# Psicologia e Matematica Perché sbagliamo le nostre decisioni 

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## Henri Poincaré

Nancy, 29 aprile 1854 - Parigi, 17 luglio 1912How does it happen there are people who do not understand mathematics?If mathematics invokes only the rules of logic, such as are accepted by all normal minds; if its evidence is based on principles common to all men, and that none could deny without being mad, how does it come about that so many persons are here refractory?
$\square$ And further: how is error possible in mathematics? A sane mind should not be guilty
 of logical fallacy, and yet there are very fine minds who do not trip in brief reasoning such as occurs in the ordinary doings of life, and who are incapable of following or repeating without error the mathematical demonstrations which are longer, but which after all are only an accumulation of brief reasonings wholly analogous to those they make so easily. Need we add that mathematicians themselves are not infallible?

## Alan Turing



## $\square$ Mechanical intelligence

$\square$ Alcuni esperimenti di fallimento della nostra capacità di ragionamento

L' Esperimento di Wason (1966)


Quali carte occorre girare per verificare se la regola seguente è valida?

## REGOLA

"Se una carta ha un A da un lato, allora ha un 2 dall'altro lato"
$\square$ "Immagina di essere un poliziotto inglese in servizio. Il tuo ruolo è di assicurare che le persone rispettino regole e leggi. Entri in un bar e vedi sul fondo quattro persone sedute ad un
tavolo. Il barista usa quattro carte per indicarti l'età e le bevande che stanno consumando
$\square \quad$ Le carte di fronte a te ti danno informazioni sulle persone sedute al tavolo: da un lato della carta c'è l'età della persona, dall'altro lato quello che la persona beve. La legge stabilisce che:
$\square$ SE UNA PERSONA BEVE BIRRA DEVE ESSERE MAGGIORENNE
$\square \quad$ Scegli le carte che devi girare per determinare se degli avventori violano la legge

$\square$ The deontic format seemed therefore to be the key to explain the successful performance, as capable of eliciting the right reaction:Leda Cosmides (1989) argued that this deontic representation elicits a domain-specific human capacity, the ability to detect cheaters; if applied to Cox and Griggs version of the selection task, this capacity leads to an "automatic" selection of the right cards; in this particular version, in fact, subjects searching for cheater turn the cards that should cover up a violation of the rule, i.e. "drinking beer" and "16 years old".
$\square$ Cosmides suggested an evolutionary explanation of the cheating detection mechanism: she argued that, for cooperation to have stabilized during human evolution, humans must have developed reciprocal altruism and - in the same time - domain-specific cognitive capacities that allowed them to detect cheaters. She argued that the cognitive capacities in question consisted in a social contract module allowing detect parties that were not respecting the terms of the contract. Moreover she argued that not all deontic rules elicit correct selections, but only those which are processed by means of a underlying evolved modules such as the social contract algorithm

## L'esperimento di Maurice Allais

$\square \quad$ Preferisci A oppure B?


ㅁ A
Certezza di ricevere 100 milioni
$\square \quad B$
Vinci 100 milioni con probabilità $10 \%$
500 milioni con probabilità $89 \%$
Non vinci nulla con probabilità $1 \%$
$\square \quad$ Preferisci C oppure D?
C
Vinci 100 milioni con probabilità $11 \%$
Non vinci nulla con probabilità $89 \%$
$\square$ D
Vinci 500 milioni con probabilità $10 \%$
Non vinci nulla con probabilità $90 \%$

## Allais' experiment : violation of cancellation

$\square$ If Cancellation were justified, the preference $A \boldsymbol{B}$ should entail C 6 D

ㅁ Proof:If $A \boldsymbol{\Theta} B$, then
$\mathrm{U}(100)>0.10 \mathrm{U}(500)+0.89 \mathrm{U}(100)+0.01 \mathrm{U}(0)$
Rearranging this expression gives
$0.11 \mathrm{U}(100)>0.10 \mathrm{U}(500)+0.01 \mathrm{U}(0)$;
And adding $0.89 \mathrm{U}(0)$ to each side yields
$0.11 \mathrm{U}(100)+0.89 \mathrm{U}(0)>0.10 \mathrm{U}(500)+0.90 \mathrm{U}(0)$

- That means COD!


