UNIVERSITY OF CAMBRIDGE

Entanglement behaviour in systems with three particle interactions

Jiannis K. Pachos

Enrique Rico

Christian D'Cruz

Angelo Carollo

Martin Plenio

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Introduction

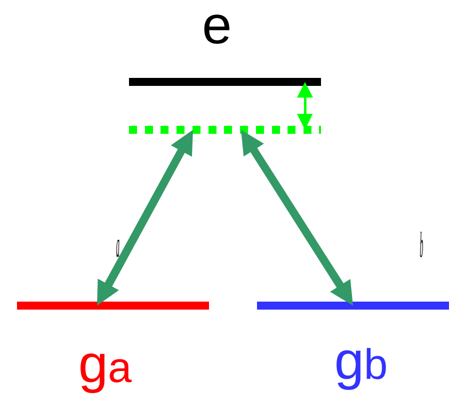
Ultra cold atoms and optical lattices

- From bosonic system to effective *three spin interactions*
- Great variety of criticalities and phase transitions
- Possible to simulate chiral and topological effects
- Geometric phases and quantum phase transitions

Entanglement measures for multipartite systems

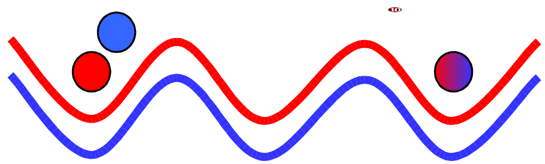
- Two and higher point correlations
- Concurrence (Wootters, Nielsen, ...)
- Entropic entanglement (Lattore, Vidal, ...)
- Localisable entanglement (Verstraete, Cirac, ..)

The physical system



- Raman transition: $J_i^{R\dagger} \frac{\quad a \quad b}{2}$

- Optical lattice model: $n_a^1 n_b^1 ; n_a^2 n_b^2$



Tunnelling transitions (J) and collisions (U)

- Bose-Hubbard Hamiltonian: $\langle a_i, a_j \rangle \quad \langle b_i, b_j \rangle \quad ij$

$H \quad J_i^a a_i \quad a_i \quad J_i^b b_i \quad b_i \quad J_i^R a_i \quad b_i \quad H.c.$

$\frac{U_{aa}}{2} \quad a_i \quad a_i^2 \quad U_{ab} \quad a_i \quad a_i b_i \quad b_i \quad \frac{U_{bb}}{2} \quad b_i \quad b_i^2$

Optical Lattice operations

Controlled Phase Gate

- Lowering only the b-mode couples |01;01> to |02;00> & |00;02>

$$H_1 = \begin{pmatrix} 0 & -J^b & -J^b \\ -J^b & U_{bb} & 0 \\ -J^b & 0 & U_{bb} \end{pmatrix}$$

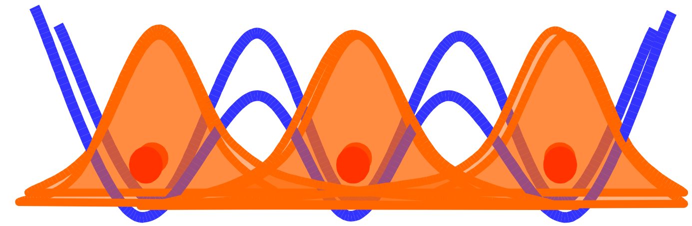
- For J<<U : |01;01> $\longrightarrow \ell^{i}$ |01;01>

where

$$\varphi = -2\int_0^T \frac{J^{b\,2}}{U_{bb}}\,dt$$

Logical Table		
|00>	$\longrightarrow$	|00>
|01>	$\longrightarrow$	|01>
|10>	$\longrightarrow$	|10>
|11>	$\longrightarrow \ell^{i}$	|11>

