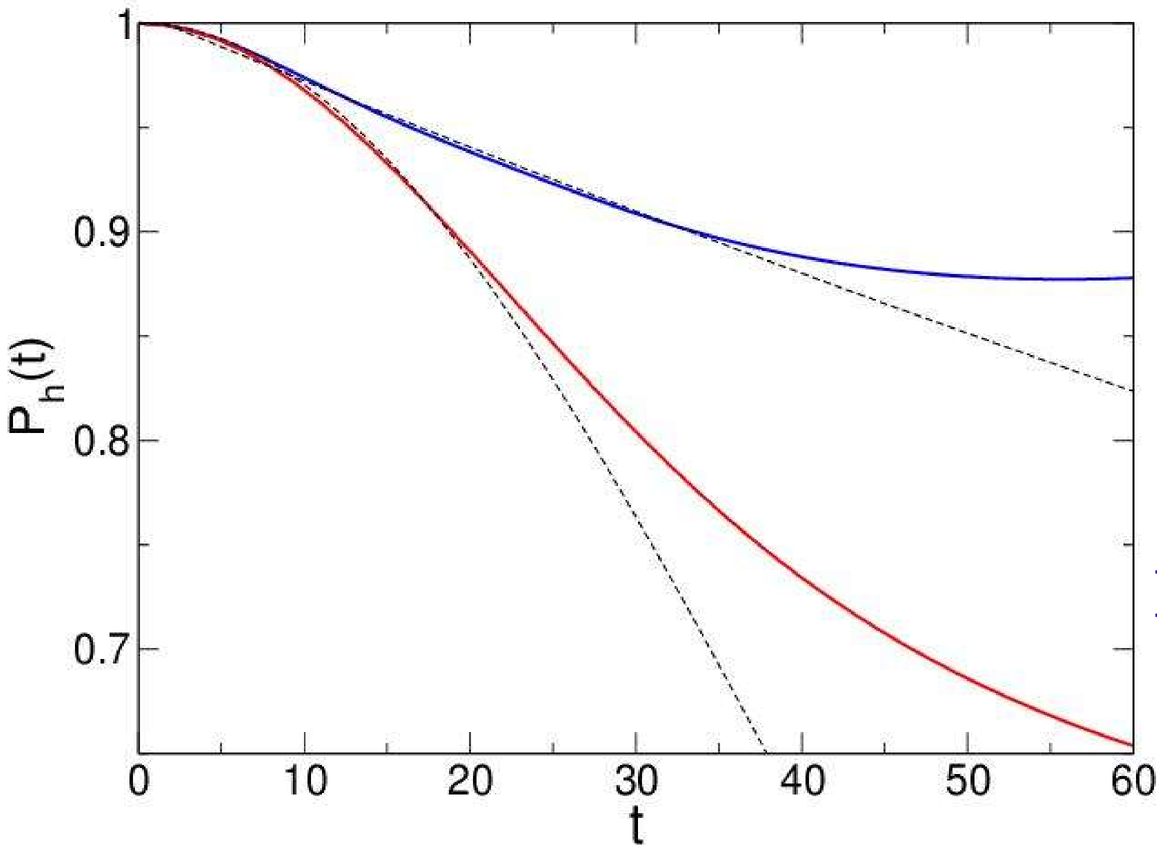
CREATION OF ENTANGLEMENT

INITIAL SEPARABLE STATE



A $t >> t_c$