## Effetto Framing

- Problema 1

Vinci 300 Euro alla lotteria ed un bonus. Con il bonus puor

- A) ottenere 100 Euro in regalo.
- B) ottenere un biglietto della lotteria con $50 \%$ di probabilità di vincere 200 Euro e $50 \%$ di non vincere nulla

ㅁ Problema 2
Vinci 500 Euro alla lotteria , ma hai un debito da pagare di 100Euro
Il debitore ti offre di :

- C) Pagare.
- D) lanciare una moneta: se viene testa non paghi nulla, se viene croce paghi 200 Euro
$\square$ «Lorsqu'a la loterie de France un numéro n'est pas sorti depuis longtemps, la foule s'empresse de le couvrir de mises. Elle juge que le numéro resté longtemps sans sortir doit, au premier tirage, sortir de préférence aux autres. Une erreur aussi commune me parait tenir à une illusion par laquelle on se reporte involontairement à l'origine des événements. Il est, par exemple, très peu vraisemblable qu'au jeu de croix ou pile on amènera croix dix fois de suite. Cette invraisemblance qui nous frappe encore, lorsqu'il est arrivé neuf fois, nous porte à croire qu'au dixième coup pile arrivera. Cependant le passé, en indiquant dans la pièce une plus grande pente que pour pile, rend le premier dé ces événements plus probable que l'autre; il augmente, comme on l'a vu, la probabilité d'amener croix au coup suivant. » (Laplace 1814, introduction, CXIII)


## Framing Effect : violation of Invariance

$\square$ Problem $(N=150)$. Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer.
$\square$ Decision (i) Choose between:
$\square$ A) a sure gain of \$240 [84\%]
$\square$ B) $25 \%$ chance to gain $\$ 1000$ and $75 \%$ chance to gain nothing [16\%]
$\square$ Decision (ii) Choose between:
$\square$ C) a sure loss of $\$ 750$ [13\%]
$\square$ D) $75 \%$ chance to lose $\$ 1000$ and $25 \%$ chance to lose nothing [87\%]

Framing Effect : violation of invariance
$\square$ Because the subjects considered the two decisions simultaneously, they expressed, in effect, a preference for the portfolio A and D over the portfolio $B$ and C. However, the preferred portfolio is actually dominated by the rejected one! The combined options are as follows.
$\square$ A \& D: 25\% chance to win $\$ 240$ and $75 \%$ chance to lose \$760.
$\square \quad B \& C: 25 \%$ chance to win $\$ 250$ and $75 \%$ chance to lose $\$ 750$.
$\square$

## Sbagliano anche gli esperti?

"From its earliest days, the research that Tversky and I conducted was guided by the idea that intuitive judgments occupy a position - perhaps corresponding to evolutionary history - between the automatic operations of perception and the deliberate operations of reasoning. Our first joint article examined systematic errors in the casual statistical judgments of statistically sophisticated researchers (Tversky \& Kahneman, 1971). Remarkably, the intuitive judgments of these experts did not conform to statistical principles with which they were thoroughly familiar."

## Kahneman Nobel Lecture



## Perceptual illusions

What to observe
The neighbouring image is immediately seen as a cube, wiggling a bit. It is not a normal cube, but one corner is missing (grey faces). If you work on it, you can see an alternate interpretation: there is a smaller grey cube attached to the corner, in front of the larger cube - and it rotates inversely to the large cube! There is a third alternative view: imagine you're looking at a room corner, and a cube is placed in that corner.
It may take a while.
What to do
Once you have seen the effect, you can mentally flip it over. Interestingly, you can't hold one interpretation for longer than, say, 10 s, another similarity to the Necker cube.

Dipendenza dal contesto

$$
12 B 14
$$

Dipendenza dal contesto

$$
A B C
$$

contesto: I'ambiguità è
ppressa dalla percezione
$A B C$
$12 B 14$
$\square$ The Figure illustrates the effect of context on accessibility. An ambiguous stimulus that is perceived as a letter in a context of letters is seen as a number in a context of numbers. The figure also illustrates another point: the ambiguity is suppressed in perception. This aspect of the demonstration is spoiled for the reader who sees the two versions in close proximity, but when the two lines are shown separately, observers will not spontaneously become aware of the alternative interpretation. They 'see' the interpretation that is the most likely in its context, but have no subjective indication that it could be seen differently.

