Vortices in Josephson Arrays and Optical Lattices

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In collaboration with

- Allan H. MacDonald
- Marco Polini
- Mario P. Tosi
- Patrizia Vignolo

M. Polini, R. Fazio, A. Mac Donald and M.P. Tosi,
Phys. Rev. Lett. **95**, 010401 (2005)

P. Vignolo, R. Fazio and M.P. Tosi,
Phys. Rev A (in press)

Strongly Correlated Systems

- High Temperature Superconductivity

- Heavy Fermions

- Low dimensional Magnetism

- Superconductor-Insulator transition

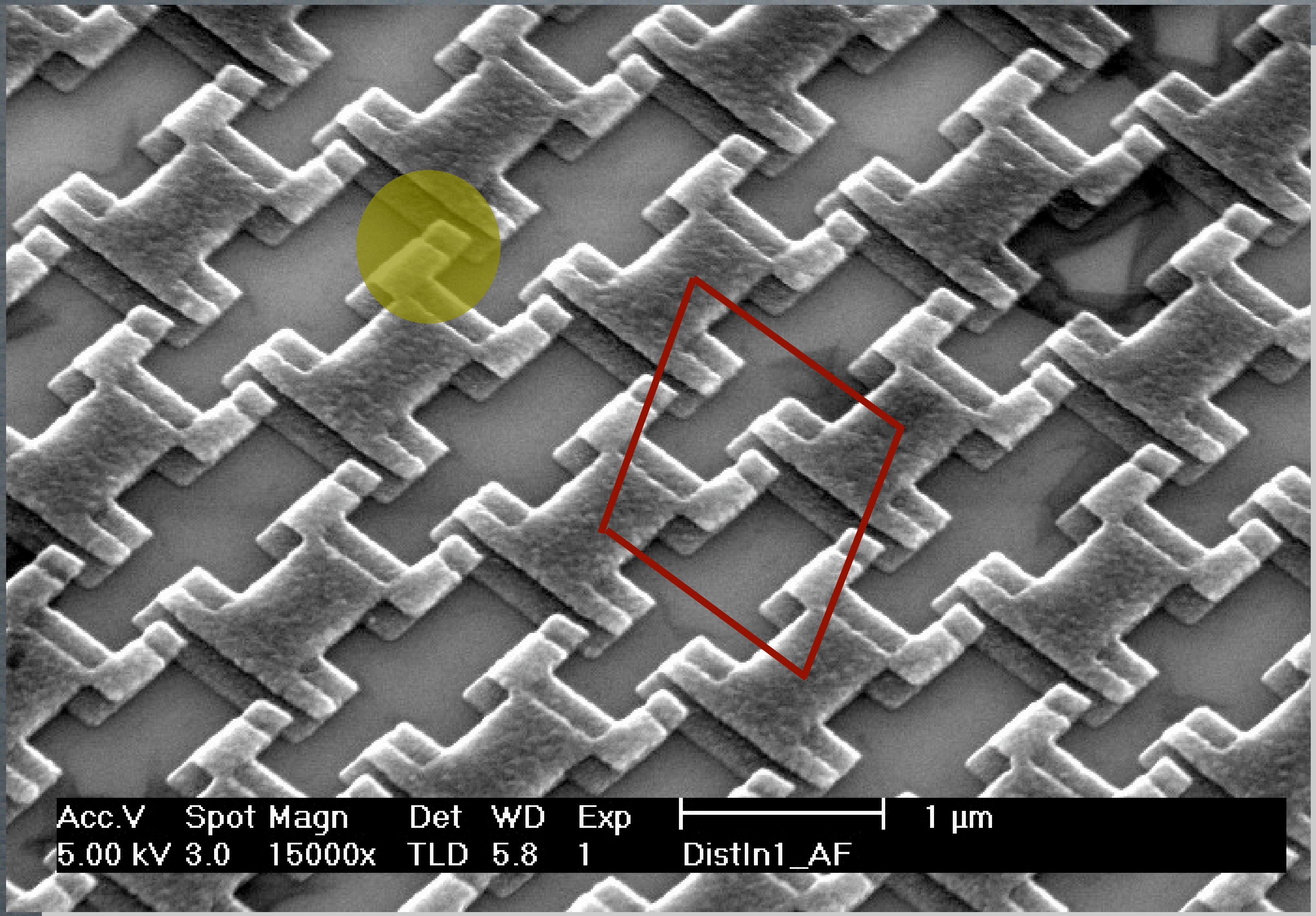
- ...

Simulate via controlled quantum systems

Requirements

- “Artificially” fabricated structures
- Controllable couplings
- Controllable topology
- “Easy” to measure

Bosonic systems - Josephson Junction arrays



Josephson junction

Square lattice

JJAs offer an opportunity to study a variety of classical and quantum phase transitions, effect of external fustration, dynamics of topological defects

What can be measured

- Transport properties (current, noise,...)
- Superfluid stiffness

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Charging Effects and Quantum Coherence in Regular Josephson Junction Arrays

L. J. Geerligs, M. Peters, L. E. M. de Groot,[a] A. Verbruggen,[a] and J. E. Mooij
Department of Applied Physics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands
(Received 17 April 1989)

Two-dimensional arrays of very-small-capacitance Josephson junctions have been studied. At low temperatures the arrays show a transition from superconducting to insulating behavior when the ratio of charging energy to Josephson-coupling energy exceeds the value 1. Insulating behavior coincides with the occurrence of a charging gap inside the BCS gap, with an S-shaped *I-V* characteristic. This so far unobserved S shape is predicted to arise from macroscopic quantum coherent effects including Bloch oscillations.

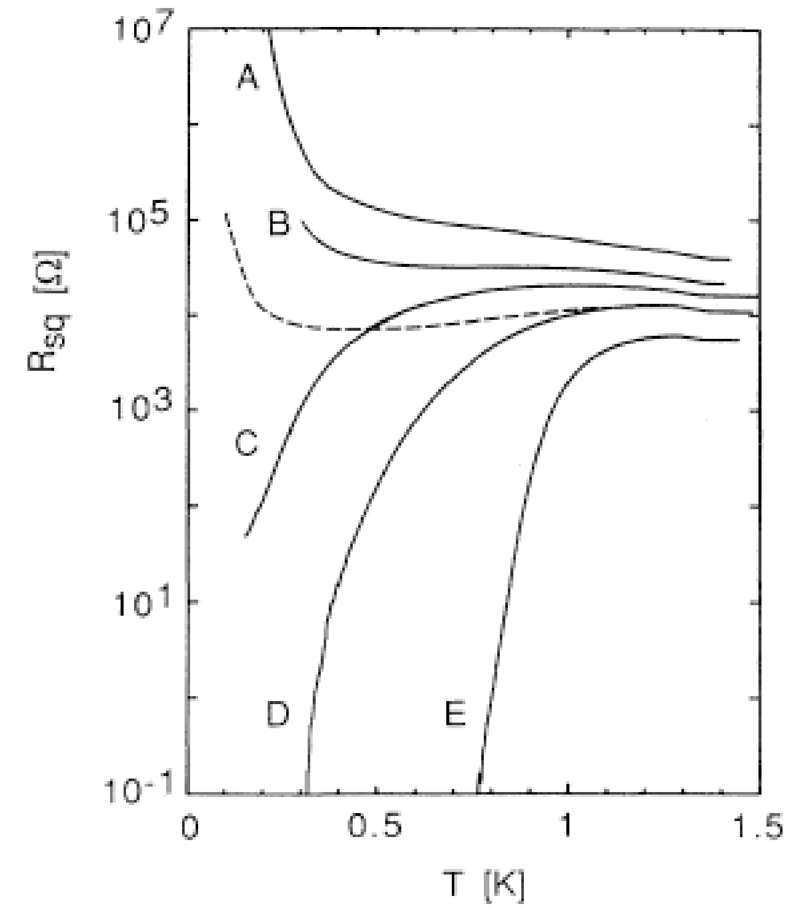
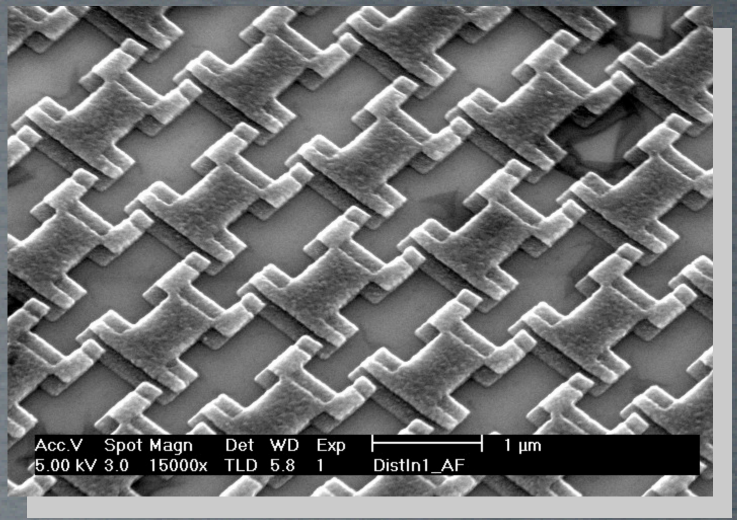


FIG. 1. $R(T)$ curves for arrays of 0.01-μm^2 junctions ($E_C \approx 0.84$ K). R_{sq} is the resistance divided by the length/width ratio 3.14. Each solid curve corresponds to an array with a particular normal-state resistance R_n in zero field. The dashed curve is for array D with $f \approx \frac{1}{2}$. Values of R_n in kΩ, E_J/k_B in K, and $x = E_J/E_C$ are, sample A: 36, 0.22, 3.9; B: 15.3, 0.51, 1.8; C: 14.1, 0.55, 1.5; D: 9.7, 0.80, 1.0; E: 4.8, 1.6, 0.53.