PURITY EVOLUTION



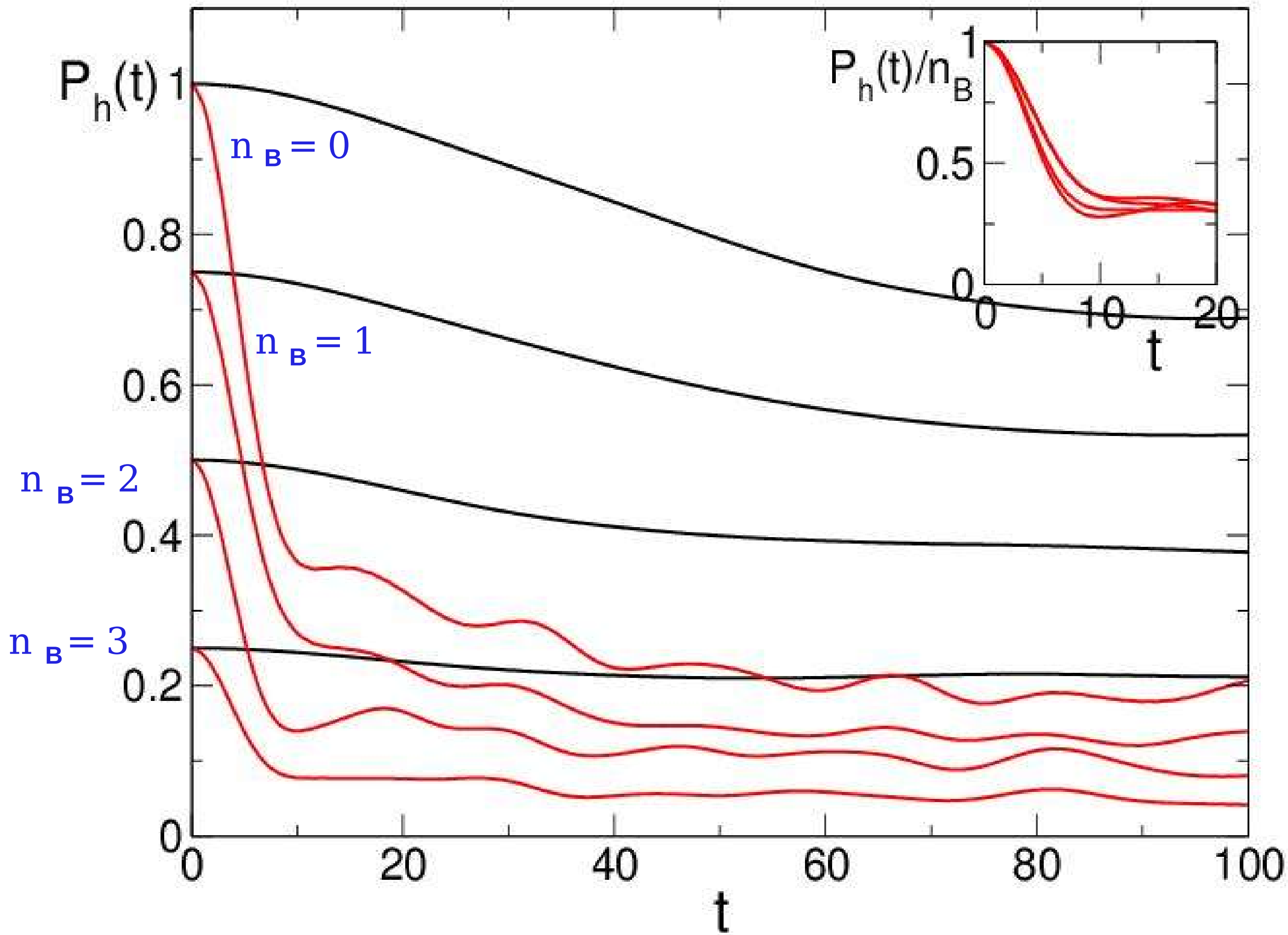
$|\Psi_0\rangle = |01..01..01\rangle$

Perturbative analisys:

