Conference PISA - 30 June 2014 - 5 July 2014

Name A. Adimurthi
University TIFR, Bangalore, India
Title & Abstract *Uniqueness and non degeneracy of positive solutions for critical exponent problem in R^2*
Brezis raised the question of uniqueness of positive solutions of critical
exponent problem in a Ball. This problem was settled in dimensions greater than or
equal to three. It was an open problem in two dimensions because of ineffectiveness of
Pohozaev's identity and the critical exponent is of exponential nature. In this talk we will
prove that the solutions are unique and non degenerate in balls of small radius for a
large class of non linearities.

Name S. Bandyopadhyay
University IISER, Kolkata, India
Title & Abstract *Calculus of Variations with Differential Forms*
In this lecture, we outline the functional framework relevant for calculus of variations
with differential forms. In this context, appropriate notions of convexity, namely ext.
one convexity, ext. quasiconvexity and ext. polyconvexity are introduced.
We study their relations, give several examples and counterexamples. We finally
conclude with an application to a minimization problem. It is a joint work with
Bernard Dacorogna and Swarnendu Sil.

Name F. Bethuel
University University Paris VI, France
Title & Abstract The Jacobian, the square root and the set ∞
We discuss the problem of prescribing the Jacobian determinant in dimension two: we restrict
ourselves to the case the datum is a finite sum of Dirac masses with integer multiplicity. The
main point here is to show that we may relate this problem to the search of harmonic maps into a
singular space which is shaped as the symbol ∞ (or the number 8). The later problem in turn is
closely linked to meromorphic functions. A large part of the presentation is devoted to a
presentation of the various mathematical objets and their connections.

Name	L. Boccardo
University	University of Rome, Italy
Title & Abstract	

A SCHRÖDINGER-MAXWELL SYSTEM

LUCIO BOCCARDO

In this talk Ω is a bounded open set in $\mathbb{R}^N$, $N \geq 2$, $M(x)$ is a symmetric measurable matrix such that, for $\alpha, \beta \in \mathbb{R}^+$,

$$\alpha|\xi|^2 \leq M(x)\xi\xi, \quad |M(x)| \leq \beta.$$

1. Paper dedicated to Benci-Fortunato on his "130th" birthday

Let $A, r > 0$ and $f(x) \in L^m(\Omega)$, $m \geq \frac{2N}{N+2}$ ("variational framework"), existence and "unexpected" summability properties of the solutions of

$$(1.1) \quad \begin{cases} 0 \leq u \in W_0^{1,2}(\Omega) : -\mathrm{div}(M(x)Du) + A\varphi u^r = f(x), \\ 0 \leq \varphi \in W_0^{1,2}(\Omega) : -\mathrm{div}(M(x)D\varphi) = u^{r+1}, \end{cases}$$

are studied in a recent paper. Moreover

$$\varphi u^r \in L^1(\Omega), \qquad u^{r+1} \in L^1(\Omega).$$

If $A = r+1$ and M is symmetric, a method for the proof of the existence is the study of the critical points of

$$\frac{1}{2}\int_\Omega MDvDv - \frac{1}{2}\int_\Omega MD\psi D\psi + \int_\Omega v|\psi|^{r+1} - \int_\Omega fv$$

thanks to a theorem by Benci-Rabinowitz (work in progress with D. Arcoya and T. Leonori).

2. Work in progress with L. Orsina

In a work in progress a "surprising" (existence of solutions in $W_0^{1,2}(\Omega)$ even outside of "variational framework") existence result is proved if $m = \frac{r+1}{r} < \frac{2N}{N+2}$, $r > \frac{N+2}{N-2}$.

