

From order to chaos

Pisa, CRM “De Giorgi”, 9–13 April 2018

Monday

- 11.30–12.20: DMITRY DOLGOPYAT, *Local Limit Theorem for Dynamical Systems and Applications*.

It is a hallmark of chaos that many ergodic sums of many dynamical systems satisfy the Central Limit Theorem. The local limit theorem requires the Gaussian asymptotics to be valid for probabilities of hitting small intervals. Those questions are much less understood compared to the Central Limit Theorem for dynamical systems. In my talk I will describe several recent results and open questions.

- 14.30–15.20: KONSTANTIN KHANIN, *Schrödinger operators with quasi-periodic potentials and the Aubry-Mather theory*.

Schrödinger operators with quasi-periodic potentials were intensively studied in the last few decades. Their spectral properties depend on the value of the coefficient in front of the potential, so called coupling constant. For small values of the coupling constant the spectrum is absolutely continuous, while for large coupling constants the spectrum is pure point. Natural families of Schrödinger operators with quasi-periodic potentials appear in the context of the Aubry-Mather theory. We shall discuss the transition from the absolutely continuous to the pure point spectrum for such families of Schrödinger operators.

- 16.00–16.50: URSULA HAMENSTÄDT, *Simplicity of the Lyapunov spectrum for flat bundles*.

We present a new method to show simplicity of the Lyapunov spectrum for a cocycle over a geometric flow defined by a flat bundle. This applies to the Kontsevich-Zorich cocycle over a stratum of abelian or quadratic differentials with respect to the invariant Lebesgue measure and also to cocycles over geodesic flows of compact negatively manifolds. We discuss some applications.

Tuesday

- 09.10–10.00: PASCAL HUBERT, *Rigidity of square-tiled interval exchange transformations*.

In this talk, I will explain when square tiled interval exchange transformation are rigid (in the sense of ergodic theory). I will also give new examples where one can prove/disprove rigidity (one of them is a famous example by Veech from 1968). This is a joint work with Sébastien Ferenczi.

- 10.30–11.20: SÉBASTIEN GOUËZEL, *Ruelle resonances for linear pseudo-Anosov maps*.

The Ruelle resonances of a dynamical system are spectral characteristics of a

system, describing the precise asymptotics of correlations. While their existence can often be shown by abstract spectral analysis arguments, it is in general not possible to compute them exactly. I will explain that, in the case of linear pseudo-Anosov maps, they can be completely described in terms of the action of the map on cohomology. Joint work with Frédéric Faure and Erwan Lanneau

- 11.30–12.20: YANQI QIU, *Determinantal point processes, Lyons-Peres completeness conjecture and Palm equivalence.*

In this talk, I will talk about the conditional measures of determinantal point processes and its application in resolving the Lyons-Peres completeness conjecture (joint with Alexander Bufetov and Alexander Shamov). The formalism of these conditional measures also allow us to obtain the Palm equivalence of determinantal point processes with Bergman kernels in very general domains of any dimensional complex spaces (joint with Alexander Bufetov and Shilei Fan).

- 14.30–15.20: KONSTANTIN MATVEEV, *Boundary of the Young graph with the Macdonald multiplicities and Kerov's conjecture.*

I will talk about the recent proof of the Kerov's conjecture (1992) classifying the homomorphisms from the algebra of symmetric functions to reals with non-negative values on the Macdonald functions. This allows to describe the boundary of the Young graph with the Macdonald multiplicities. For the special case of the Schur functions this is equivalent to classifying totally non-negative infinite Toeplitz matrices, and the result was first proved by Schoenberg, Edrei, et.al. in the beginning of the 1950s. Their motivation came from Analysis, but in the 1960s Thoma has discovered a connection with the representation theory of the infinite symmetric group. Some other special cases of the Kerov's conjecture are also connected to asymptotic representation theory. Our proof is a combination of two methods. 1) Developing in the Macdonald generality the "pole elimination" argument developed for the Schur case by Schoenberg. 2) A new method based on showing certain diffusivity in the branching graph of the Macdonald functions. I will explain all the relevant notions.