(Controlled)^2 Phase Gate



$$\int_0^T \frac{J^{b\,3}}{U_{bb}^2}\,dt = \pi$$

$$|111> \rightarrow -|111> \qquad C^2P$$

Exchange Interaction

- Consider tunneling in a and b-modes

Evolution: effective exchange interaction

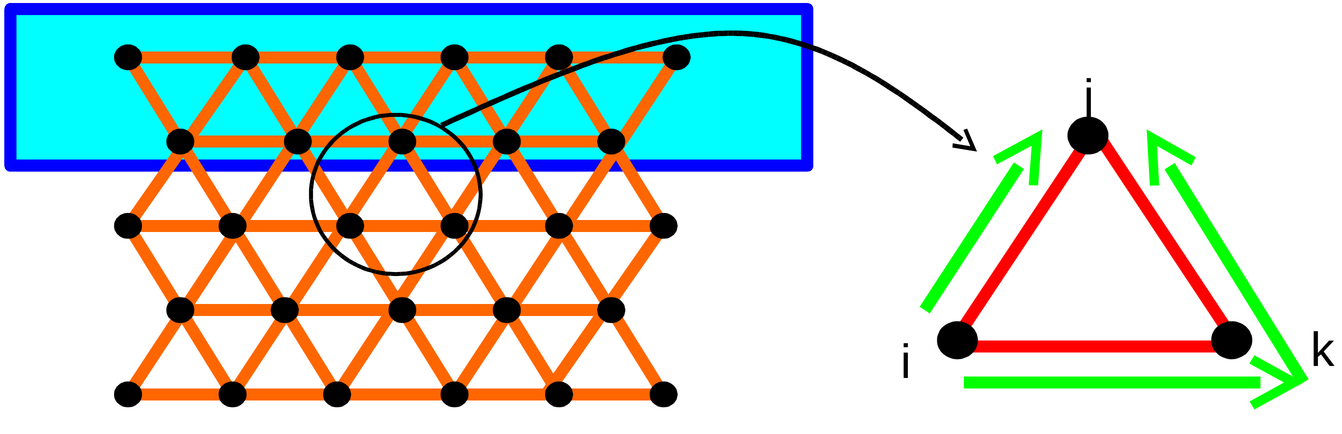
Heff =-K(|10><01|+|01><10|)

$$K \dagger 2\frac{J^a J^b}{U_{ab}}$$

[JKP and P. L. Knight, PRL (2003); A. Kay and JKP, JMP (2004)]

Three spin interactions

- Consider the next order in perturbation $J^3 \Rightarrow U^2$
- You can obtain with three sites:



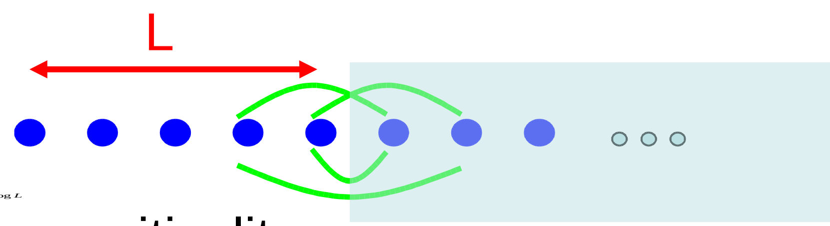
Critical points and quantum phase transitions

- Criticality: Infinite correlation length
- Perform numerical study
- Correlations:

: criticality

: non-critical

- Entanglement of reduced density matrix



: criticality

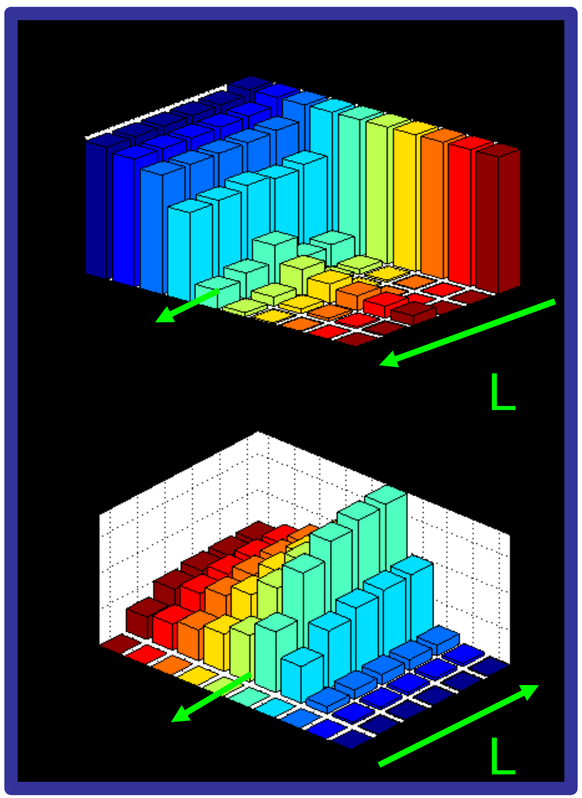
: non-critical

von Neumann entropy

[J. Latorre *et al*., quant-ph/0304098]

