Abstracts

Complex ODEs: Asymptotics, Orthogonal Polynomials and Random Matrices

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The correlation kernel for a new Double Aztec Diamond with blocks

We study a Double Aztec Diamond, which is two Aztec Diamonds glued together appropriately and we either add or subtract from both sides symmetrically and simultaneously a few blocks. The calculation of the correlation kernel for this case leads to a singular Szego determinant, which enormously complicates the process of taking the large size limit, as opposed to previous situations. The appropriate large size limit is conjectured to be universal and agree with a large size limit for non-convex lozenge tilings.

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Jacobi matrices on trees and multiple orthogonal polynomials

Multiple orthogonal polynomials are known to satisfy recurrence relations on the lattice $(\mathbb{Z}^+)^d$. We use these relations to construct self-adjoint operator (Jacobi matrix) on the tree. This tree is a homogenous infinite rooted tree with homogeneity degree which is equal to $d + 1$ (i.e., each vertex has one "parent" (incoming) edge and d "children" (outgoing) edges). The case $d = 1$ gives the polynomials orthogonal on the real line, the tree becomes $\mathbb{Z}^+$, and the Jacobi matrix is the standard three-diagonal matrix. We shall discuss multiple orthogonal polynomials approach to the spectral theory of multidimensional discrete Schrödinger operator and corresponding discrete integrable systems. It

is a joint work with Sergey Denisov and Maxim Yattselev. The talk is based on the joint work [1].

References

[1] *Aptekarev A.I., Kaliaguine V.A. and Saff E.B.* Higher-Order Three-Term Recurrences and Asymptotics of Multiple Orthogonal Polynomials // Constr. Approx.,, V. 30(2). 2009. P. 175–223.

Árpád Baricz

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Zeros of orthogonal polynomials near an algebraic singularity of the measure

The local zero behavior of some orthogonal polynomials around an algebraic singularity is studied, when the measure of orthogonality is supported on $[-1, 1]$ and behaves like $h(x)|x-x_0|^\lambda dx$. It is shown that the so-called fine zero spacing, which is known for $\lambda = 0$,unravels in the general case, and the asymptotic behavior of neighbouring zeros around the singularity can be described with the zeros of the linear combination of Bessel functions of the first kind. Moreover, using SturmLiouville theory, the behavior of the zeros of this linear combination of Besselfunctions is studied. The talk is based on a recent paper with Tivadar Danka (University of Szeged, Hungary).

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A two parameter extension of the Urbanik semigroup

A Stieltjes moment sequence is a sequence of non-negative numbers of the form

$$s_n = \int_0^\infty t^n \, d\mu(t), \quad n \in \mathbb{N}_0 := \{0, 1, 2, \ldots\}, \tag{1}$$

where μ is a positive measure on $[0, \infty)$ such that $t^n \in L^1(\mu)$ for all $n \in \mathbb{N}_0$. The sequence (s_n) is called *normalized*, if $s_0 = \mu([0, \infty)) = 1$, and it is called S-*determinate* (resp. S-*indeterminate*) if (1) has exactly one (resp. several) solutions μ as positive measures on $[0, \infty)$. A Stieltjes moment sequence (s_n) is called *infinitely divisible*, if (s_n^c) is a Stieltjes moment sequence for any $c > 0$.

Examples of Stieltjes moment sequences are

$$1/(n + 1), \quad n!, \quad (2n - 1)!!, \quad C_n = \binom{2n}{n}/(n + 1), \quad \frac{2^{n+1} - 1}{n + 1}$$

and while the first four are infinitely divisible, the last one is not.

That $(n!)^c$ is a Stieltjes moment sequence for any $c > 0$ was proved by Urbanik in (1992) by finding a product convolution semigroup of probability densities $e_c, c > 0$ on the half-line with these moments. It holds that $(n!)^c$ is