Name	P. Bousquet
University	University Aix-Marseille, France
Title & Abstract	The divergence equation and Hardy inequalities for L^1 vector fields. We present an existence theorem for the divergence equation which extends to the whole scale of Sobolev spaces a limiting estimate due to Bourgain and Brezis. We also discuss new Hardy type inequalities

Name	G. Csato
University	TIFR - India
Title & Abstract	Non-Existence Theorems for Harmonic Fields on Riemannian Manifolds

Abstract

A simple example of a harmonic field is a vector field $u \in C^1(\overline{\Omega}; \mathbb{R}^3)$ on a domain $\Omega \subset \mathbb{R}^3$ which satisfies

$$\mathrm{div}\, u = 0 \quad \text{and} \quad \mathrm{curl}\, u = 0 \quad \text{in } \Omega.$$

If, moreover, ν is the outward unit normal on the boundary $\partial\Omega$ and $u_N = \langle \nu, u \rangle \nu = 0$ on $\partial\Omega$, we say that u has vanishing normal component u_N. I will talk about conditions on Ω which imply that harmonic fields with vanishing normal or tangential part have to be identically 0. Interesting is the case when Ω is replaced by a Riemannian manifold with boundary and the curvature properties of Ω and $\partial\Omega$ play an important role.

Name	A. Farina
University	University of Picardie, Amiens, France
Title & Abstract	Some new results on entire solutions of an elliptic system arising in phase separation

We study the positive solutions of the elliptic system

$$\begin{cases} \Delta u = uv^2 & \text{in } \mathbb{R}^N \\ \Delta v = vu^2 & \text{in } \mathbb{R}^N, \end{cases}$$

where $N \geq 2$.

We present some new results concerning the monotonicity and the one-dimensional symmetry of solutions with algebraic growth at infinity.

Name	A. Figalli
University	University of Texas, Austin / USA
Title & Abstract	A transportation approach to random matrices

Optimal transport theory is an efficient tool to construct change of variables between probability densities. However, when it comes to the regularity of these maps, one cannot hope to obtain regularity estimates that are uniform with respect to the dimension except in some very special cases (for instance, between uniformly log-concave densities).

In random matrix theory the densities involved (modeling the distribution of the eigenvalues) are pretty singular, so it seems hopeless to apply optimal transport theory in this context. However, ideas coming from optimal transport can still be used to construct approximate transport maps (i.e., maps which send a density onto another up to a small error) which enjoy regularity estimates that are uniform in the dimension. Such maps can then be used to show universality results for the distribution of eigenvalues in random matrices.

The aim of this talk is to give a self-contained presentation of these results.

Name	N. Fusco
University	University of Naples, Italy
Title & Abstract	Motion of elastic thin films by anisotropic surface diffusion with curvature regularization.

Short time existence for a surface diffusion evolution equation with curvature regularization is proved in the context of epitaxially strained three-dimensional films. This is achieved by implementing a minimizing movement scheme, which is hinged on the H^{-1}-gradient flow structure underpinning the evolution law. Long-time behavior and Liapunov stability in the case of initial data close to a flat configuration are also addressed.

Name	T. Gallouët
University	University Aix-Marseille, France
Title & Abstract	On the stationary compressible Stokes equations

abstract : A way to prove the existence of a solution for the
stationary compressible Stokes equations with a quite general Equation Of State is presented. The main interest of the method
is that it can be adapted to prove the convergence (as the discretization parameter
goes to zero) of an approximated solution given by a convenient numerical scheme.
Generalizations to the Navier-Stokes equations and to the evolution equations are also
given.

Name	N. Katzourakis
University	University of Reading, UK
Title & Abstract	On a new theory of weak solutions for fully nonlinear PDE systems
	I will present some basics of a new approach in PDE theory which allows to define and handle non-differentiable solutions of systems of equations which are in non-divergence form and perhaps fully nonlinear, being a duality-free counterpart of linear distributions. I will present some existence results for linear and nonlinear systems of 1st and 2nd order, and also applications to vectorial Calculus of Variations in the L-Infinity norm.

Name	L. Keller
University	EPFL, Lausanne, Switzerland
Title & Abstract	Minimization of the Willmore functional under isoperimetric constraint
	The Willmore energy of an immersion of an abstract oriented surface is given by the squared mean curvature integrated with respect to the induced volume form.
	We study the minimization of this energy under the constraint of given isoperimetric ratio, i.e. under prescribed area and prescribed enclosed volume and for underlying abstract surfaces of positive genus. For this problem we present existence and regularity results.

Name	O. Kneuss
University	University of Zürich, Switzerland
Title & Abstract	Bi-Lipschitz Stretching of Measurable Sets in the Plane and Applications to Nonlinear Elasticity
	We construct a bi-Lipschitz mapping stretching a given measurable subset of the plane with small Lebesgue measure by some chosen factor close to 1 and control, at the same time, its Lipschitz constant uniformly. We then apply our result to a model problem in nonlinear elasticity. This is a joint work with J. Fischer.