Perceptual illusions



## Perceptual illusions



Figure 5. Reference-Dependence in tie Perception of Brightness


## Rotazione in senso orario oppure

 antiorario?
## Silouette

## L'effetto belief-bias per dimostrare il Dualismo

$\square$ Nessun cane poliziotto è drogato
$\square$ Alcuni cani altamente addestrati sono drogati
$\square \quad$ Dunque, alcuni cani altamente addestrati non sono cani poliziotti

- Nessun oggetto nutritivo è gratuito
$\square$ Alcune tavolette di vitamine sono gratuite
$\square$ Dunque, alcune tavolette di vitamine non sono nutritive
- Nessun additivo è gratuito
- Alcune sigarette sono gratuite
$\square$ Dunque, alcuni additivi non sono sigarette
- Nessun milionario è un duro lavoratore
$\square \quad$ Alcune persone ricche sono duri lavoratoriDunque alcuni milionari non sono ricchi
$\square$ Goel and Dolan, using fMRI techniques, discovered that the resolution of such conflict problems in favour of either logic or belief was differentiated with respect to associated neurological activity. Specifically, on trials where the logically correct decision was made, responses were associated with activation of the right inferior prefrontal cortex. By contrast, incorrect, belief-biased responses were associated with activation of the ventral medial prefrontal cortex. This provides very strong evidence for long standing claims of dual-process theorists that different mental processes are competing for control of the response to these problems [66]. Goel and Dolan conjecture that the right prefrontal cortex is critical in detecting and resolving conflict, a key aspect of System 2 functioning, also supported in their earlier study [27]. They also cite a range of neuropsychological studies to support their contention that the ventral medial prefrontal cortex is associated with a range of intuitive or heuristic responses of the kind typically characterised as emanating from System 1.




## Brain's regions involved in automatic / in deliberate / thinking

Automatic and controlled processes can be very roughly distinguished by where they occur in the brain (Lieberman, Gaunt, Gilbert and Trope 2002). Regions that support cognitive automatic activity are concentrated in the back (occipital), top (parietal) and side (temporal) parts of the brain (see Figure 1). The amygdala, buried below the cortex, is responsible for many important automatic affective responses, especially fear. Controlled processes occur mainly in the front (orbital and prefrontal) parts of the brain. The prefrontal cortex (pFC) is sometimes called the "executive" region, because it draws inputs from almost all other regions, integrates them to form near and long-term goals, and plans actions that take these goals into account (Shallice and Burgess, 1998). The prefrontal area is the region that has grown the most in the course of human evolution and which, therefore, most sharply differentiates us from our closest primate relatives (Manuck, Flory, Muldoon and Ferrell 2003).
$\square$ Kahneman emphasizes the essential role of the "framing" effect for understanding the origin of biases in decision making and reasoning; Kahnemann suggest that framing must be considered a special case of the more general phenomenon of dependency from the representation: the question is how to explain the fact that different representations of the same problem yield different human decisions.

## Tetris

## NAIUE



## Accessibility

Accessibility is defined as "the ease with which particular mental contents come to mind" (Higgins, 1996).
"A defining property of intuitive thoughts is that they come to mind spontaneously, like percepts. To understand intuition, then, we must understand why some thoughts are accessible and others are not. ..... Category labels, descriptive dimensions (attributes, traits), values of dimensions, all can be described as more or less accessible, for a given individual exposed to a given situation at a particular moment. (KNL)

## Euristica della rappresentatività e Sostituzione di attributo: Linda

$\square$ Linda ha 31 anni, è single, schietta e molto brillante. Ha una laurea in filosofia.quando era studentessa era molto interessata ai temi della discriminazione e della giustizia sociale ed ha partecipato a dimostrazioni antinucleari.
$\square$ Adesso è più probabile che sia
$\square$ impiegata di banca
$\square$ impiegata di banca ed attiva nel movimento femminista

Ragionamento automatico e ragionamento deliberato : Gobet and Simon (1996) : pattern matching \& weighing of costs and benefits

$\square$ The acquisition of skill gradually increases the accessibility of useful responses and of productive ways to organize information, until skilled performance becomes almost effortless. This effect of practice is not limited to motor skills. A master chess player does not see the same board as the novice, and visualizing the tower in an array of blocks would also become virtually effortless with prolonged practice. (Kahneman)
$\square$ Often when one works at a hard question, nothing good is accomplished at the first attack. Then one takes a rest, longer or shorter, and sits down anew to the work. During the first half-hour, as before, nothing is found, and then all of a sudden the decisive idea presents itself to the mind. It might be said that the conscious work has been more fruitful because it has been interrupted and the rest has given back to the mind its force and freshness. But it is more probable that this rest has been filled out with unconscious work and that the result of this work has afterwards revealed itself to the geometer just as in the cases I have cited; only the revelation, instead of coming during a walk or a journey, has happened during a period of conscious work, but independently of this work which plays at most a role of excitant, as if it were the goad stimulating the results already reached during rest, but remaining unconscious, to assume the conscious form.

Grazie per l'attenzione