S-determinate if and only if $c \leq 2$. The S-determinacy for $c \leq 2$ follows easily by Carleman's criterion, but the S-indeterminacy for $c > 2$ is more delicate and was settled by Berg in (2005) using asymtotic properties of one-sided stable distributions going back to Prohorov. In 2015 Berg and López found the behaviour of $e_c(t)$, $t \to \infty$ by asymptotic methods, and we decided to call $e_c, c > 0$ the Urbanik semigroup. The numbers C_n are integers called the Catalan numbers and they were proved to be infinitely divisible by Lin (ArXiv:1711.01536). In a recent paper on ArXiv:1802.00993 we prove an extension of Urbanik and Lin's results by proving that for any $a, b > 0$

$$s_n(a,b) = \Gamma(an+b)/\Gamma(b)$$

is a normalized infinitely divisible Stieltjes moment sequence and that $(s_n(a,b)^c)$ is S-determinate iff $ac \leq 2$. The numbers $s_n(a,b)^c$ are the moments of a product convolution semigroup of probability densities $e_c(a,b)$ on the half-line given by

$$e_c(a,b)(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} t^{ix-1}[\Gamma(b-iax)/\Gamma(b)]^c\,dx, \quad t > 0.$$

In particular we find the asymptotic behaviour of $e_c(a,b)(t)$ for $t \to \infty$.

Mattia Cafasso

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Non-commutative Painlevé equations and systems of interacting particles

I will discuss some recent results on non-commutative generalisations of the Painlevé equations, the relation with the theory of matrix orthogonal polynomials and applications to systems of interacting particles of Calogero-Moser type. These results had been obtained in collaboration with M.Bertola, M. D. de la Iglesia and V. Roubstov.

Kenier Castillo

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On zeros of paraorthogonal polynomials on the unit circle

The main purpose of this talk is to extend in a simple and unified way the known results on interlacing of zeros of paraorthogonal polynomials on the unit circle.

Reference: K. Castillo and J. Petronilho, Refined interlacing properties for zeros of paraorthogonal polynomials on the unit circle, Proc. Amer. Math. Soc. (2018). In press.

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Asymptotic gap probability distributions of the Gaussian unitary ensembles and the Jacobi unitary ensembles

Ncl. Phys. B, 926 (2018) 639–670.

In this paper, we address a class of problems in unitary ensambles. Specifically, we study the probability theta gap symmetric about 0, i.e., $(-a, a)$ is found in the Gaussian unitary ensembles (GUE) and the Jacobi unitary ensembles(JUE) (where in the JUE, we take the parameters $\alpha = \beta$). By exploiting the even parity of the weight, a doubling of the interval to (a^2, ∞) for the GUE, and $(a^2, 1)$, for the (symmetric) JUE shows that the gap probabilities maybe determined as the product of the smallest eigenvalue distributions of of the LUE with parameter $\alpha = -1/2, \alpha = 1/2$ and the (shifted) JUE with weights $x^{(}1/2)(1-x)^{\beta}$ and $x^{(}-1/2)(1-x)^{\beta}$. The sigma function, namely, the derivative of the log of the smallest eigenvalue distributions of the finite-n LUE or the JUE, satisfies the Jimbo-Miwa-Okamoto sigma form for PV and PVI, although in the shifted Jacobi case, with the weight $x^{\alpha}(1-x)^{\beta}$, the β parameter does not show up in the equation. We also obtain the asymptotic expansions for the smallest eigenvalue distributions of the LUE, and the JUE after appropriate double scalings, and obtained the constants in the asymptotic expansion of the gap probabilities, expressed in term of the Barnes G-function evaluated at special point.

Yang CHEN (Macau), Shulin Lyu (Guangzhou), and Engui FAN (Shanghai).

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The smallest eigenvalue of large Hankel matrices

We investigate the large N behavior of the smallest eigenvalue, λ_N, of an $(N+1)\times(N+1)$ Hankel (or moments) matrix H_N, generated by the weight $w(x) = x^\alpha(1-x)^\beta$; $x \in [0;1]$; $\alpha > -1; \beta > -1$. By applying the arguments of Szegö, Widom and Wilf, we establish the asymptotic formula for the orthonormal polynomials $P_n(z)$; $z \in \mathbb{C} \setminus [0;1]$, associated with $w(x)$, which are required in the determination of λ_N. Based on this formula, we produce the expressions for λ_N, for large N.

Using the parallel algorithm presented by Emmart, Chen and Weems, we show that the theoretical results are in close proximity to the numerical results for sufficiently large N.

This is the joint work with Mengkun Zhu, Niall Emmart and Charles Weems.

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On the probability that all eigenvalues of random matrices lie within an interval.

