Stochastic Processes and Stochastic Calculus
with application to Economics and Finance.

San Miniato, September 12-16, 2015

The course will be devoted to an elementary but rigorous introduction to the theory of stochastic processes.

After briefly recalling notation and essential definitions in probability theory (random variables, laws, conditional expectations and independence), Markov chains will be considered as basic example of discrete-time processes. Then, we will study continuous time Markov jump processes, with applications (Poisson process, birth-death processes, queuing theory).

We will then introduce Brownian motion (or Wiener process), as the fundamental example of a diffusion process, and stochastic calculus, focusing on stochastic integration and Ito formula.

As applications, we will study stochastic differential equations and their link with partial differential equations, Girsanov theorem and the martingale representation theorem.

We will conclude with a short overview of option pricing theory (Black-Scholes formulas and extensions) and on interest rate models.

Most of the lectures will focus on mathematical aspects, but examples of applications as well as exercises, motivated by economics or finance will be delivered throughout.

Schedule of the course

Monday
Review of Basic Concepts of Probability: Conditional Probability and Independence; Random Variables and their Laws; Conditional Expectation.

Tuesday
Discrete-time Markov Chains: Classification of States; Stationary Distributions. Markov Jump Processes: Basics of Queuing Theory; Poisson Process.

Wednesday
Discrete and continuous-time Martingales. Brownian motion in one and several dimensions. Stochastic Integration and Ito Formula.

Thursday
Applications of Stochastic Calculus: Girsanov Theorem; the Martingale Representation Theorem; Stochastic Differential Equations and their links with Partial Differential Equations.

Friday
Option Pricing Models (Samuelson-Black-Scholes); interest rate Models.

Prerequisites:
Multivariate differential and integral calculus. Knowledge of basic notions in probability theory is not mandatory, but strongly recommended.

Teachers:

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