- Ising-like model:
- Duality transformation:

[L. Turban, J. Phys. C (1982)]



: criticality

: non-critical

: criticality

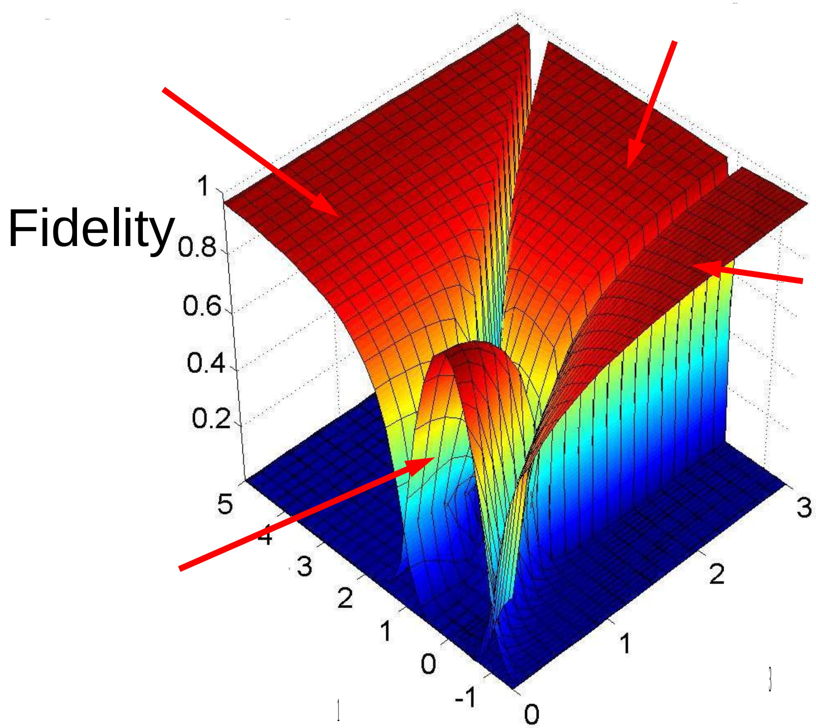
: non-critical

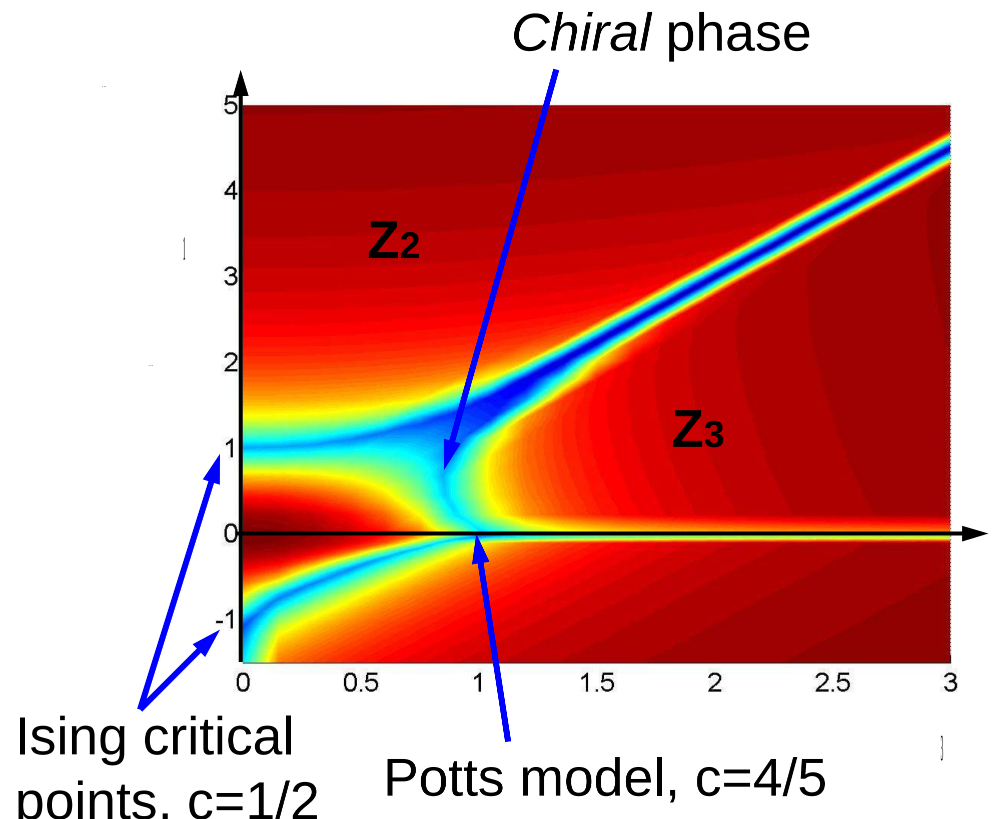
Proportionality factor gives c=4/5

=>three state Potts model

Density orders of period *two* and *three*

- Realise in optical lattice:





[Sachdev *et al*., PRA (2004)]

Cluster Hamiltonian