We present the probability that all eigenvalues of Gaussian, Wishart and double Wishart random matrices lie within the bulk given by the Marcenko-Pastur or the semicircle laws, and show that it tends to the universal limiting values of 0.6921 and 0.9397 in the real and complex cases, respectively. We also derive algorithms for calculating the exact probability that all eigenvalues lie within an interval for some random matrices of finite (large) dimensions, with application to information theory and signal processing.

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Generalized Hahn polynomials of type I

We will present some properties of the polynomials orthogonal with respect to the weight function

$$\rho(x) = \frac{(a_1)_x (a_2)_x}{(b+1)_x} \frac{z^x}{x!}, \quad x = 0, 1, \ldots.$$

These polynomials were introduced in [1], as part of the classification of the discrete semiclassical orthogonal polynomials of class 1.

References

[1] D. Dominici and F. Marcellán. Discrete semiclassical orthogonal polynomials of class one. *Pacific J. Math.*, 268(2):389–411, 2014.

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Resurgence monomials and moduli of dynamical systems

We shall describe families of resurgent functions introduced by J. Ecalle, named as resurgence monomials for the simple and explicit behaviour they display with respect to alien derivations and implement them for the classification of some analytic dynamical systems with irregular singularities.

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Noise analysis of eigenlevel sequences in quantum chaotic systems

The power spectrum analysis of stochastic spectra has emerged as a powerful tool for studying both system-specific and universal properties of complex systems. In the context of complex quantum systems, it reveals whether the corresponding classical dynamics is regular or chaotic, or a mixture of both, and encodes a degree of chaoticity. In combination with other long- and short-range spectral fluctuation measures, it provides an effective way to identify system symmetries, determine a degree of incompleteness of experimentally measured spectra, and get the clues about systems internal structure. Yet, the theoretical foundations of the power spectrum analysis of stochastic spectra have not been settled. In this talk, I shall formulate a non-perturbative approach [Phys. Rev. Lett. 118, 204101 (2017)] to the power spectrum of energy level fluctuations in fully chaotic quantum structures. In the particular case of broken time-reversal symmetry, our theory produces a parameter-free prediction for the power spectrum expressed – in the domain of its universality – in terms of a fifth Painlevé transcendent. Finally, I shall present fair evidence that a universal Painlevé V curve can be observed in the spectrum of Riemannium. [Joint work with V. Al. Osipov (University of California, Irvine) and R. Riser (University of Haifa, Israel)].

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Gevrey multiscale expansions of singular solutions of PDEs with cubic nonlinearity

Joint work with S. Malek.

We study a singularly perturbed PDE with cubic nonlinearity depending on a complex perturbation parameter ϵ. This is the continuation of the precedent work [1] by the first author. We construct two families of sectorial meromorphic solutions obtained as a small perturbation in ϵ of two branches of an algebraic slow curve of the equation in time scale. We show that the nonsingular part of the solutions of each family shares a common formal power series in ϵ as Gevrey asymptotic expansion which might be different one to each other, in general.

References

[1] S. Malek, *On singular solutions to PDEs with turning point involving a quadratic nonlinearity*, Abstract and Applied Analysis, vol. 2017, Article ID 9405298, 2017.

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Non-Abelian integrable Hierarchies: Matrix biorthogonal polynomials and spectral perturbations

In this presentation Geronimus–Uvarov perturbations for matrix orthogonal polynomials in the real line are studied and then applied to the analysis of non Abelian integrable hierarchies. The orthogonality is understood in full generality, i.e., in terms of a nondegenerate continuous sesquilinear form that is determined by a quasidefinite matrix of bivariate generalized functions with a well defined support. We derive Christoffel type formulas that give the perturbed matrix biorthogonal polynomials and its norms in terms of the original ones. The keystone is the Gauss–Borel factorization of the Gram matrix. Geronimus–Uvarov transformations are considered in the context of the 2D non–Abelian Toda lattice and noncommutative KP hierarchies. The interplay between transformations and integrable flows is discussed. Miwa shifts, τ-ratio matrix functions and Sato formulas are given. Bilinear identities, involving Geronimus–Uvarov transformations, first for the Baker functions, second for the biorthogonal polynomials and its second kind functions as well as for the τ-ratio matrix functions are deduced.

