

## Titles & Abstracts

**Dario Bambusi** (Università degli Studi di Milano)

Title: On the spectrum of the Schrödinger operator on  $T^d$ : a normal form approach

Abstract: I will present a new method based on normal form and pseudodifferential calculus to get spectral results on Schrödinger type operators on  $T^d$ . In the one dimensional case one obtains in a very simple way the classical result that the eigenvalues of a Schrödinger operator come in couple which are well separated one from the others and such that the two eigenvalues in a couple have the same asymptotic. In the higher dimensional case I will show how to obtain the asymptotic behavior of a large part of the spectrum of Schrödinger operators and prove some properties similar to those just described for the one dimensional case.

Joint work with Beatrice Langella and Riccardo Montalto

**Joackim Bernier** (Université de Rennes 1)

Title: Rational normal forms and stability of small solutions to nonlinear Schrödinger equations

Abstract: Considering general classes of nonlinear Schrödinger equations on the circle with nontrivial cubic part and without external parameters, I will present the construction of a new type of normal forms, namely rational normal forms, on open sets surrounding the origin in high Sobolev regularity. With this new tool we prove that, given a large constant  $M$  and a sufficiently small parameter  $\varepsilon$ , for generic initial data of size  $\varepsilon$ , the flow is conjugated to an integrable flow up to an arbitrary small remainder of order  $\varepsilon^{M+1}$ . This implies that for such initial data  $u(0)$  we control the Sobolev norm of the solution  $u(t)$  for time of order  $\varepsilon^{-M}$ . Furthermore this property is locally stable: if  $v(0)$  is sufficiently close to  $u(0)$  (of order  $\varepsilon^{3/2}$ ) then the solution  $v(t)$  is also controlled for time of order  $\varepsilon^{-M}$ . (This is a joint work with Erwan Faou and Benoît Grébert).

**Massimiliano Berti** (SISSA)

Title: Long time dynamics of water waves

Abstract: I shall present recent results about the complex dynamics of the water waves equations of a bi-dimensional fluid under the action of gravity and eventually capillary forces, with space periodic boundary conditions. This is an infinite dimensional Hamiltonian system. We shall discuss both long time existence results as well as bifurcation of small amplitude time quasi-periodic solutions.

**Jacques Féjoz** (Université Paris Dauphine & IMCCE, Observatoire de Paris)

Title: Remarks on quasiperiodic solutions in the N-body problem

Abstract: I will review some old and newer means to find quasiperiodic solutions in the N-body problem.

**Benoit Grébert** (Université de Nantes)

Title : Long time behavior of the solutions of NLW on the d-dimensional torus

Abstract:

We consider the non linear wave equation (NLW) on the d-dimensional torus

$$i\partial_t u - \Delta u + \mu u + f(u) = 0 \quad x \in \mathbb{T}^d$$

where  $f = \partial_u F$  is analytic on a neighborhood of the origin and which is at least of order 2 at the origin.

Let  $u(t)$  be a solution corresponding to a small initial datum  $u(0) \in H^s(\mathbb{T}^d)$ .

We prove that we control  $\|u(t)\|_s$  that mix the  $H^s$  norm of the  $\epsilon^{-\beta(r)}$  lower

Fourier modes of the solution  $u$  and the energy norm of the remaining higher modes during long times of order  $\epsilon^{-r}$ .

Our general strategy applies to any Hamiltonian PDEs whose linear frequencies satisfy only a first Melnikov condition. In particular it also applies to the Hamiltonian Boussinesq  $\beta\alpha\gamma\delta$  system and the Whitham-Boussinesq system in water waves theory.

Joint work with Joackim Bernier and Erwan Faou.

**Zaher Hani** (University of Michigan)

Title: On the kinetic description of the long-time behavior of dispersive PDE.

Abstract: Wave turbulence theory claims that at very long timescales, and in appropriate limiting regimes, the effective behavior of a nonlinear dispersive PDE on a large domain can be described by a kinetic equation called the "wave kinetic equation". This is the wave-analog of Boltzmann's equation for particle collisions. We shall consider the nonlinear Schrodinger equation on a large box with periodic boundary conditions, and explore some of its effective long-time behaviors at time scales that are shorter than the conjectured kinetic time scale, but still long enough to exhibit the onset of the kinetic behavior. (This is joint work with Tristan Buckmaster, Pierre Germain, and Jalal Shatah).