What can be controlled



$t \quad (E_J)$ By changing the properties of the insulating barrier

U By changing the dimensions of the islands

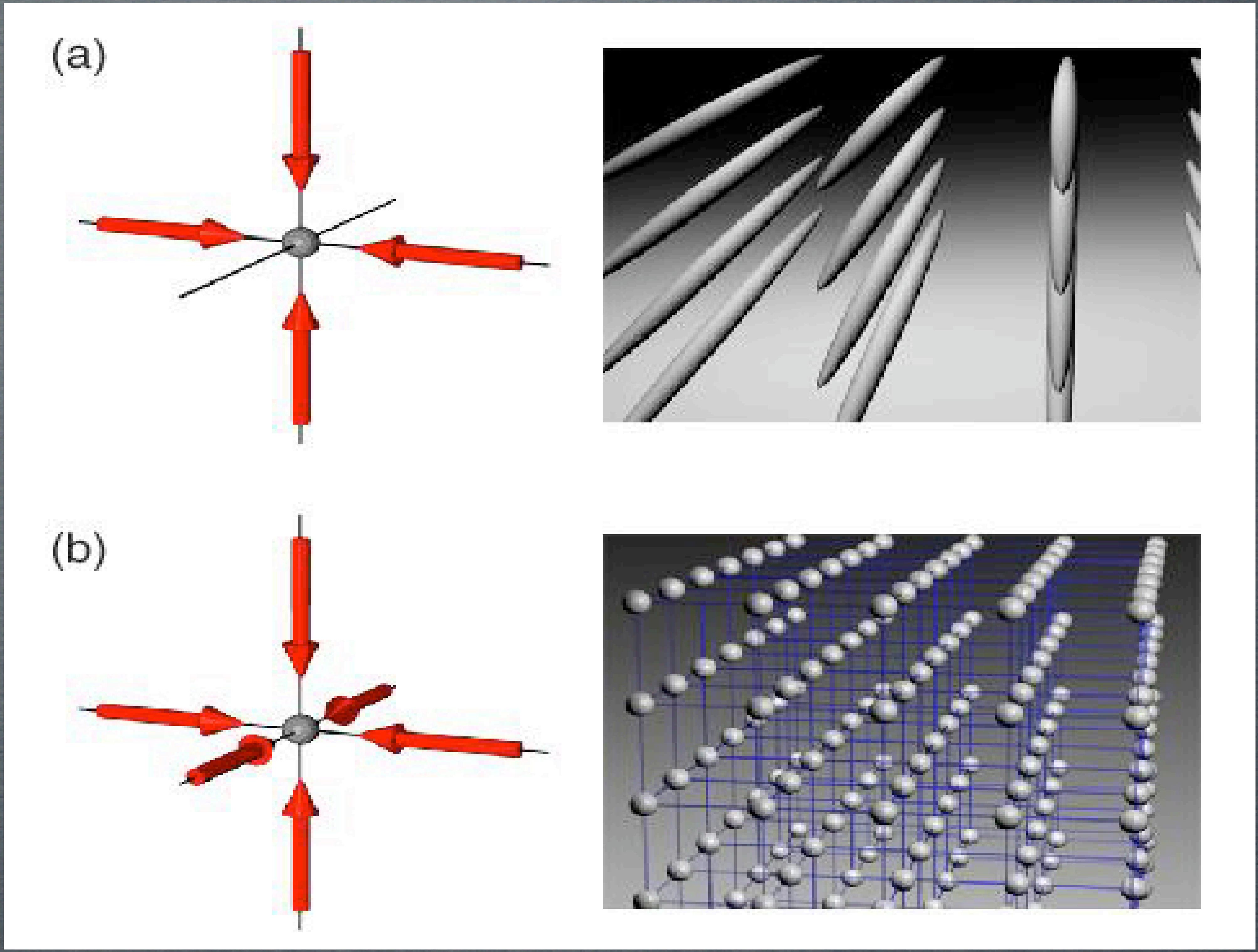
$\mu \quad (q_x)$ By changing a gate potential

- ✓ **Very flexible system**
- ✓ **Easy to implement frustration effects**
- ✓ **Possibilities to study quantum to classical crossover**

Disadvantages

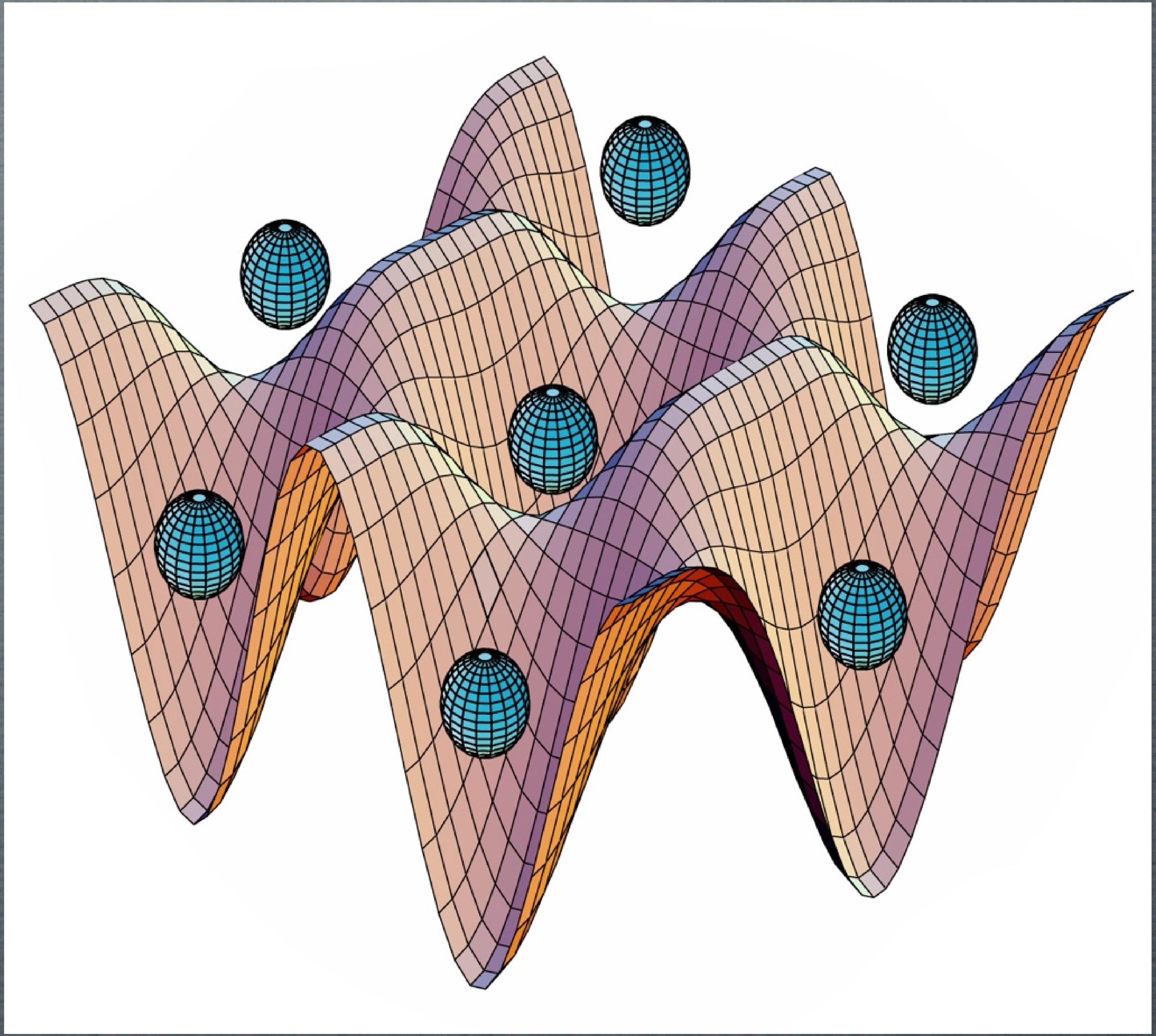
- Different systems should be fabricated differently
- Errors due to the fabrications processes