- Realise in optical lattice:

- Duality transformation:

- Complete study of the critical exponents shows, it is in the same universality class as the Ising model.

- Ground state for is a 1-dim cluster state :

stabilizer group

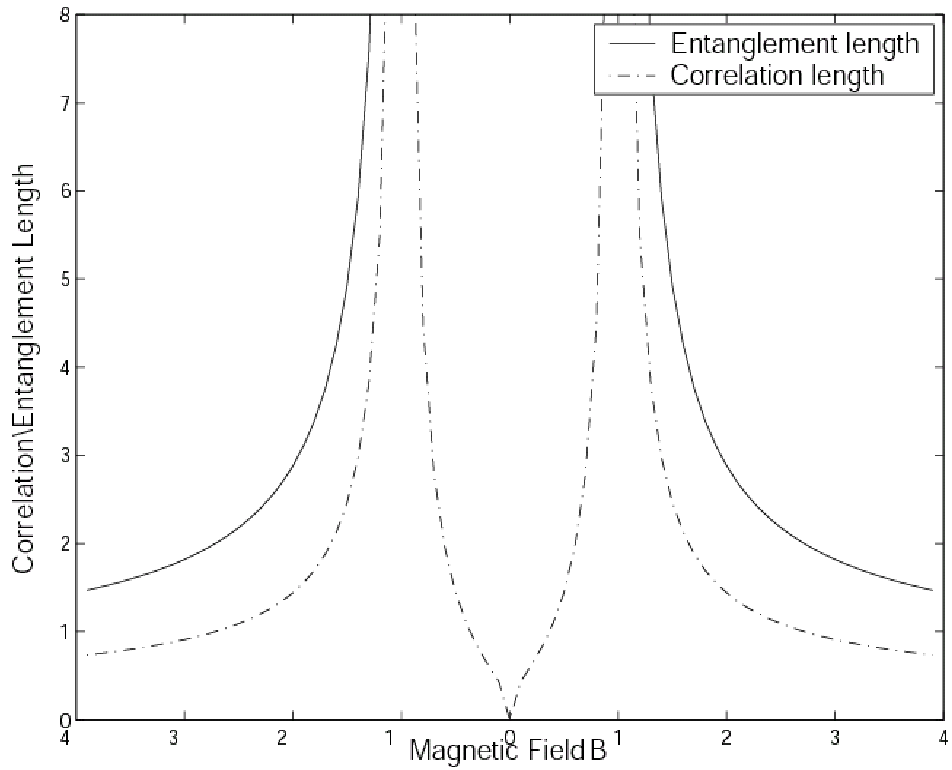
[JKP and M. B. Plenio, PRL (2004); JKP and E. Rico, PRA (2004)]

- Infinite localizable entanglement length, **but** finite (zero) correlation length at

