Spectral gap and asymptotic strong Feller property for Reaction Diffusion equation driven by a Levy-type noise.

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We are concerned with Reaction Diffusion equation with Dirichlet boundary conditions.

$$\begin{cases} du + \nu(-\Delta)u\,dt + u^3\,dt - u\,dt = dM + f\,dt, \\ \quad u(t,0) = u(t,1) = 0, \quad \text{for} \quad t > 0, \\ \quad u(0,x) = u_0(x), \quad \text{for} \quad x \in (0,1), \end{cases} \tag{0.1}$$

where the noise is of Levy-type

$$dM = \int_z F(u(t^-,x),x,z)\widetilde{N}(dtdxdz),$$

or

$$dM = \int_z F(u(t^-),z)(x)\widetilde{N}(dtdz).$$

We denote by $(\mathcal{P}_t)_t$ the Markov transition semi-group associated to the solutions of (0.1).

Combining results of Mattingly, Hairer and Fournier based on Malliavin calculus, we prove the asymptotic strong Feller property of $(\mathcal{P}_t)_t$

$$|\nabla\mathcal{P}_t\psi(u_0)| \le C\,|\psi|_\infty + e^{-t}\,|\nabla\psi|_\infty\,. \tag{0.2}$$

Combining this result an irreducibility argument, it follows from results of Hairer and Mattingly that $(\mathcal{P}_t)_t$ has the spectral gap property. It means that, for some distance d, we have

$$d\left(\mathcal{P}_t^*\mu_1, \mathcal{P}_t^*\mu_2\right) \le Ce^{-\beta t}d(\mu_1,\mu_2),$$

for any probability measures (μ_1,μ_2).

An immediate consequence is the uniqueness of the invariant measure μ and the exponential mixing of $(\mathcal{P}_t)_t$

$$\left|\mathcal{P}_t\psi(u_0) - \int f(u)\,\mu(du)\right| \le Ce^{-\beta t}\left(|\psi|_\infty + |\nabla\psi|_\infty\right). \tag{0.3}$$