This is a joint work with G. Ariznabarreta and M. Mañas (Universidad Complutense de Madrid, Spain) and J. C. García Ardila (Universidad Rey Juan Carlos, Spain).

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Lozenge tilings of non-convex regions and asymptotic fluctuations: a new universality class

Tilings of polygonal regions having two opposite non-convexities (cuts) lead to a new kernel and a new statistics for the asymptotic fluctuations, when the size of the region and the cuts gets large under an appropriate scaling. The limiting statistics has been observed in very different circumstances.

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Extreme events versus extreme random matrices

Using the introduced by us "thinning method", we explain the link between classical Fisher-Tippett-Gnedenko classification of extreme events and their free analogue obtained by Ben Arous and Voiculescu in the context of free probability calculus. In particular, we present explicit examples of large random matrix ensembles, realizing free Weibull, free Frechet and free Gumbel limiting laws, respectively. We also explain, why these free laws are identical to Balkema-de Haan-Pickands limiting distribution for exceedances, i.e. why they have the form of generalized Pareto distributions and derive a simple exponential relation between classical and free extreme laws.

Emmanuel Paul

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The Painlevé II equation from the isomonodromic point of view

It is known that all the Painlevé equations turn out to be the isomonodromic condition for a family of linear – may be irregular – connections. Therefore we can study Painlevé equations as a foliation on a moduli space of connections. This point of view gives us natural compactification of the basis (the time variable). We will present this description for the Painlevé II equation. Poisson structures and orbifolds are here essential ingredients underlying this description.

Jean-Pierre Ramis

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The character variety of the q-analog of the sixth Painlevé equation

The talk is based upon a joint work with Yousuke Ohyama and Jacques Sauloy. For the q-analog of the Painlevé equations, according to H. Sakai work, "everything" is known on the side of the q analog of the Riemann-Hilbert map (the varieties of initial conditions), but the other side (the q-analog of the character varieties) remained quite mysterious. We will present a complete description of the character variety of $q - PVI$. It is an elliptic surface and we will explicit some parametrizations. This surface is analytically, but not algebraically isomorphic to the Sakai surface.

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Painlevé equations, resurgence, and quantization

We will overview recent work on Painlevé and discrete-Painlevé equations, dealing with the construction of solutions and tau-functions via a resurgent transseries approach. Time allowing we will also briefly discuss on-going work on connecting the (two parameter) tau-function transseries to PT-symmetric quantum hamiltonians.

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The order of indeterminate moment problem

For an indeterminate moment problem let p_n denote the corresponding orthonormal polynomials. We study the relation between the growth of the function $P(z) = \sum |p_n(z)|^2$ and the summability properties of the sequence $\{p_n(z)\}$. Under certain assumptions on recurrence coefficients we show in particular that the function $P(z)$ is of order α (i.e. $P(z) \leq Ce^{Kx^{\alpha}}$) with $0 < \alpha < 1$ if and only if the sequence $\{p_n(z)\}$ is absolutely summable with any power greater than 2α. Other types of growth like logarithmic order, as well as much slower orders, are also studied. By specifying the recurrence coefficients we can achieve any prescribed growth of the function $P(z)$.

The talk is based on two papers joint with Christian Berg.

1. C. Berg, R. Szwarc, On the order of indeterminate moment problem, *Adv. Math* **250** (2014), 105-143.

2. C. Berg, R. Szwarc, Symmetric moment problems and a conjecture of Valent, *Mat. Sb.* **208** (2017), 28-53.

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Computing indices of variance, cumulants of mutual information entropy and allied statistics in Random Matrix Theory

Two examples, amongst many others, of statistics arising in RMT are the variance of the index and low order cumulants of the mutual information entropy. These can be computed explicitly in terms of special functions such as hypergeometric functions, primarily because of two reasons: the first is that their generating functions are τ-functions of the Painlevé type, and secondly that the zeroth order τ-function is a trivial function. The derivation of these results will be explained. Another particularly useful feature is that for the finite rank ensembles this τ-function is the normalisation for an (bi-)orthogonal polynomial (OP) system with a weight deformed from one of the classical cases in the Askey Table. An algorithmic procedure exists which allows for the derivation of all the characterising equations for such (b)OP systems, and it is these that are employed to compute the aforementioned statistics. Some of this work has been in collaboration with Peter Forrester, and some with Liu Wei.

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