Optical Lattices



Jaksch *et al*, 1998
M. Greiner *et al*, 2002

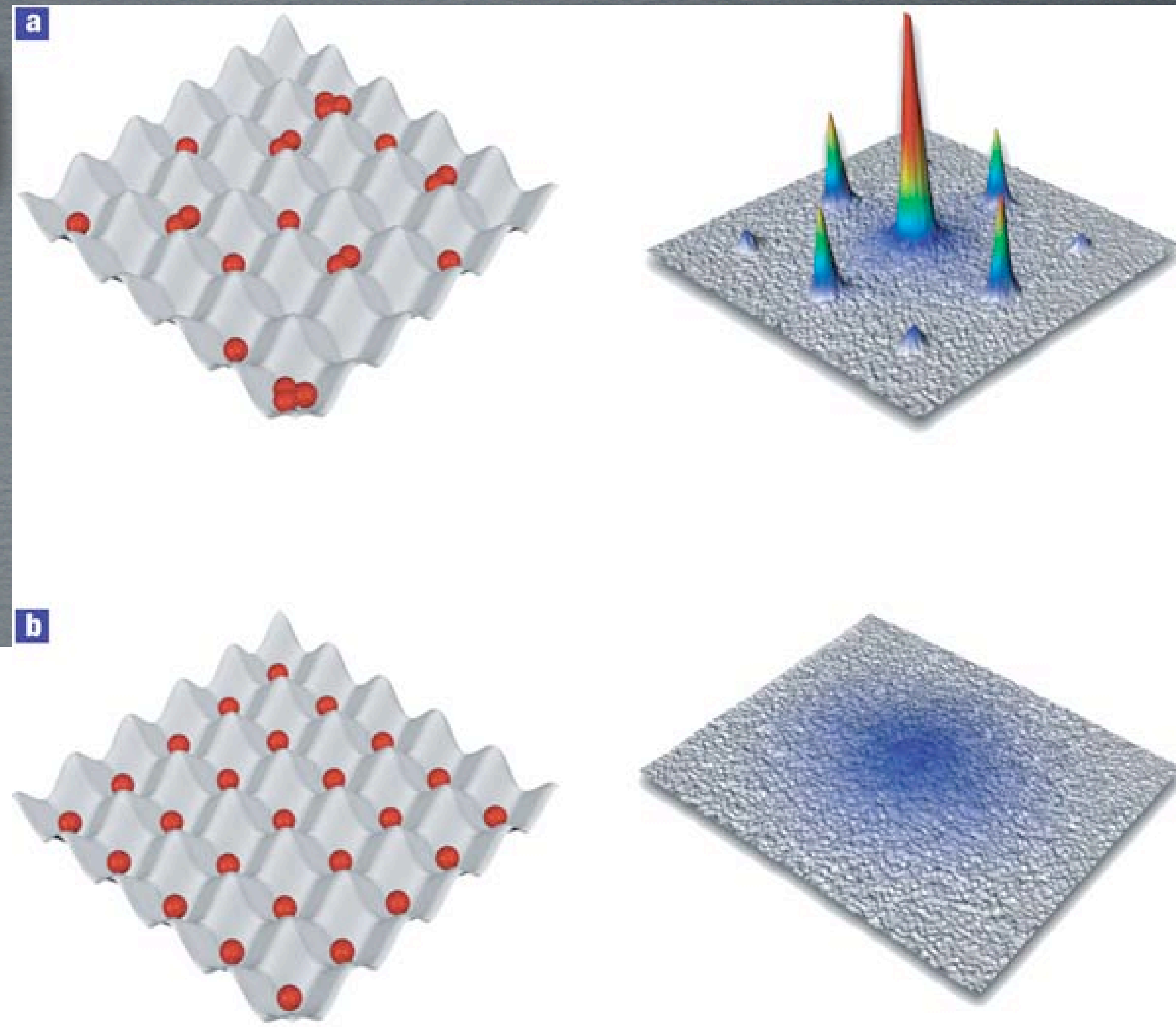
Optical Lattices

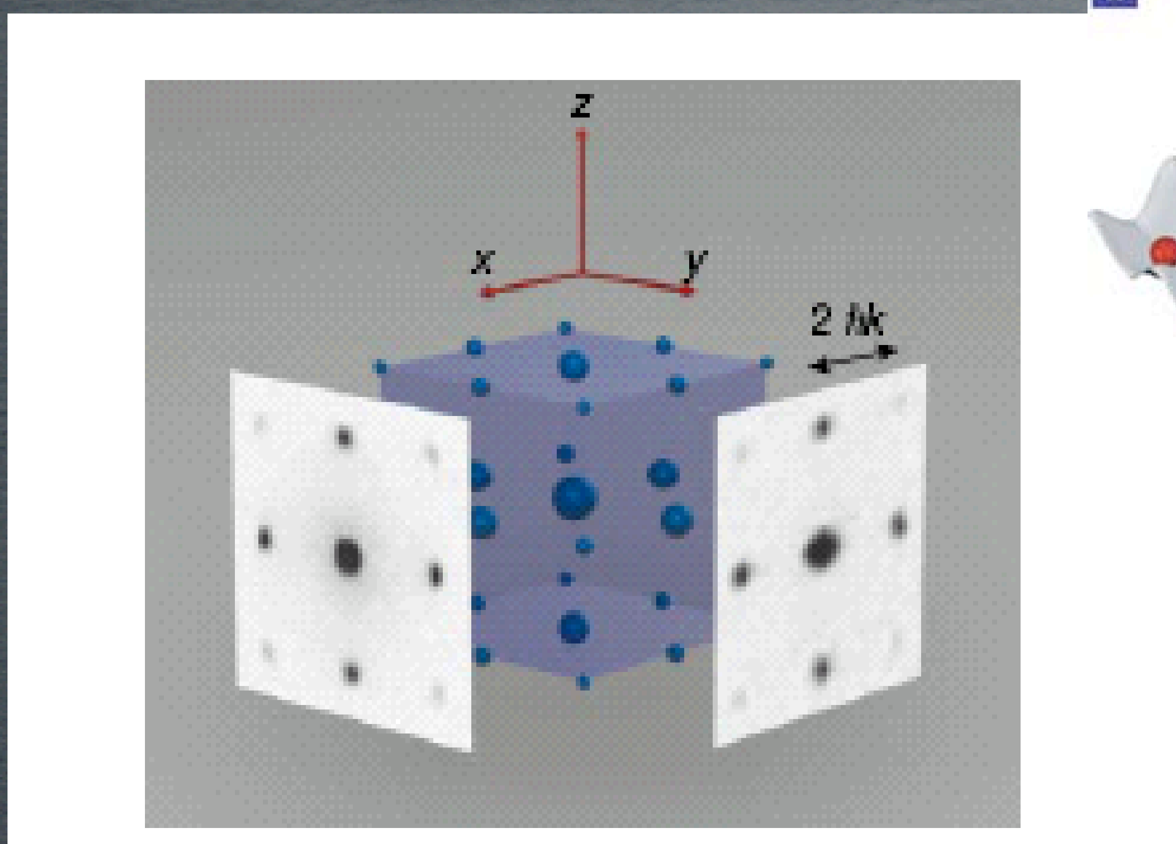


Jaksch *et al*, 1998, M. Greiner *et al*, 2002

- ✓ By varying the intensity of the lasers it is possible to control both the hopping and on-site repulsion
- ✓ Disorder is absent
- ✓ The atomic species loaded in the lattice can be changed (fermions, bosons, fermion-boson mixtures, ...)
- ✓ Time-dependent phenomena

Optical Lattices





Measurement of correlation functions

M. Greiner *et al.*, Nature 415, 39 (2002)

Statics and Dynamics of topological defects

Bose-Hubbard Hamiltonian

$$H = \frac{1}{2}\sum_{ij} n_i U_{ij} n_j - \mu \sum_i n_i - \frac{t}{2}\sum_{\langle ij \rangle} b_i^\dagger b_j + \text{h.c.}$$

$$b_i \sim e^{-i\phi_i}$$

Quantum Phase Model

$$H = \frac{1}{2}\sum_{i,j} (q_i - q_{\mathrm{x}})\, U_{ij}\, (q_j - q_{\mathrm{x}}) - E_J \sum_{<i,j>} \cos(\phi_i - \phi_j)$$

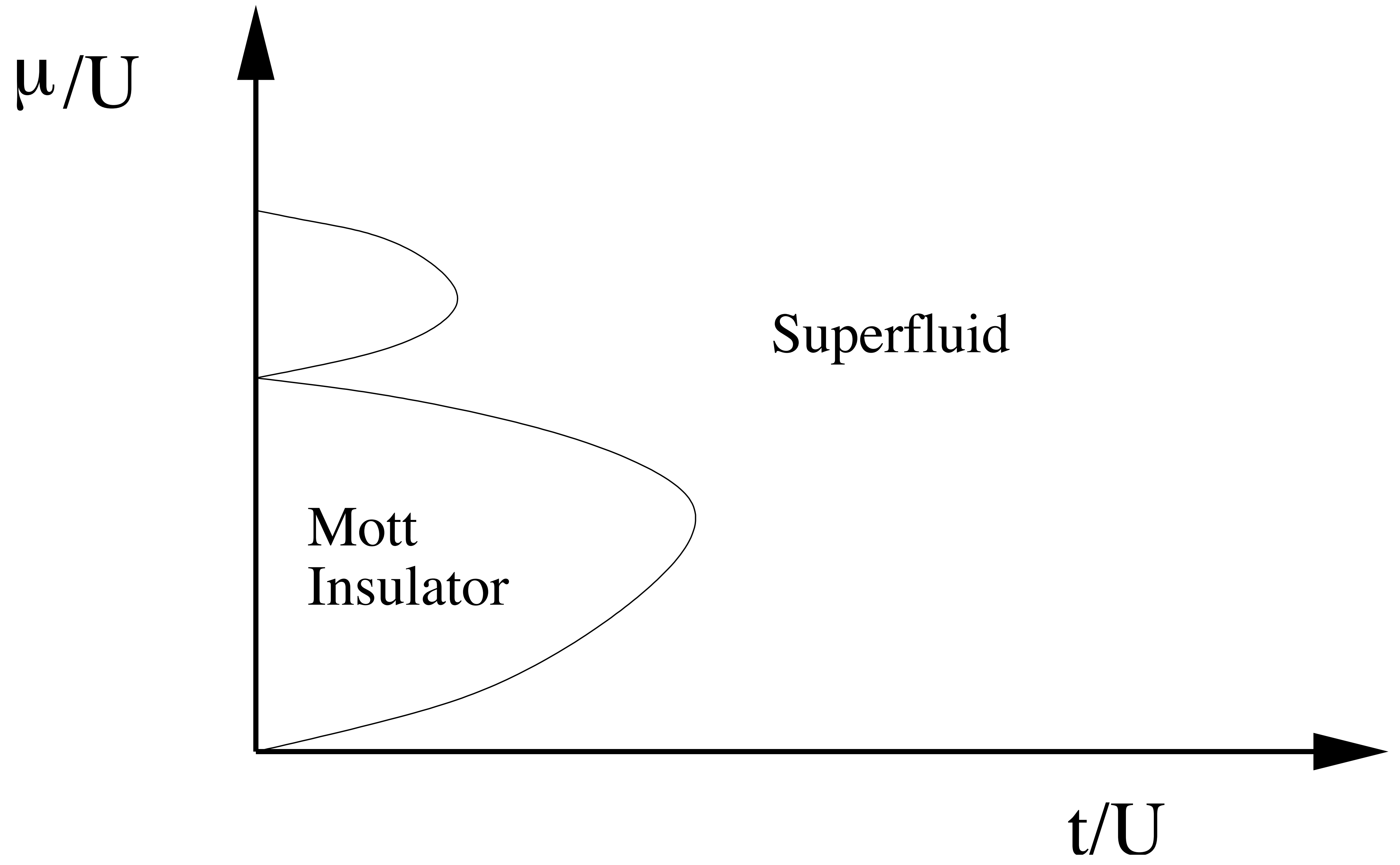
FRUSTRATION

$$\sum_{<ij>} \cos(\phi_i - \phi_j) \longrightarrow \sum_{<ij>} \cos(\phi_i - \phi_j - A_{ij})$$

