Mañé's Theorem and Heteroclinics in Lagrangian Dynamics.

In 1989, see Ann. Inst. H. Poincaré Anal. Non Linéaire 6 (1989)) p. 21–90, Paul Rabinowitz showed how to obtain by direct variational methods heteroclinic orbits for Hamiltonian systems with a classical Hamiltonian

$$\frac{1}{2}\|p\|^2 + V(x),$$

where V is a potential on \mathbf{R}^k which is \mathbf{Z}^k periodic. This has generated a lot of further work.

For more general Lagrangians, Sergei Bolotin showed later the role played Aubry-Mather sets, see Amer. Math. Soc. Transl. **168** p. 21–90. We show how to obtain these results in a simple way from a theorem of Mañé that in Lagrangian Dynamics, see C. R. Acad. Sci. Paris Sé r. I Math. **326** (1998), p. 1213–1216, and also the work by G. Contreras & G. Paternain in Topology **41** (2002), p. 645–666.