$P_h(t) \approx |\mathcal{F}[\rho_W(E-Ei)]|^4$

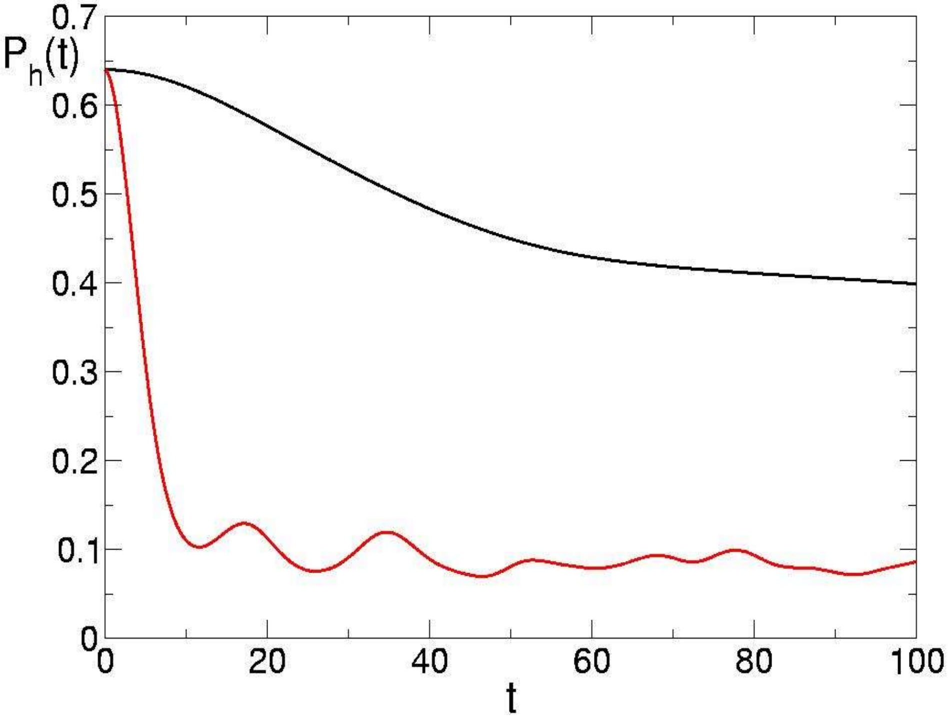
Generalized Entanglement w.r.t. Local Algebra : $P_h(t) \approx \sum_{\substack{k=x,y,z \\ i=1..n}} |\langle\Psi|\sigma_i^k|\Psi\rangle|^2/n$

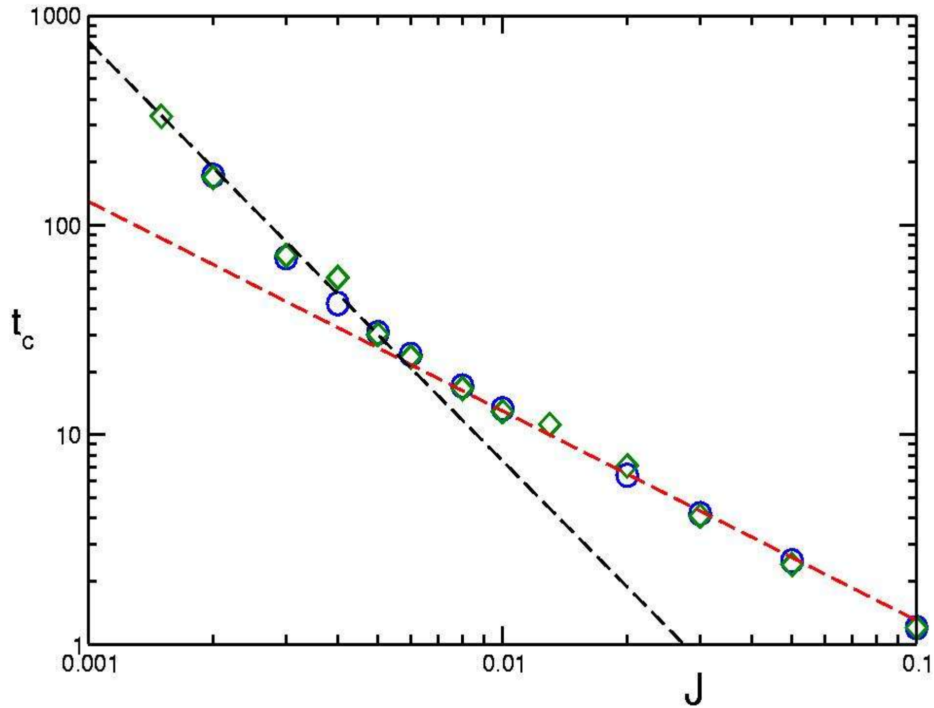
MULTI-BIPARTITE ENTANGLEMENT



$|\Psi_0> \prod (|01>+|10>)^{n_B}$ ™ $|01..01..01>$

MULTIPARTITE ENTANGLEMENT





W state: $|\Psi_0> \prod \sigma\, |0..010..0>$ $\Delta = 0$

CONCLUSIONS

- Dynamical regimes strongly influence QI processing.
- SI are more dangerous than dynamical ones, which eventually will average to zero.
- A quasi-static regime exits in which SI description holds.
- Entanglement decay follows fidelity decay.
- Pairwise entanglement is not influenced by system size.
- Generalized entanglement: more information, clearer view, flexibility...
- Further work: spin=1, different algebras...