$$f = \frac{2\pi}{\Phi_0} \sum_P A_{ij}$$

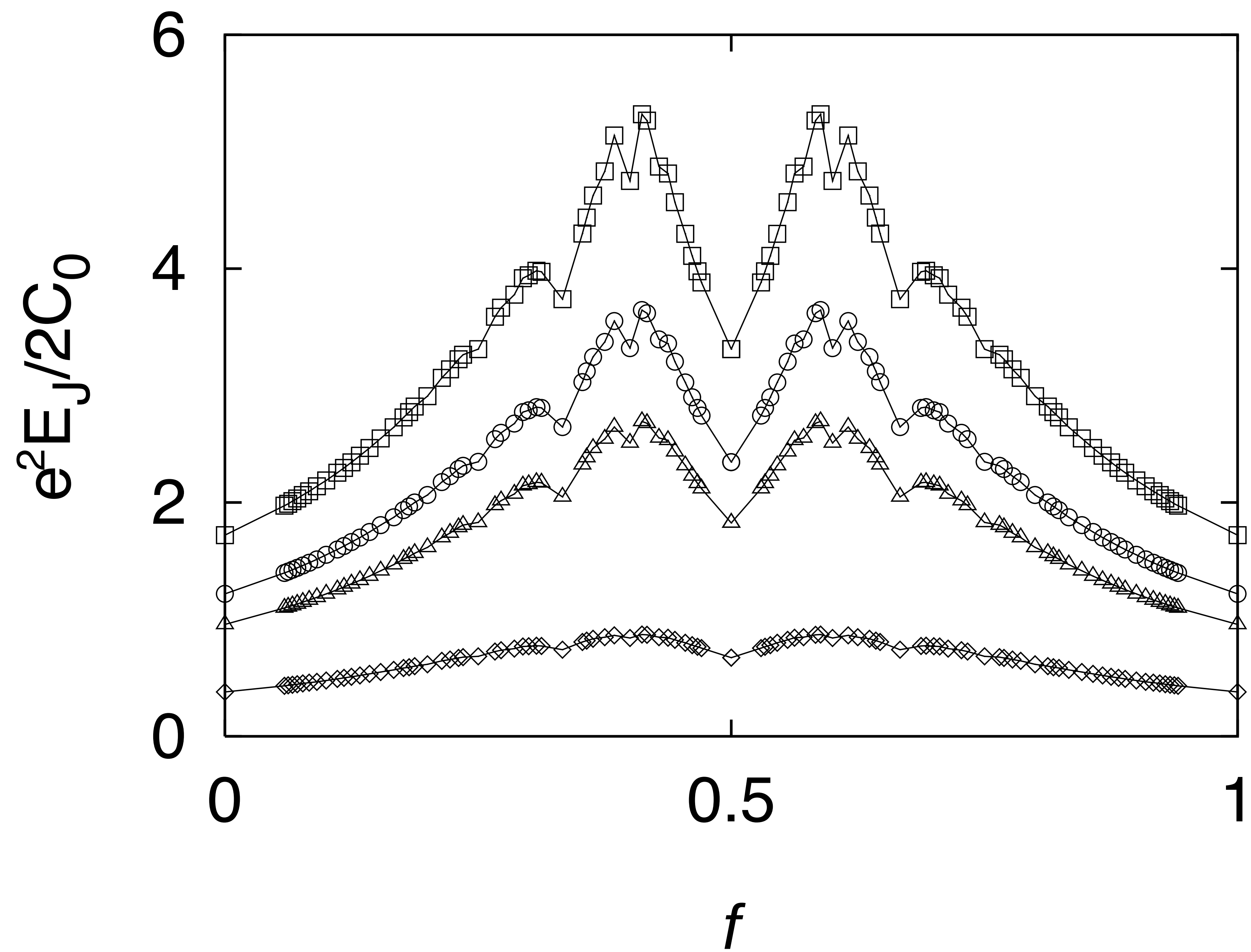
Phase Diagram

T=0



Phase Diagram

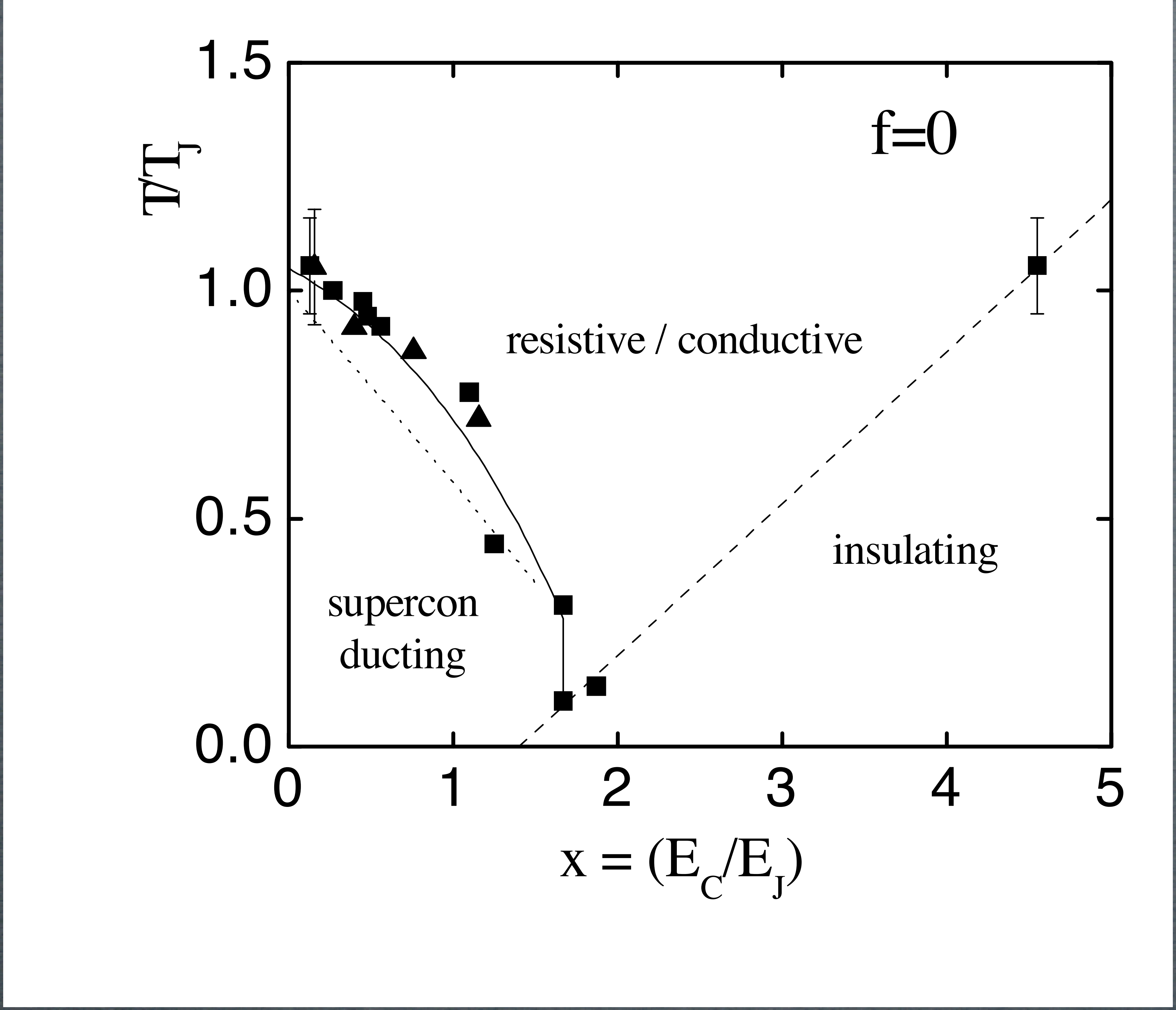
T=0



Kim *et al* '98

Phase Diagram

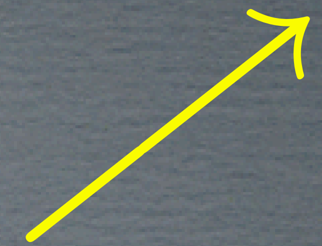
T≠0



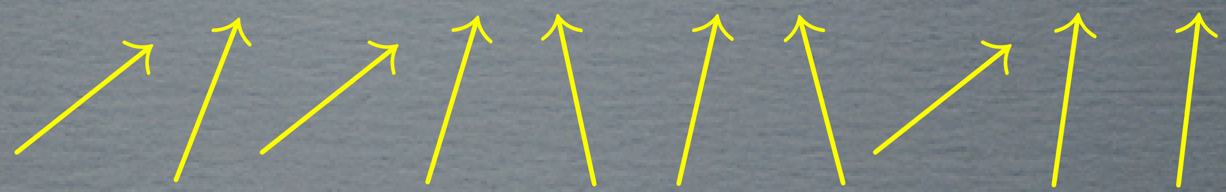
H. van der Zant *et al* '91

Excitations

$$e^{i\varphi_i}$$



- “Spin” waves



- “Charges”

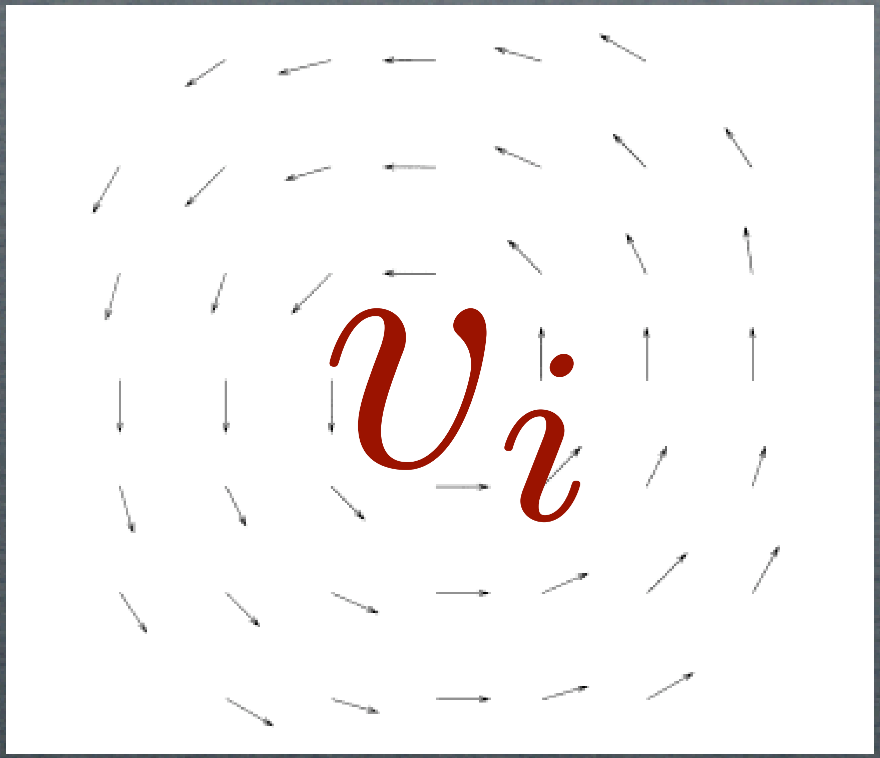
- Vortices

Excitations

- "Spin" waves

- "Charges"

- Vortices

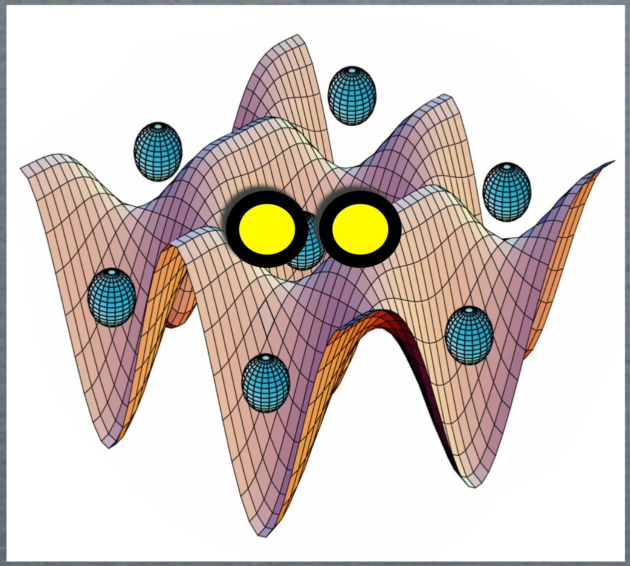


$$\phi_i = \pm\arctan\left(\frac{y_i - y}{x_i - x}\right)$$

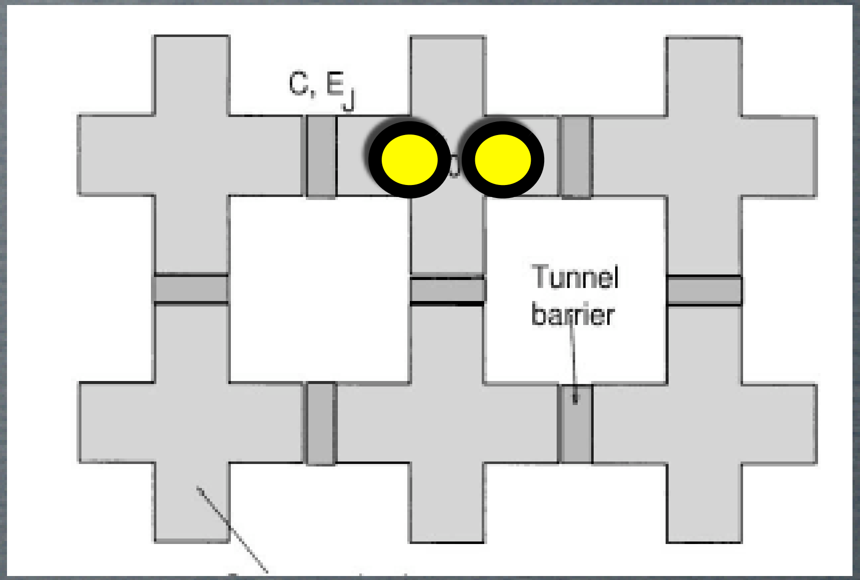
Excitations

- "Spin" waves

- "Charges"



q_i



- Vortices

From the Quantum Phase Model

$$H = \frac{1}{2}\sum_{i,j}(q_i - q_{\mathrm{x}})\,U_{ij}\,(q_j - q_{\mathrm{x}}) - E_J \sum_{<i,j>} \cos\left(\phi_i - \phi_j\right)$$