Name	J. Kristensen
University	University of Oxford, UK
Title & Abstract	Rank--one convexity and integral estimates
	It is known that many interesting questions about sharp integral estimates involving derivatives of mappings can be reformulated as questions about quasiconvexity properties of associated integrands. In this talk we discuss the strong convexity properties of one--homogeneous rank--one convex integrands and some of their consequences for integral estimates.
	The talk is based on joint work with Bernd Kirchheim (Leipzig).

Name	G. Leoni
University	Carnegie Mellon University, USA
Title & Abstract	Second order Gamma convergence for the Modica Mortola functional
	I will discuss some recent results on the second order asymptotic development by Gamma convergence of the Modica-Mortola functional.

Name	P. Marcellini
University	University of Florence, Italy
Title & Abstract	A variational approach to parabolic systems

We consider a purely variational approach to time dependent problems,
yielding the existence of global parabolic minimizers.
These evolutionary variational solutions are obtained as limits of maps
depending on space and time minimizing certain convex variational functionals.
In the simplest situation, with some growth conditions on f, the method provides the existence
of global weak solutions to Cauchy-Dirichlet problems of parabolic systems.
This is a joint collaboration with V. Bögelein and F. Duzaar.

Name	G. Mingione
University	University of Parma, Italy
Title & Abstract	Regularity for double phase variational problems

I am going to describe a few sharp regularity theorems for minimizers of a
family of functionals whose integrand switches between two different types
of elliptic phases. This is joint work with Maria Colombo (Pisa).

Name	F. Murat
University	University Paris VI, France
Title & Abstract	A semilinear elliptic problem with a singularity in u = 0

In this joint work with Daniela Giachetti (Rome) and Pedro J. Martinez Aparicio (Cartagena, Spain) we consider the problem

$$-\mathrm{div}\,A(x)Du = F(x,u) \text{ in } \Omega,$$

$$u = 0 \text{ on } \partial\Omega,$$

(namely an elliptic semilinear equation with homogeneous Dirichlet boundary condition), where the non-linearity $F(x,u)$ is singular in $u = 0$, with a singularity of the type

$$F(x,u) = \frac{f(x)}{u^\gamma} + g(x)$$

with $\gamma > 0$ and f and g non negative (which implies that also u is non negative).

The main difficulty is to give a convenient definition of the solution of the problem, in particular when $\gamma > 1$. We give such a definition and prove the existence and stability of a solution, as well as its uniqueness when $F(x,u)$ is non increasing en u.

We also consider the homogenization problem where Ω is replaced by Ω^ε, with Ω^ε obtained from Ω by removing many very small holes in such a way that passing to the limit when ε tends to zero the Dirichlet boundary condition leads to an homogenized problem where a "strange term" μu appears.

This work was inspired by the paper of Lucio Boccardo and Luigi Orsina *Semilinear elliptic equations with singular nonlinearities*, Calc. Var. Partial Differential Equations, 37, (2010), pp. 363–380.

Name	I. Peral
University	University of Madrid, Spain
Title & Abstract	Some elliptic and parabolic problems of fourth order related to a variational epitaxial growth model

We study a model related to *epitaxial growth* of crystals that involves equations of the type,

$$u_t + \Delta^2 u = \det\left(D^2 u\right) + \mu f(x, u), \quad x \in \Omega \subset \mathbb{R}^2,$$

Ω smooth bounded domain, $\mu \geq 0$ and with several boundary conditions.

We analyze the associated stationary equation and the evolution equation.