**Vadim Kaloshin** (University of Maryland)

Title: On dynamical spectral rigidity of planar domains

Abstract: Consider a convex domain on the plane and the associated billiard inside. The length spectrum is the closure of the union of perimeters of all period orbits. The length spectrum is closely related to the Laplace spectrum, through the wave trace and the well-known question: "Can you hear the shape of a drum?"

A domain is called dynamically spectrally rigid if any smooth deformation preserving the length spectrum is an isometry.

During the talk I will discuss recent results on dynamical spectral rigidity of convex planar domains.

**Raphael Krikorian** (Université de Cergy Pointoise)

Title: On the divergence of Birkhoff Normal Forms.

Abstract: A real analytic hamiltonian or a real analytic exact symplectic diffeomorphism admitting a non resonant elliptic fixed point is always formally conjugated to a formal integrable system, its Birkhoff Normal Form (BNF). Siegel proved in 1954 that the formal conjugation reducing a hamiltonian to its BNF is in general divergent and Hakan Eliasson has asked whether the BNF itself could be divergent. Perez-Marco proved in 2001 that for any fixed non resonant frequency vector the following dichotomy holds: either any real analytic hamiltonian system admitting this frequency vector at the origin has a convergent BNF or for a prevalent set of hamiltonians admitting this frequency vector the BNF generically diverges. It is possible to exhibit examples of hamiltonian systems with diverging BNF (X. Gong 2012 or the recent examples of B. Fayad in 4 degrees of freedom). The aim of this talk is to give a complete answer to the question of the divergence of the BNF (in the setting of exact symplectic diffeomorphisms): for any non resonant frequency vector, the BNF of a real analytic exact symplectic diffeomorphism admitting this frequency vector at the origin, is in general divergent.

This theorem is the consequence of the remarkable fact that the convergence of the formal object that is the BNF has dynamical consequences, in particular an abnormal abundance of invariant tori.

**Jean-Pierre Marco** (UPMC Sorbonne Universités)

Title: Geodesics of an infinite dimensional ellipsoid : an intermediate between finitely dimensional integrable systems and integrable PDE's.

Abstract: The goal of the talk is to show that the usual methods (first integrals and spectral curve)

used to integrate the geodesic flow of finite dimensional ellipsoids

generalize in a natural

way to infinite dimensional ellipsoids of the Hilbert space  $\ell^2(\mathbb{N})$ .

We will discuss the

relevance of Birkhoff normal forms and Arnold-Liouville theorem in this context.

**Riccardo Montalto** (Università degli Studi di Milano)

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Title: Normal form coordinates for the KdV equation having expansions in terms of pseudo-differential operators

Abstract: Near an arbitrary compact family of finite dimensional tori, left invariant under the KdV flow, we construct a real analytic, 'normal form transformation' for the KdV equation having the following main properties:

(1) When restricted to the family of finite dimensional tori, the transformation coincides with the Birkhoff map.

(2) Up to a remainder term, which is smoothing to any given order, it is a pseudo-differential operator of order 0 in the normal directions with principal part given by the Fourier transform.

(3) It is canonical and the pullback of the KdV Hamiltonian is a paradifferential operator which is in normal form up to order three.

Such coordinates are a key ingredient for studying the stability of finite gap solutions of arbitrary size (periodic multisolitons) of the KdV equation under small, quasi-linear perturbations. This is a joint work with Thomas Kappeler.

**Laurent Niederman** (Université Paris XI Orsay & IMCCE, Observatoire de Paris)

Title: Quasi periodic coorbital motions (joint work with Philippe Robutel and Alexandre Pousse)

Abstract: The motions of the satellites Janus and Epimetheus around Saturn are among the most intriguing in the solar system since they exchange their orbits every four years.

In [1], we give a rigorous proof of the existence of quasi-periodic orbits of this kind in the three body plane planetary problem thanks to KAM theory.

We will discuss extensions of these results to get lagrangian tori in the same problem.