To an effective action only in terms of the topological defects

Dual transformations

Effective Action

$$Z = \sum_{[q,v]} e^{-S\{q,v\}}$$

$$S\{q,v\} = \int_0^\beta d\tau \sum_{ij} \{q_i(\tau)U_{ij}q_j(\tau) + \pi E_J v_i(\tau)G_{ij}v_j(\tau)$$
$$+ i\, q_i(\tau)\Theta_{ij}\dot{v}_j(\tau) + \frac{1}{4\pi E_J}\dot{q}_i(\tau)G_{ij}\dot{q}_j(\tau)\Big\}$$

$$G_{ij} \sim -\frac{1}{2}\ln r_{ij}$$

$$\Theta_{ij} = \arctan\left(\frac{y_i - y_j}{x_i - x_j}\right)$$

Effective Vortex Action

$$t \gg U$$
$$E_J \gg U$$

- Introduce the vortex trajectory $v_{i,\tau} = v\delta(\mathbf{r}_i - \mathbf{r}(\tau))$

- Integrate out the charges

$$S_{eff} = \frac{1}{2}\int_{\tau\tau'} \dot{\mathbf{r}}^a(\tau)\mathcal{M}_{ab}[\mathbf{r}(\tau) - \mathbf{r}(\tau'), \tau - \tau']\dot{\mathbf{r}}^b(\tau')$$

$$\mathcal{M}_{ab} = \sum_{jk} \nabla_a \Theta(\mathbf{r}(\tau) - \mathbf{r}_j)\langle q_{j\tau} q_{k\tau'} \rangle \nabla_b \Theta(\mathbf{r}_k - \mathbf{r}(\tau'))$$

Effective Vortex Action

<u>In the adiabatic limit the effective action can further simplified</u>

- Vortex mass

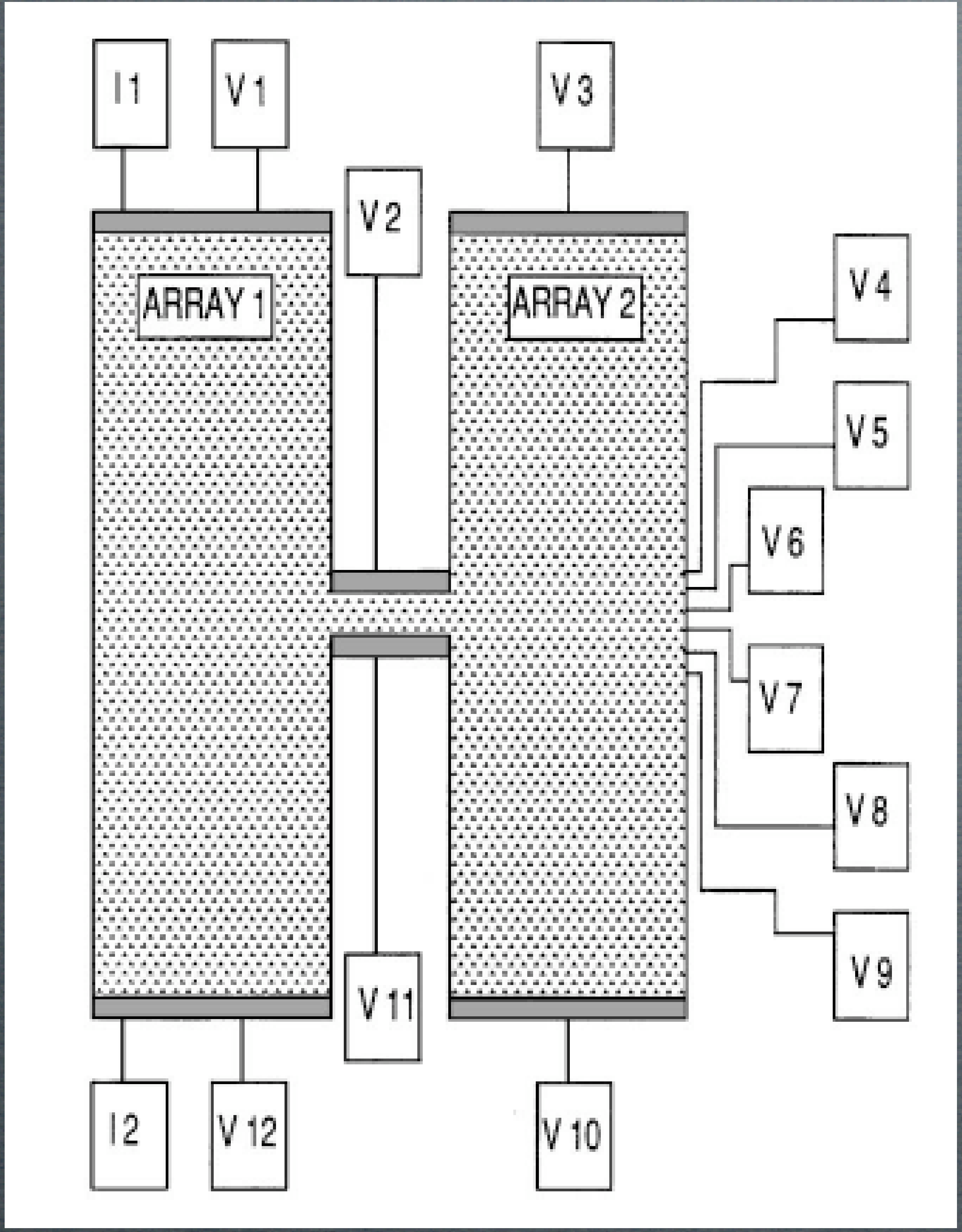
- Damping (spin waves)

