ORBITS OF N-EXPANSIONS WITH A FINITE SET OF DIGITS

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Abstract. For $N \in \mathbb{N}$ and $\alpha \in \mathbb{R}$ such that $0 < \alpha \leq \sqrt{N} - 1$, the continued fraction map $T_\alpha : [\alpha, \alpha+1] \to [\alpha, \alpha+1)$ is defined as $T_\alpha(x) := \frac{N}{x} - d(x)$, where $d : [\alpha, \alpha+1] \to \mathbb{N}$ is defined by $d(x) := \left\lfloor \frac{N}{x} - \alpha \right\rfloor$. For $N \geq 7$ there are α, intervals $(a, b) \subset [\alpha, \alpha+1]$ and $n_0 \in \mathbb{N}$ such that $T_\alpha^n([\alpha, \alpha+1]) \cap (a, b) = \emptyset$ for all $n \geq n_0$, save for *fixed points* under T_α of (a, b). These *gaps* (a, b) are investigated in the square $\Upsilon_\alpha := [\alpha, \alpha+1] \times [\alpha, \alpha+1)$, where the *orbits* $T_\alpha^k(x), k = 0, 1, 2, \ldots$ of numbers $x \in [\alpha, \alpha+1]$ are represented as cobwebs. The squares Υ_α are the union of *fundamental regions*, which are related to the cylinder sets of the map T_α, according to the finitely many values of d in T_α. In the case of two cylinders there may be none, one or two gaps on $[\alpha, \alpha+1]$; in the case of three cylinders, there are either none, one, two or three gaps, depending both on N and α. In the case of four cylinders there are usually no gaps, except for the rare cases that there is one, very wide gap. In the case of five or more cylinders no gaps occur.

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