Ref: On the co-orbital motion in the three-body problem: existence of quasi-periodic horseshoe-shaped orbits, arXiv 1806.07262

**Michael Orioux** (SISSA)

Title: Optimality for minimum time affine control systems: a Hamiltonian approach

Abstract: When one's interested in the minimum time control of mechanical systems, and more generally of dynamics that are affine in the control, necessary conditions give the optimal trajectory as the projection of the integral curves of an Hamiltonian system defined on the cotangent bundle of the phase space. Those curves are called extremal, and the time minimisation induces a lack of regularity: the Hamiltonian is not smooth, and has codimension 2 singularities. In this talk we will prove sufficient conditions for optimality of these singular extremals. Our method uses techniques from symplectic geometry, which consist in building a Lagrangian submanifold on which the canonical projection of the extremal flow is invertible. Then one can compare final times of neighboring trajectories by lifting them to the cotangent bundle and evaluate the Poincaré-Cartan form along their lifts. The main difficulty is the definition of these objects without the required regularity, and an extended study of the extremal flow is necessary.

**Gabriella Pinzari** (Università di Padova)

Title: On the co-existence of maximal and whiskered KAM tori in the three-body problem

Abstract: In the phase space of the three-body problem there is a small region where maximal and whiskered tori seem to co-exist. I shall discuss a strategy of proof of this. The talk is based on a paper published on JMP, 2018 and current work in progress.

**Nicola Visciglia** (Università di Pisa)

Title: On the growth of high order momenta to NLS

Abstract: we provide an asymptotic description of high order momenta of solutions to NLS. The result is a combination of suitable modified energies and scattering theory. The method is robust and works also in case that a linear potential perturbation is added to NLS.

This is a joint work with N. Tzvetkov (Cergy-Pontoise)

**Eric Paturel** (Université de Nantes) eric.paturel at univ-nantes.fr

On reducibility of quantum harmonic oscillator on  $\mathbb{R}^d$

Abstract :

We prove that a linear d-dimensional Schrödinger equation on  $\mathbb{R}^d$  with harmonic potential  $|x|^2$  and small  $t$ -quasiperiodic potential

$$i\partial_t u - \Delta u + |x|^2 u + \varepsilon V(t, \omega, x) u = 0, \quad x \in \mathbb{R}^d$$

reduces to an autonomous system for most values of the frequency vector  $\omega \in \mathbb{R}^n$ . As a consequence any solution of such a linear PDE is almost periodic in time and remains bounded in all Sobolev norms.

**Michela Procesi** (Università di Roma Tre)

Title: Almost-periodic solutions for the NLS with parameters

Abstract: I shall discuss a recent result with L. Biasco and J. Massetti on the existence of almost-periodic solutions for the NLS on the circle with external parameters. After discussing the (very few) known results I shall describe our strategy, which is quite flexible and can be applied also for the construction of non maximal tori.

**Jacopo De Simoi** (Toronto University)

Title: Spectral determination of analytic dispersing billiards

Abstract: In this talk we will see how it is generically possible, in the analytic category, to recover the geometry of some class of dispersing billiards with some symmetries (three obstacles systems) from the purely dynamical data encoded in their Marked Length Spectrum.

This is part of an ongoing project with V. Kaloshin, M. Leguil and P. Bálint.

**Xiaoping Yuan** (Fudan University)

Title: Several KAM theorems for Hamiltonian partial differential equations

Abstract: In this talk, I will present our team's recent works on KAM theory. (1) It is proved that there are many infinitely dimensional KAM tori of prescribed Diophantine frequency for a class of infinite dimensional lattice system of short range. (2) We give out a criterion to decide whether or not there are KAM tori of finite dimension for a degenerate Hamiltonian system of infinite

dimension. (3) It is shown that there are many KAM tori for Pochhammer-Chree equation where the normal frequencies have a finite limit.

**Jangong You** (Chern Institute of Mathematics, Nankai University)  
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Title: Arithmetic version of Anderson localization via reducibility

Abstract: We give a new method for proving the arithmetic version of Anderson localization. Our method is based on the reducibility and an arithmetic version of Aubry duality which gives explicit arithmetic description both of the frequency and of the localization phase for a class of quasi-periodic long-range operators on  $\ell^2(\mathbb{Z}^d)$ .