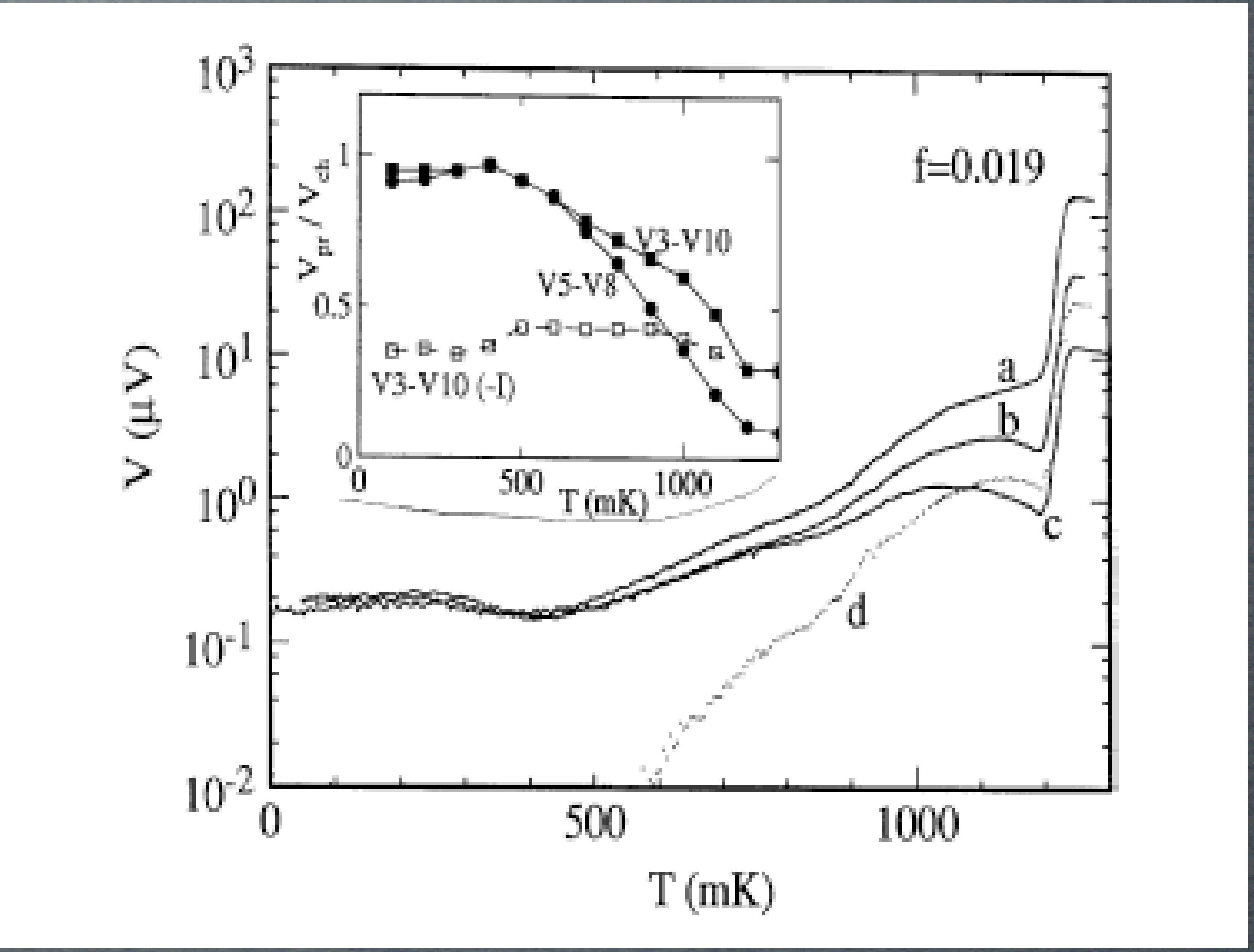
- Moving in a periodic potential

- Quantum properties

R. Fazio and H. van der Zant, Phys. Rep. '01

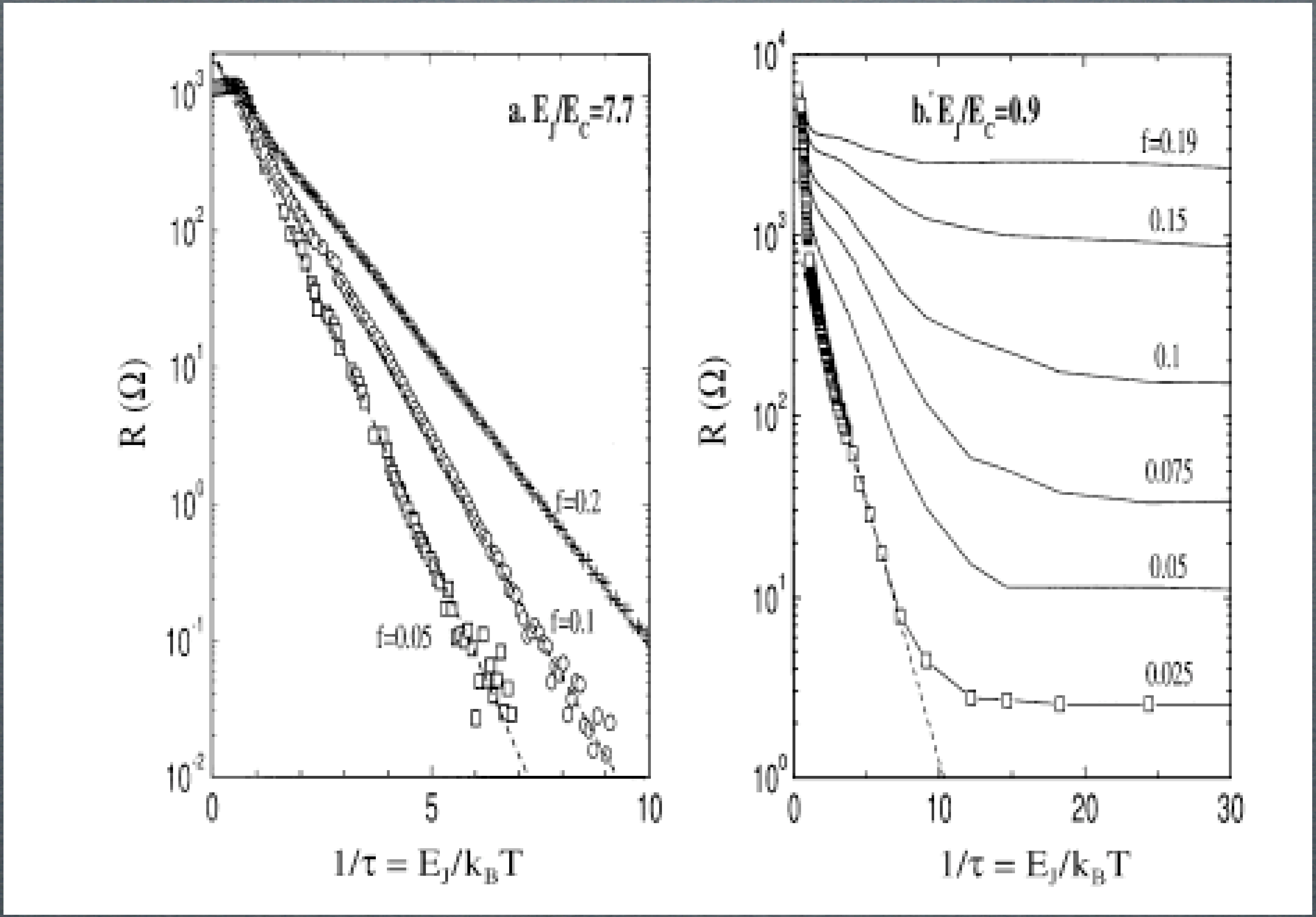
Ballistic Motion





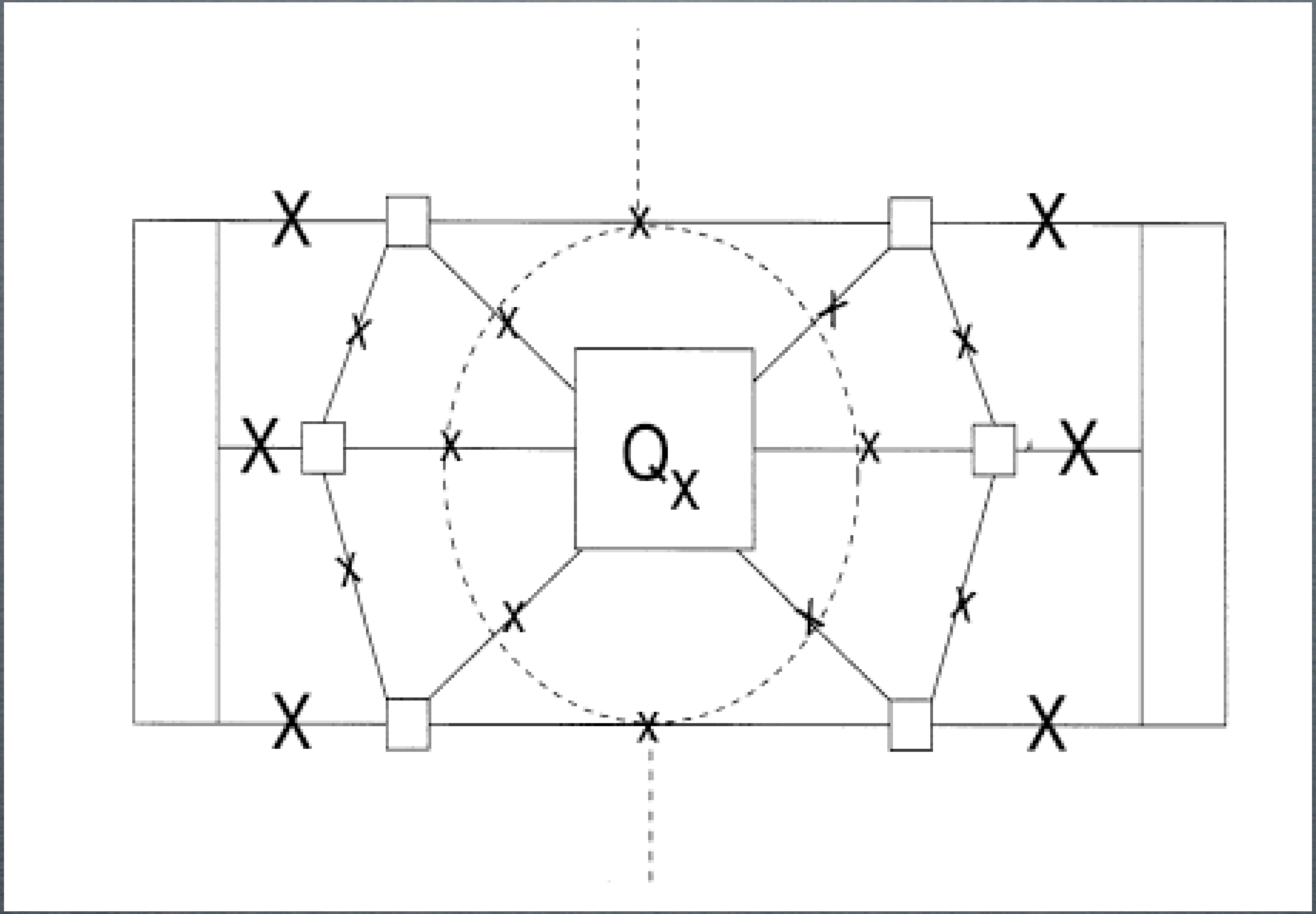
H. van der Zant *et al*

Quantum tunneling



H. van der Zant *et al*

Aharonov-Casher effect



W. Elion *et al*

VORTICES IN OPTICAL LATTICES

P. Vignolo, R. Fazio and M.P. Tosi,
Phys. Rev A (in press)

“Magnetic ” frustration can be achieved by:

- Rotating lattices

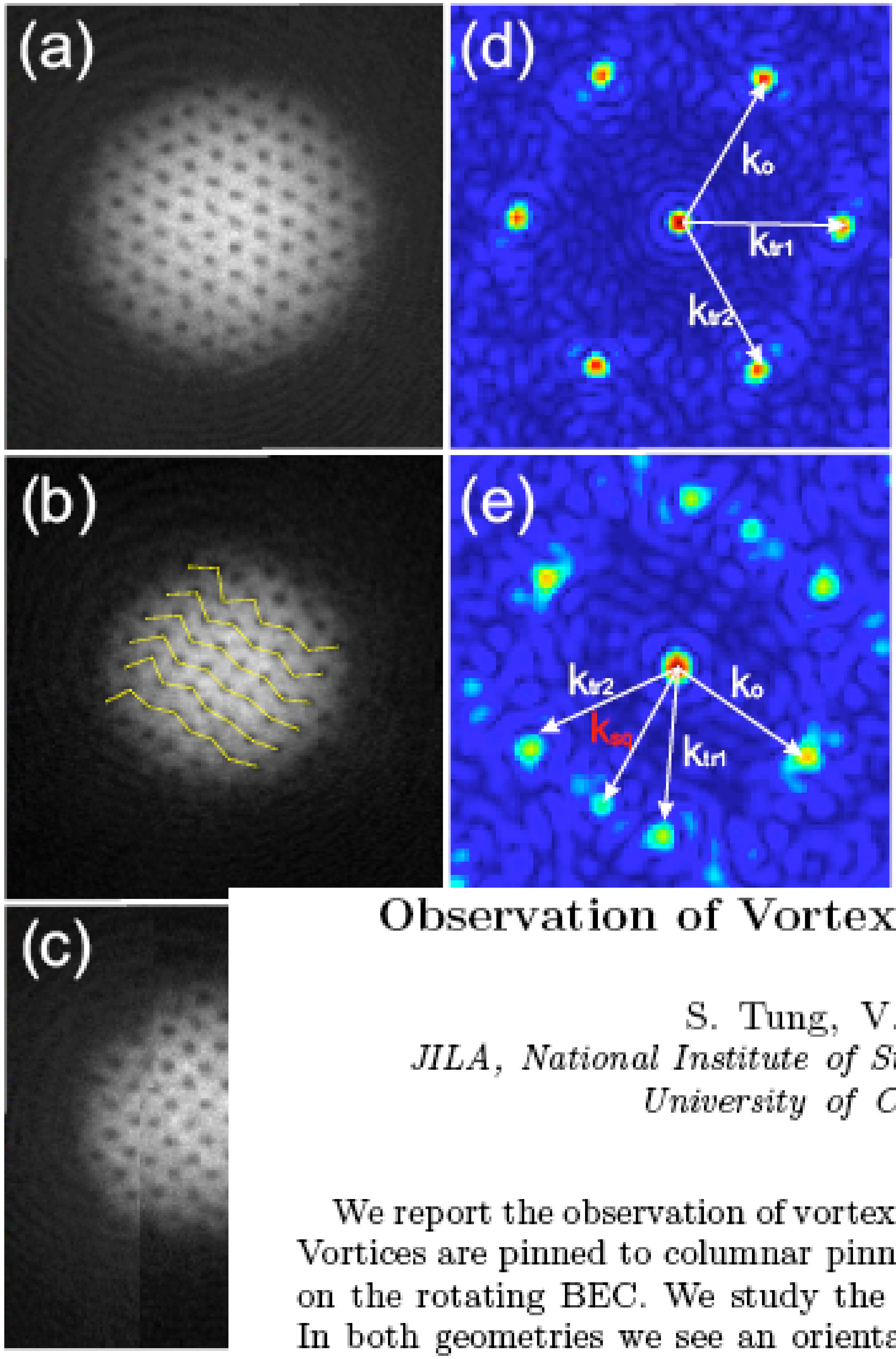
Polini *et al*(2004)

