INdAM Intensive period: Perspectives in Lie Theory

Pisa, December 2014 - February 2015

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Through the past century, Lie theory has developed in several directions, finding important applications in other areas of mathematics and physics. There exist deep connections among these directions. First of all, techniques and tools are often very similar. Secondly, some problems that arise in one context naturally induce questions that find their solution in another one. The combinatorics of root systems and Coxeter groups, for instance, is one of the fundamental ingredients in Lie theory, and some problems not directly connected with this language eventually amount to combinatorial statements. This is the case, for instance, of the problem of classifying finite and infinite-dimensional Lie algebras, studying symmetric and spherical varieties, as well as topological properties of hyperplane arrangements. In the same vein, some of the most significant results in the combinatorics of Coxeter groups are handled via the geometry of the above varieties, or the representation theory of Lie algebras. We believe that the collaboration between Lie theorists that have distinct education and interests may constitute a fundamental ingredient for our goals. In this special period we would like to focus mainly on the following themes:

- Homogeneous varieties and their compactifications
- Wonderful models of subspace and toric arrangements.
- Configuration and loop spaces.
- Combinatorics and topology of toric and hyperplane arrangements and representations of Artin groups.
- Combinatorics of Coxeter groups and groups generated by pseudoreflections.
- Infinite-dimensional Lie algebras, their generalizations and applications.
- Representation theory and Kazhdan-Lusztig conjecture.
- Affine groups and algebras, geometrical aspects and representation theory.

This INDAM Intensive Period will be divided in three sessions on different topics:

1. "Vertex algebras, W-algebras, and applications" from December 9th to December 20th 2014 and from January 12th to January 18th 2015;
2. "Lie Theory and Representation Theory" from January 19th to February 6th 2015;

3. “Algebraic topology, geometric and combinatorial group theory” from February 8th 2015 to February 28th 2015.

The idea of the Intensive Period is to have some interesting mini courses for PhD students and researchers and regular talks, but also time for discussions and collaboration in the beautiful setting of the city of Pisa.

Two satellite workshop will be part of this INDAM Intensive Period:

- INdAM Meeting “Configuration spaces: Geometry, Topology and Representation Theory”, in Cortona from August 31st to September 6th 2014;
- INdAM Workshop “Aspects of Lie Theory”, in Rome from January 7th to January 10th 2015.

First session: Vertex algebras, W-algebras, and applications

Since the 60’s, the theory of semisimple Lie algebras has been generalized in many directions. Vertex algebras have been introduced by Borcherds in the 80’s, drawing from Conformal Field Theory in physics, in the study of representations of the Fischer monster group. Lie superalgebras, and more recently n-superalgebras, are a natural generalization of Lie algebras and have gained growing importance in mathematics and theoretical physics, in the last few decades. Finally, quantizations of the enveloping algebra of a Lie algebra, the so-called quantum groups, have been introduced by Drinfeld and Jimbo in the 80’s as an important tool in the study of classical problems, and have found applications in distant areas of mathematics. In this special period we want to focus on the application of such structures towards problems in mathematical physics. In particular, we will be interested in the important problem of classifying Hamiltonian structures and the corresponding completely integrable systems via Poisson vertex algebras.

Second session: Lie Theory and Representation Theory

The study of geometry and representations of affine Lie groups and algebras achieved an increasing importance in the last 30 years starting with the connection with modular representation theory (Lusztig conjecture) and to the study of vector bundles on curves (solution of the KP equation and proof of Verlinde’s formula).

In certain cases the theory for affine algebras, even if presenting many more technical subtleties, follows closely the finite case. Indeed, it is possible to generalize some of the constructions and results which hold for semisimple Lie algebras: the theory of highest weight representations, topological properties of the singular locus of Schubert varieties in connection with Hecke algebras, the standard monomial theory applied to the description of the algebraic geometric aspects of the affine Grassmannian and Schubert varieties. Further aspects, while well-understood in the finite-dimensional case, turn out to give rise to new interesting phenomena, as for the Springer fibers. Finally, for this class of algebras completely new questions arise, such as the theory of critical level representations.

The theory of affine Lie algebras and affine Lie groups find applications to many subjects in Lie theory and algebraic geometry. We have already mentioned the relations with modular representation theory and the moduli space of vector bundles on curves, but they are also the central object in the geometric Langlands program, and they are intimately connected with the local models of Shimura varieties.

The intent of finding an algebraic proof for the Kazhdan-Lusztig conjecture on characters of irriducible modules for semisimple and affine Lie algebras, motivated Soergel conjecture. This consists in a purely algebraic statement, in terms of ranks of certain bimodules, which Soergel proved to imply the Kazhdan-Lusztig conjecture. Soergel's conjecture has been recently proven by Elias and Williamson, by adapting to this setting ideas from the work of de Cataldo and Migliorini and making use of categorification techniques.

Third session: Algebraic topology, geometric and combinatorial group theory

An important part of the present project is centered around the study of the geometry, topology and combinatorics of subspace and toric arrangements and of their wonderful models. We will focus on the role which arrangements, configurations spaces and their compactifications play in several fields of mathematical research: subspace and toric arrangements, toric varieties and tropical geometry, moduli spaces of curves, configuration spaces, box splines, index theory, discrete geometry, Coxeter groups and Artin groups, mapping class groups, loop spaces.

Minicourses during this session will enlight the connections of all this topics with algebraic topology, combinatorics and geometric group theory.