- 16.00–16.50: CARLOS MATHEUS, *The difference between Lagrange and Markov after Hausdorff.*

After the remarkable works of Markov in 1879 and 1880, the Lagrange and Markov spectra (coding arithmetic properties of irrational numbers and indefinite binary quadratic forms) were studied by several authors (including Hurwitz, Perron, Hall, Freiman, Cusick, Flahive, ...). In this talk, we will discuss the complement of the Lagrange spectrum L in the Markov spectrum M . More precisely, after recalling the results of Freiman, Cusick and Flahive from the 70's showing that $M \setminus L$ contains a countable, infinite subset, we will show that the Hausdorff dimension of $M \setminus L$ is strictly between 0 and 1.

Wednesday

- 09.10–10.00: JENS MARKLOF, *Kinetic transport in the Lorentz gas: classical and quantum.*

I will survey old and new results on the Boltzmann-Grad limit of the Lorentz gas. I will in particular focus on recent results with Andreas Strombergsson

(Uppsala) for the case of quasicrystalline scatterer configurations, and also discuss the Boltzmann-Grad limit for quantum transport in the case of smooth periodic potentials, which is joint work with Jory Griffin (Queens University, Canada).

• 10.30–11.20: ANDREY DYMOV, *A functional limit theorem for the sine-process*. Determinantal random point processes form an important class of random point processes and naturally arise in different areas of mathematics. In particular, in the random matrix theory, mathematical physics and the number theory. One of the most known determinantal processes in the sine-process. It is well-known that a large class of determinantal processes including the sine-process satisfies the Central Limit Theorem. For many dynamical systems satisfying the CLT the Donsker Invariance Principle also takes place. The latter states that trajectories of the system can be approximated by trajectories of the Brownian motion, in appropriate sense. I will present my joint work with A. Bufetov, where we prove a functional limit theorem for the sine-process, which turns out to be very different from the Donsker Invariance Principle. We show that the anti-derivative of our process can be approximated by the sum of a linear Gaussian process and small independent Gaussian fluctuations which are governed in appropriate sense by the Gaussian Free Field on the plane.

• 11.30–12.20: LIVIO FLAMINIO, *Nil-sequences, multiplicative functions and asymptotic orthogonal powers*.

We prove that the affine unipotent and ergodic diffeomorphisms of nilmanifolds T have asymptotic orthogonal powers, that is all ergodic joinings of the the products $T^r \otimes T^s$ converge to the product joining, when the relatively prime integers r and s go to infinity. In collaboration with K. Frączek, J. Kułaga-Przymus and M. Lemańczyk

Thursday

• 09.10–10.00: ALEXANDRA SKRIPCHENKO, *Interval exchange transformation with flips: what do they hide?*.

Interval exchange transformation with flips is a very natural generalization of orientation preserving interval exchange transformation: we consider a piecewise isometry of an interval to itself with a finite number of jump discontinuities, reversing the orientation of at least one of the intervals of continuity. However, flips change dramatically the dynamics: due to Arnaldo Nogueira we know that generic interval exchange transformation with flips has finite orbits. I will discuss the set of minimal interval exchange transformations with flips (its Hausdorff dimension, combinatorial properties and open questions related to it). The talk is mainly based on recent joint work with Serge Troubetzkoy.

• 10.30–11.20: THEODOROS ASSIOTIS, *Stochastic dynamics for the Hua-Pickrell measures*.

The Hua-Pickrell measures form a distinguished family of unitarily invariant measures on the space of infinite Hermitian matrices, much studied in the theory of random matrices. I will describe how using techniques from integrable probability and making use of some remarkable exact solvability properties one can construct a natural Feller Markov process preserving these measures. Time permitting I will talk about further consequences of this construction and some work in progress.

- 11.30–12.20: MILTON MINERVINO, *Tree substitutions and Rauzy fractals*.

Symbolic dynamical systems generated by Pisot substitutions can be interpreted geometrically by Rauzy fractals. The Pisot conjecture states that these dynamical systems have pure discrete spectrum, which is the case if the associated Rauzy fractals tile the space where they are represented. The aim of this talk is to show how to construct certain self-similar trees which fill these fractal domains, under the additional hypothesis that the substitutions are parageometric free group automorphisms. This allows to link the dynamics of the substitutive system with that of an interval exchange transformation.