Motivated for the problem above, we will also review some results for equations of the type,

$$\Delta^2 u = S_2(\lambda_1(D^2 u), \lambda_2(D^2 u), ..., \lambda_N(D^2(u)) + \mu f(x, u), \quad x \in \Omega \subset \mathbb{R}^N,$$

where $\lambda_i(D_2 u)$ are the solution to the equation

$$\det\left(\lambda I - D^2 u\right) = 0,$$

that is, the eigenvalues of the hessian matrix of u and

$$S_2(\xi_1, ..., \xi_N) := \sum_{1 \leq i < j \leq N} \xi_i \xi_j.$$

Name	G. Pisante
University	Università di Napoli, Italy
Title & Abstract	Blow-up Results for Semilinear Filtration Problems

In this talk we will present some qualitative properties of solutions to the parabolic initial boundary value problem associated to the generalized porous medium equation. More precisely, we consider the following problem:

$$\begin{cases} u_t = \Delta K(u) + \lambda f(u), & x \in \Omega, \quad t > 0, \\ \dfrac{\partial K(u)}{\partial n} + \beta(x) K(u) = 0, & x \in \partial\Omega, \quad t > 0, \\ u(x, 0) = u_0(x) & x \in \Omega, \end{cases}$$

where $u = u(x, t)$, n is the outward pointing normal vector field on $\partial\Omega$ and Ω is a bounded domain of $\mathbb{R}^N$, with sufficiently smooth boundary $\partial\Omega$. Under suitable hypotheses on the functions K and f, the existence of local classical solutions can be proved. Aim of the talk, which is based on a joint work with K. Fellner and E. Latos, is to discuss several situations where the solution exhibits a blow-up behaviour.

Name	A. Pratelli
University	University of Erlangen-Nürnberg, Germany
Title & Abstract	On the approximation of W^{1,1} planar homeomorphisms by diffeomorphisms.

The task of approximating a homeomorphism by smooth
diffeomorphisms is a quite delicate but importat one, and it has been
extensively studied in the last decades. In particular, it is known,
thanks to a series of papers by Iwaniec, Kovalev and Onninen, that any
W^{1,p} planar homeomorphism can be approximated in the W^{1,p} norm by
diffeomorphisms, but this only worked for p>1. Only very recently we
have proved, in a paper with Hencl which makes use of a completely
different strategy, that the same result holds true for p=1. In this
talk I will describe the history of the problem and the most important
results, and then I will pass to illustrate the main ideas of the
construction of the above-mentioned result with Hencl.

Name	R. Schiattarella
University	Università di Napoli, Italy
Title & Abstract	On the approximate differentiability of the inverse mapping

Abstract. A *chain rule* for an a.e. approximatively differentiable homeomorphism $f : \Omega \subset \mathbb{R}^n \xrightarrow{\text{onto}} \Omega' \subset \mathbb{R}^n$ is proved. Namely, there exists a Borel set $B \subset \mathcal{R}_f^{\text{ap}} = \{x \in \Omega : f \text{ is approximately differentiable at } x \text{ and } J_f(x) \neq 0\}$ such that $|\mathcal{R}_f^{\text{ap}} \setminus B| = 0$, $f(B) \subset \mathcal{R}_{f^{-1}}^{\text{ap}}$ and
$$J_{f^{-1}}(f(x))J_f(x) = 1 \qquad \forall x \in B.$$
Moreover, $f(B)$ has full measure in $\mathcal{R}_{f^{-1}}^{\text{ap}}$.

In general, it is not true that $f(\mathcal{R}_f^{\text{ap}}) \subset \mathcal{R}_{f^{-1}}^{\text{ap}}$. Indeed, there exists a homeomorphism $f_0 : \Omega \subset \mathbb{R}^n \xrightarrow{\text{onto}} \Omega' \subset \mathbb{R}^n$ such that f_0 is approximately differentiable at x_0 with $J_{f_0}(x_0) \neq 0$ and f_0^{-1} is not approximately differentiable at $f_0(x_0)$.

This study was initiated in [N. Fusco, G. Moscariello, C. Sbordone: *The limit of* $W^{1,1}$*-homeomorphisms with finite distortion*, Calc. Var., 33, (2008), 377–390] and in [S. Hencl:*Sharpness of the assumptions for the regularity of a homeomorphism*, Michigan Math. J. 59 (2010), no. 3, 667–678.]

Name	M. Willem
University	Catholic University of Louvain-la-Neuve, Belgium
Title & Abstract	Critical variational problems in the space of functions of bounded variation

We prove, by a new elementary method, the existence of optimal functions for the
Poincaré inequality in BV. Using a relative isoperimetric inequality, we prove also
some estimates for the best constant of the Poincaré inequality.