- Quadrupolar time-dependent field

Sorensen, Demler & Lukin (2004)

- Atoms with different internal states in different columns

Jaksch & Zoller (2003)



Observation of Vortex Pinning in Bose-Einstein Condensates

S. Tung, V. Schweikhard, and E. A. Cornell
JILA, National Institute of Standards and Technology, and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440

(Dated: May 1, 2007)

We report the observation of vortex pinning in rotating gaseous Bose-Einstein condensates (BEC). Vortices are pinned to columnar pinning sites created by a co-rotating optical lattice superimposed on the rotating BEC. We study the effects of two types of optical lattice, triangular and square. In both geometries we see an orientation locking between the vortex and the optical lattices. At sufficient intensity the square optical lattice induces a structural cross-over in the vortex lattice.

Control of the vortex properties, via Feshbach resonances and/or the strength of the transverse confinement

Access to the direct measurement of vortex properties, such as the mass, the coupling to its environment or the pinning potential

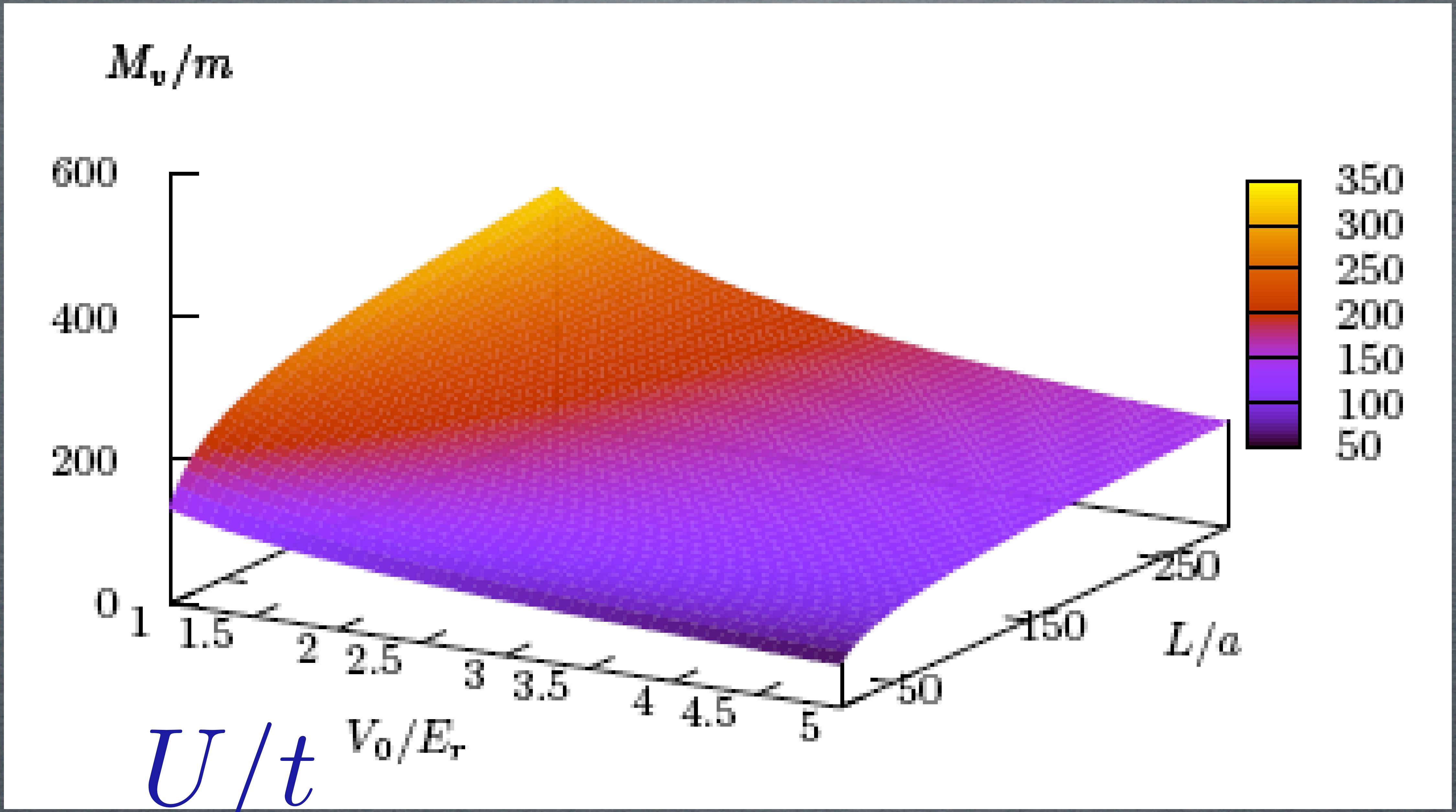
Vortex mass

Optical lattice

$$M_v = \frac{\sqrt{2\pi} l_\perp w^2}{4 a_{sc} a^2} m \ln(L/a)$$

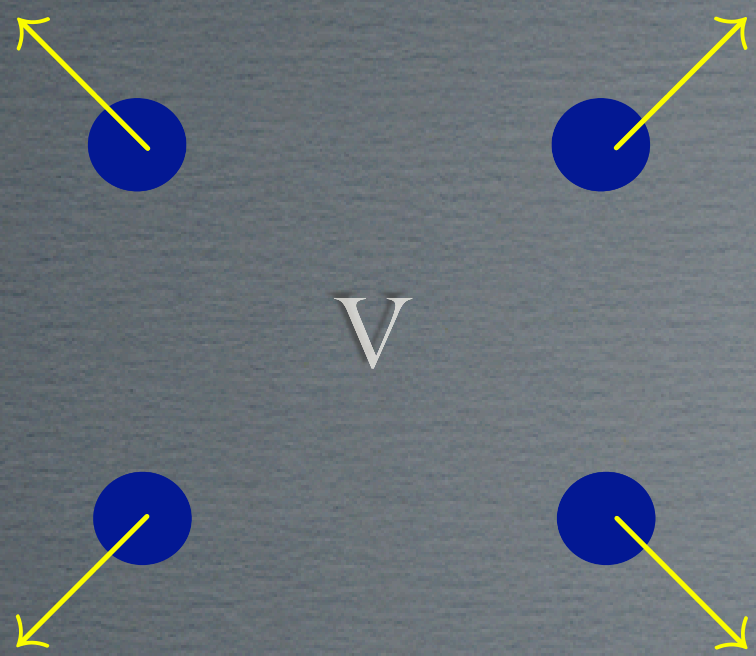
Vortex mass

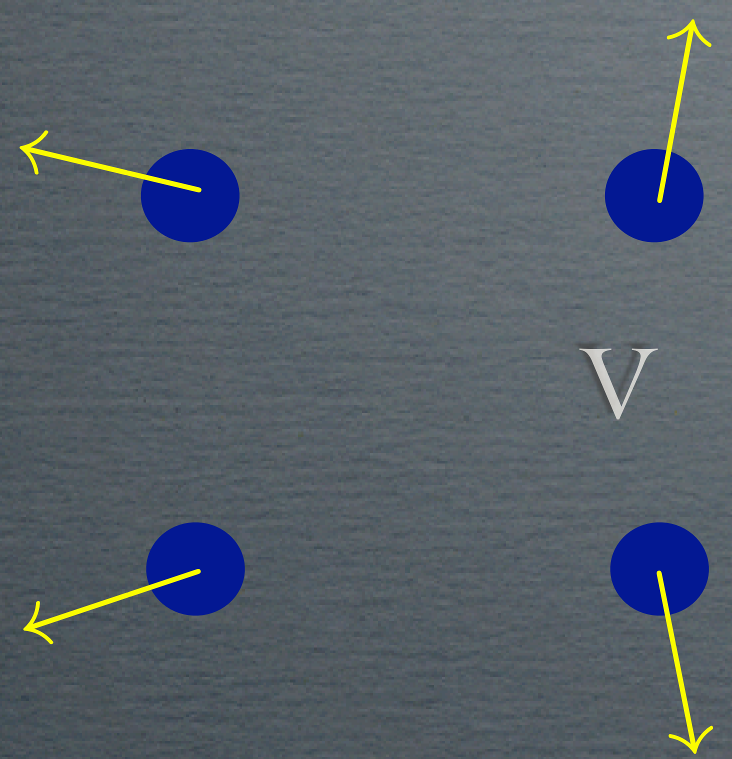
Optical lattices

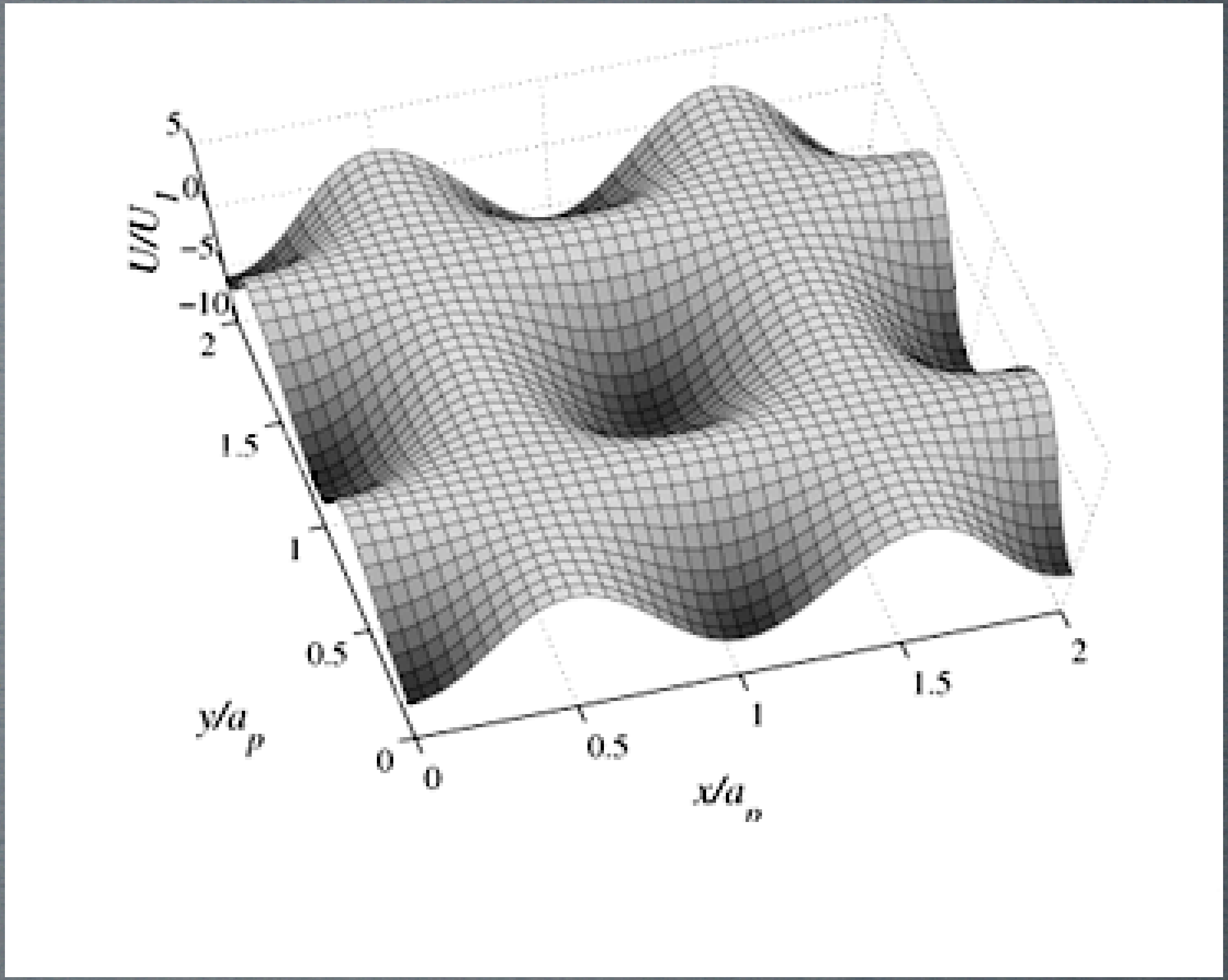


U/t

$$M_v \simeq 29m \ln(L/a) \simeq 150m \simeq 2.2 \times 10^{-20}\text{gr}$$

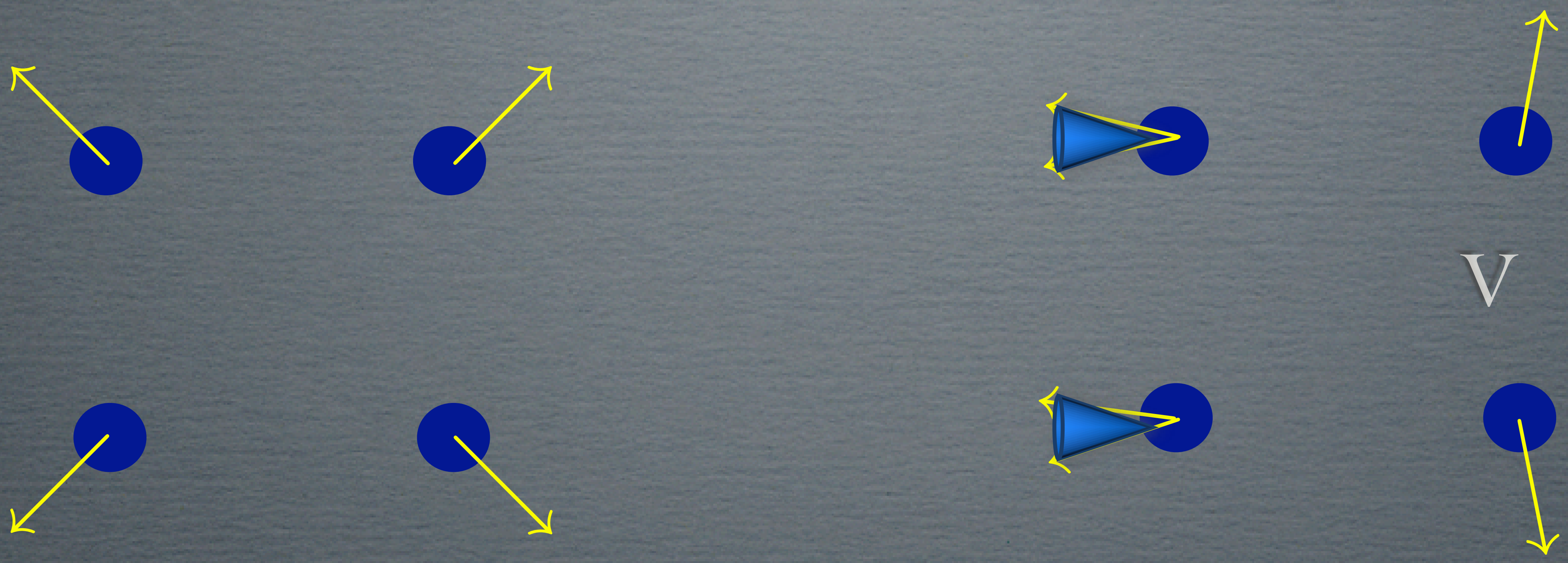






$$U_0 \sim 0.2t$$

Damping due to the excitation of "spin waves"



Bragg reflection

$$\omega_2 = \omega_1 + \omega \qquad \omega_1$$

$$\mathbf{k}_2 = \mathbf{k}_1 + \mathbf{q} \qquad \mathbf{k}_1$$

Bragg spectroscopy measures the probability of momentum transfer $\hbar\mathbf{q}$ at energy ω