- 14.30–15.20: YAROSLAV NAPRIENKO, *Convergent asymptotic expansions for Gaussian Ensembles*.

In this talk, first, I will tell about Wigner Semicircle Law and some results on asymptotic expansions. After that, I'll introduce Kopelevitch's theorem on convergent asymptotic expansion for Gaussian Unitary Ensemble. In the end, I'll tell about generalizations of this results to all Gaussian Ensembles. Also, I'll include some intermediate results: differential equations on the densities and their moment generating functions for all Gaussian Ensembles and explicit formulas for Kopelevitch's theorem.

- 16.00–16.50: CORINNA ULCIGRAI, *Shearing properties and disjointness in Arnold flows*.

Shearing is key phenomenon in the study of parabolic systems. Quantitative shearing properties were for example used by Marina Ratner and many others to study fine spectral properties of horocycle flows (such disjointness and joining rigidity). We define a new disjointness criterion based on a quantitative form of shearing and show that it can be applied to study other smooth parabolic flows, in particular to prove disjointness properties in a class of Arnold flows (locally Hamiltonian flows on tori). As a corollary, this also proves some instances of Sarnak conjecture on Möbius orthogonality. This is joint work with Adam Kanigowski and Mariusz Lemańczyk.

Friday

- 09.10–10.00: PAVEL NIKITIN, *Rigidity for determinantal and Pfaffian processes*. Determinant point processes (a classical example of such a process is the distribution of the eigenvalues of a Gaussian unitary ensemble of random matrices) often have an amazing property of rigidity: the number of particles within any bounded set is completely determined by the configuration outside of this set. We plan to recall how this property can be proved in determinantal case, and to show how we can generalize the proof to the Pfaffian sine and Bessel processes. Joint work with Alexander Bufetov and Yanqi Qiu.

- 10.30–11.20: JORDAN EMME, *Random Matrix Product and Arithmetic Applications*.

In a work with Pascal Hubert, we study limit laws for some particular random matrix products and we give an arithmetic application: Let us denote by s_2 the sum-of-digit function in base 2. We consider, for every pair of integers a and d , the asymptotic density of the set of integers n such that $s_2(n+a) - s_2(n) = d$. We denote this asymptotic density by $\mu_a(d)$. For every integer a , μ_a is a probability measure on \mathbb{Z} . We show that for every shift-invariant ergodic probability measure on $\{0, 1\}^{\mathbb{N}}$, and for every integer sequence (a_n) whose binary decom-

position follow the prefixes of a generic point in $\{0,1\}^{\mathbb{N}}$ for this measure, the probability measures (μ_{a_n}) satisfy a central limit theorem. This is done by computing the characteristic functions of these measures, which are given by random matrix products.

- 11.30–12.20: EVGENY VERBITSKIY, *Solvable models from the ergodic point of view*.

In this review talk, I will discuss a link between solvable models of statistical mechanics (dimers, spanning trees, sandpiles) and algebraic dynamical systems. Even though the question about the existence of such a link was raised almost two decades ago, this problem remained largely inaccessible. The development of the theory of symbolic covers of algebraic dynamical systems has only recently provided a suitable framework.

	Monday	Tuesday	Wednesday	Thursday	Friday
10.30–11.00		09.10–10.00	Marklof	Skripchenko	Nikitin
11.00–11.30	Registration	10.00–10.30	Coffee Break		
11.30–12.20	Coffee Break	10.30–11.20	Dymov	Assiotis	Emme
12.30–14.30	Dolgopyat	11.30–12.20	Flaminio	Minervino	Verbitskiy
14.30–15.20	Khanin	Lunch			
15.30–16.00	Coffee break	14.30–15.20		Naprienko	
16.00–16.50	Hamenstädt	15.30–16.00	Free	Coffee break	Free
		16.00–16.50	Matheus	Ulcigrai	
Evening	Poster session & Welcoming aperitive			Social Dinner	