[F. Verstraete *et al*., PRL (2004)]

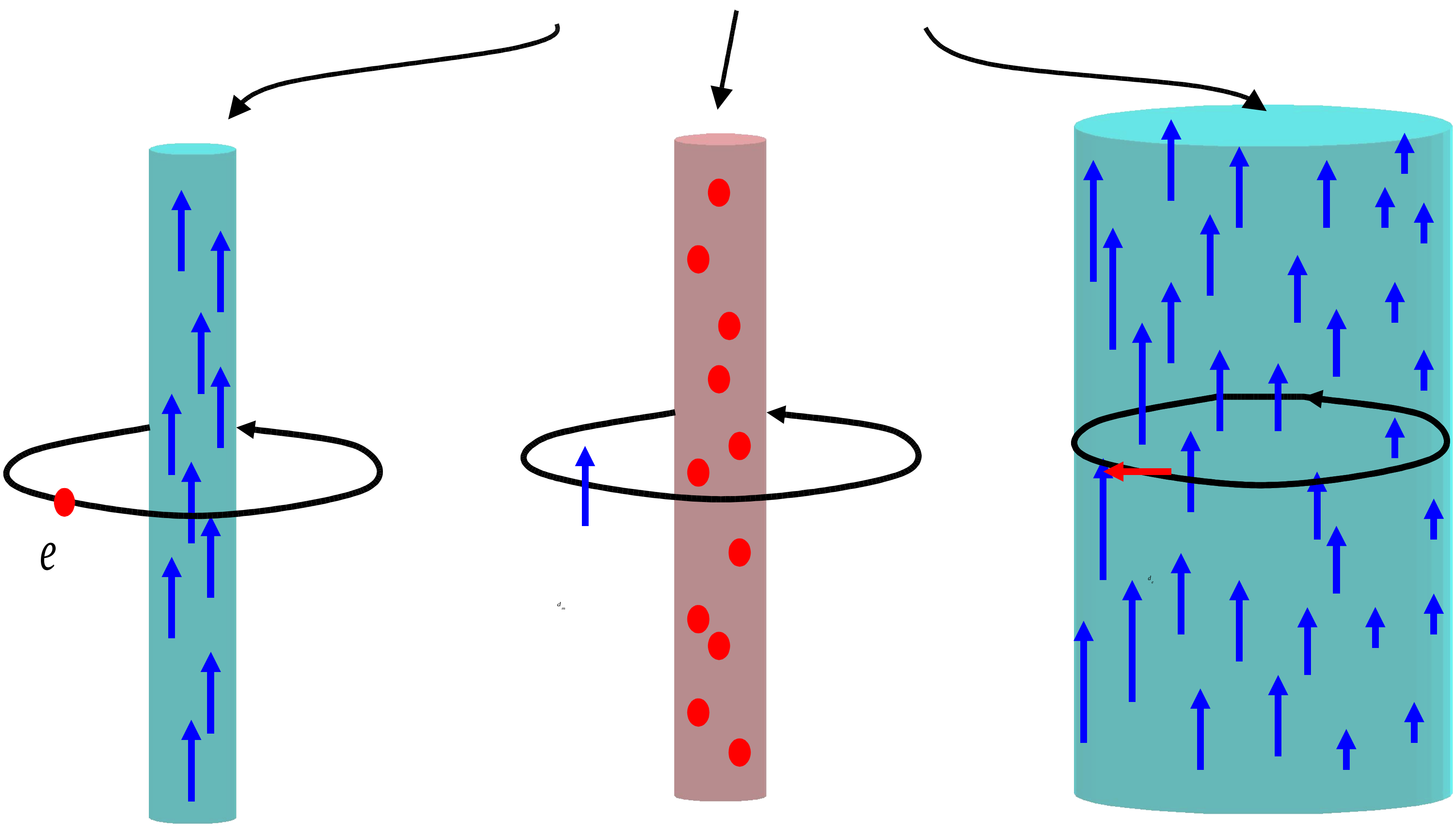
For any correlation of n neighboring sites will be non-vanishing with probability

For any correlation of n neighboring sites will be non-vanishing with much higher probability



[JKP and M. B. Plenio, PRL (2004); JKP and E. Rico, PRA (2004)]

Simulation of charge in magnetic field…



Aharonov-Bohm

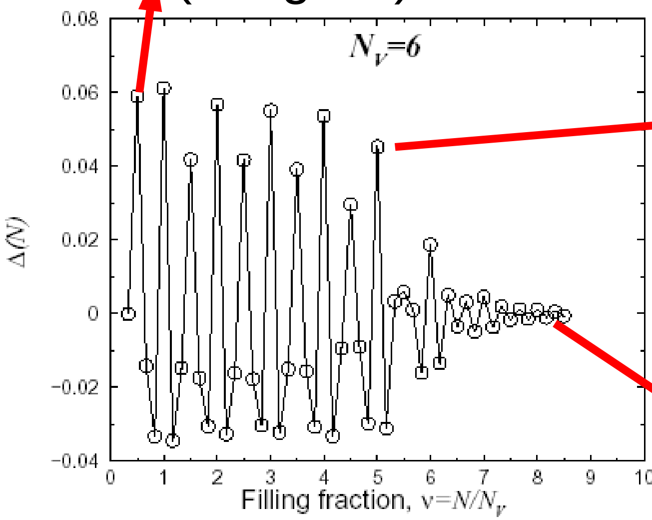
Aharonov-Casher

"Differential" A-B

..Topological Quantum Conputation…

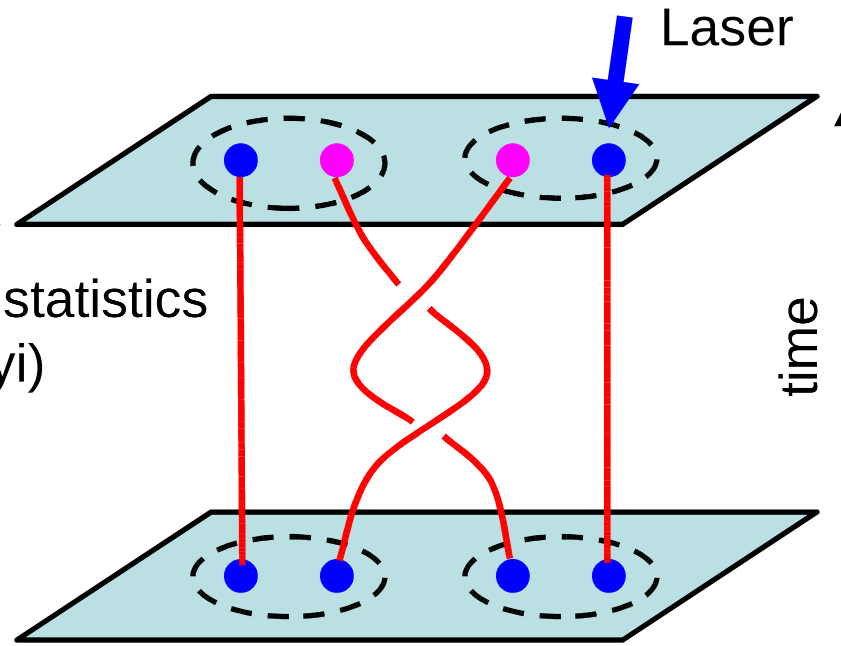
States: quasiparticle excitations in QHE **Gates: geometrical phases**

Abelian statistics
(Laughlin)



Non-abelian statistics
(Read-Rezayi)

Vortex
lattice



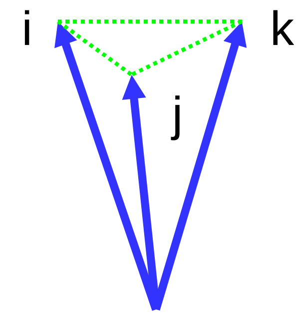
10-100 stronger
angular momentum

Exciting prospect for
atom-chip technology

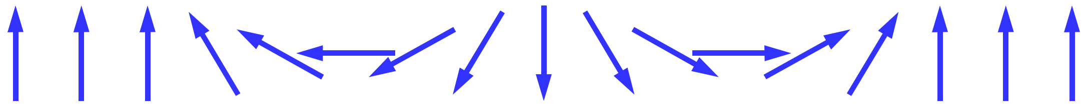
Holonomies

[N. R. Cooper *et al*., PRL (2001); JKP *et al*., PRA (2000)]

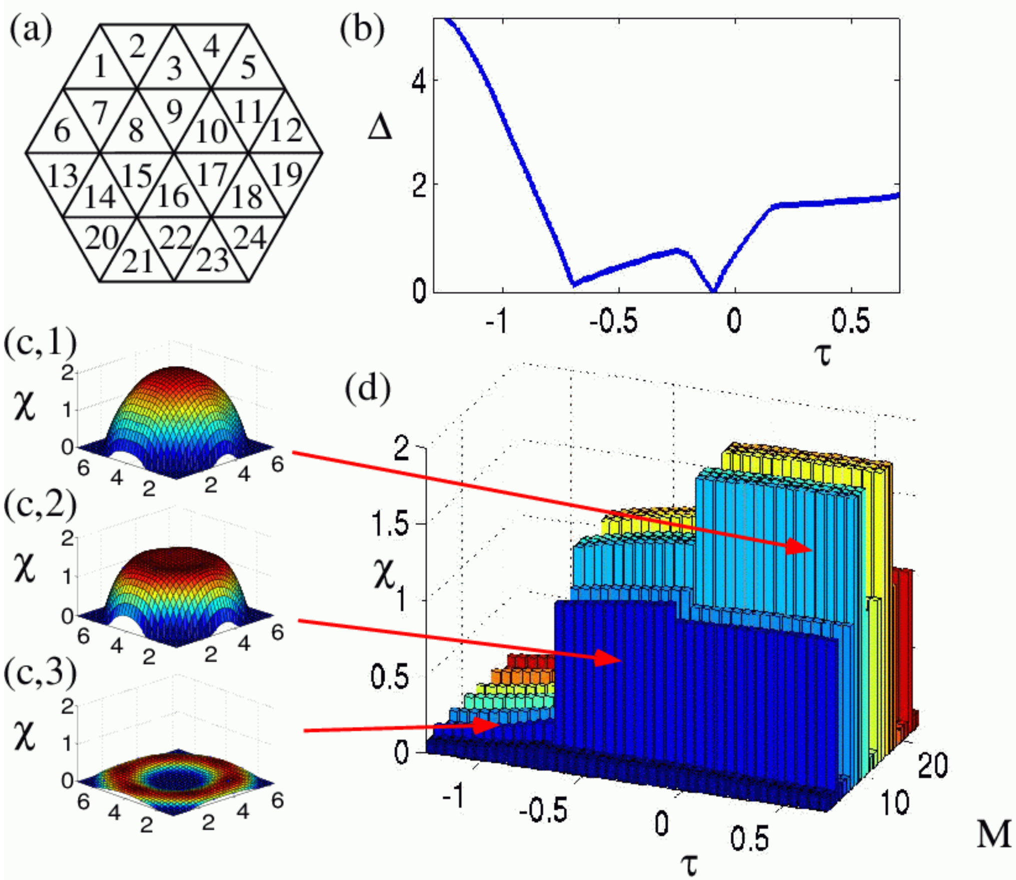
..Skyrmion Hamiltonian



Can realise:



- For   positive, minimum energy if solid angle is constant between three neighboring sites => Skyrmion



[JKP, cond-mat/0405374]

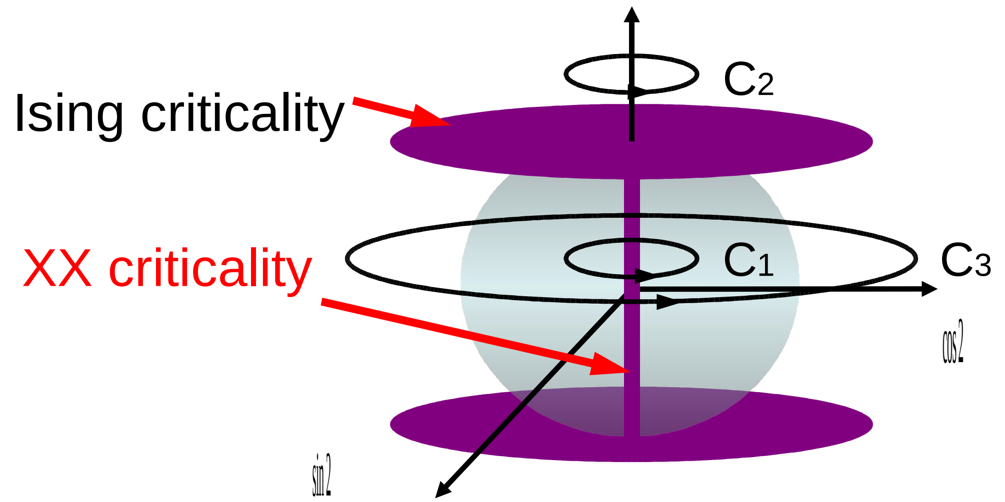
Geometric phases in many body spin systems

Rotated Anisotropic XY Model

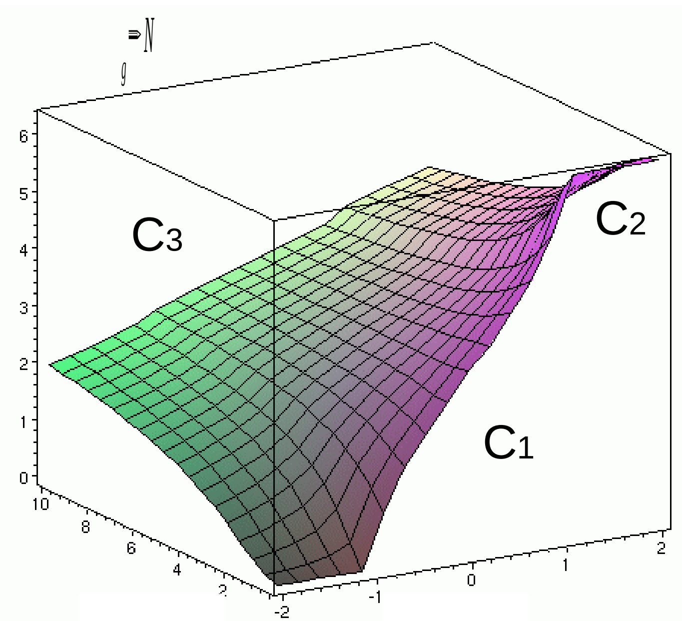
• Perform Wigner-Jordan transformation to diagonalise

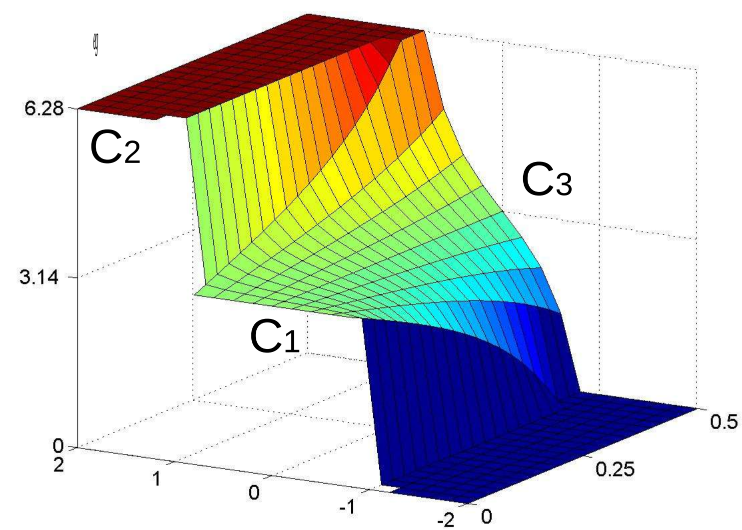
• Equivalent to N independent spins in a magnetic field with orientation

Criticality



Berry phases





The end

- Optical lattices provide a unique laboratory to study three spin interactions
- Possibility to observe phases that have not yet been physically realised
- Complete study of criticality for different values of the parameters
- Use different criticality criteria that respect three or more party entanglement
- Engineer systems with non-vanishing chirality
- New proposal for realising Topological Quantum Computation

Thank you for your attention