$$S(\mathbf{q},\omega) = \int dt e^{i\omega t} \sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} \langle \hat{n}_i(t)\hat{n}_j(0)\rangle$$

NO VORTEX

$$H = U \sum_i n_i^2 + \frac{t}{2} \sum_{\langle i,j \rangle} (\phi_i - \phi_j)^2$$

$$\hat{\phi}_{\mathbf{k}} = [U N_s/(\hbar \Omega_{\mathbf{k}})]^{1/2} \left(\hat{a}_{\mathbf{k}} + \hat{a}^{\dagger}_{-\mathbf{k}} \right) / \sqrt{2}$$

$$\hat{n}_{\mathbf{k}} = (N_s \hbar \Omega_{\mathbf{k}}/U)^{1/2} \left(\hat{a}_{\mathbf{k}} - \hat{a}^{\dagger}_{-\mathbf{k}} \right) / i\sqrt{2}$$

$$\Omega^2_{\mathbf{k}} = (2tU/\hbar^2)[2 - \cos(k_x a) - \cos(k_y a)]$$

NO VORTEX

$$S(\mathbf{q},\omega) = \frac{\hbar\Omega_{\mathbf{q}}}{2U}\delta(\omega - \Omega_{\mathbf{q}})$$

$$\Omega_{\mathbf{k}}^2 = (2tU/\hbar^2)[2 - \cos(k_x a) - \cos(k_y a)]$$

ONE - VORTEX

$$H_v = \frac{1}{2} M_v \dot{\mathbf{r}}^2 + \frac{1}{2} M_v \Omega_v^2 \mathbf{r}^2$$

The vortex is pinned to a minimum of the periodic potential

$$\langle \hat{n}_i(t_1) \hat{n}_j(t_2) \rangle = \frac{U^2}{\hbar^2} \langle \dot{\hat{\phi}}_i(t_1) \dot{\hat{\phi}}_j(t_2) \rangle$$

The phase correlator is re-expressed in terms of vortex coordinates

ONE - VORTEX

$$S_v(\mathbf{q},\omega) = \frac{\hbar^2\Omega_v}{U^2}\frac{4\pi^2\hbar}{M_v a^4 q^2}\,\delta(\omega-\Omega_v)$$

$$\Omega_v = (0.1t/M_v)^{1/2}2\pi/a$$

The presence of a vortex induces a resonance at a frequency that allows access to the vortex mass

FRUSTRATED LATTICES

M. Polini, R. Fazio, A. Mac Donald and M.P. Tosi,
Phys. Rev. Lett. **95**, 010401 (2005)

FRUSTRATION

$$\sum_{<ij>} \cos(\phi_i - \phi_j) \longrightarrow \sum_{<ij>} \cos(\phi_i - \phi_j - A_{ij})$$

$$f = \frac{2\pi}{\Phi_0} \sum_P A_{ij}$$

“Magnetic ” frustration can be achieved by:

- Rotating lattices

Polini *et al*(2004)

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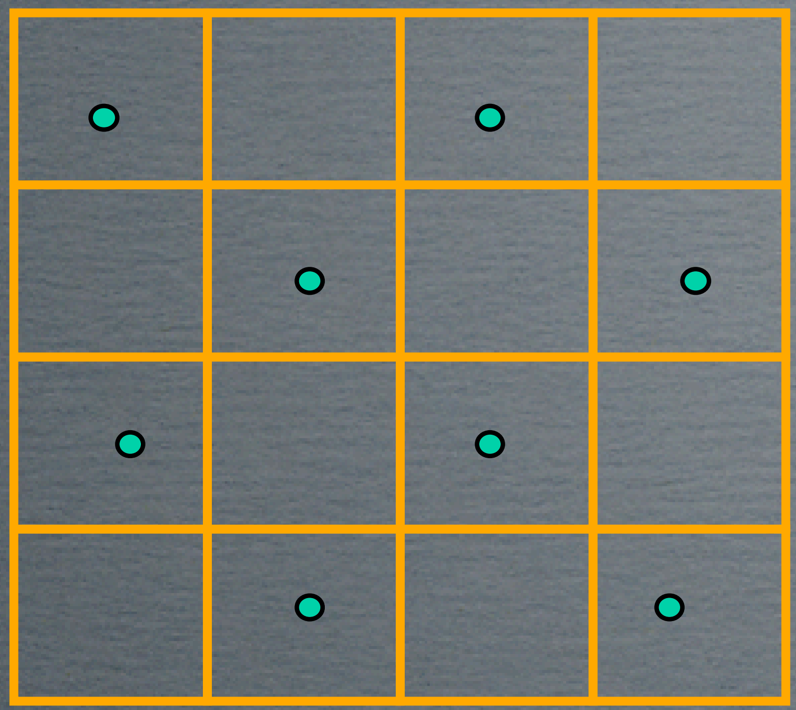
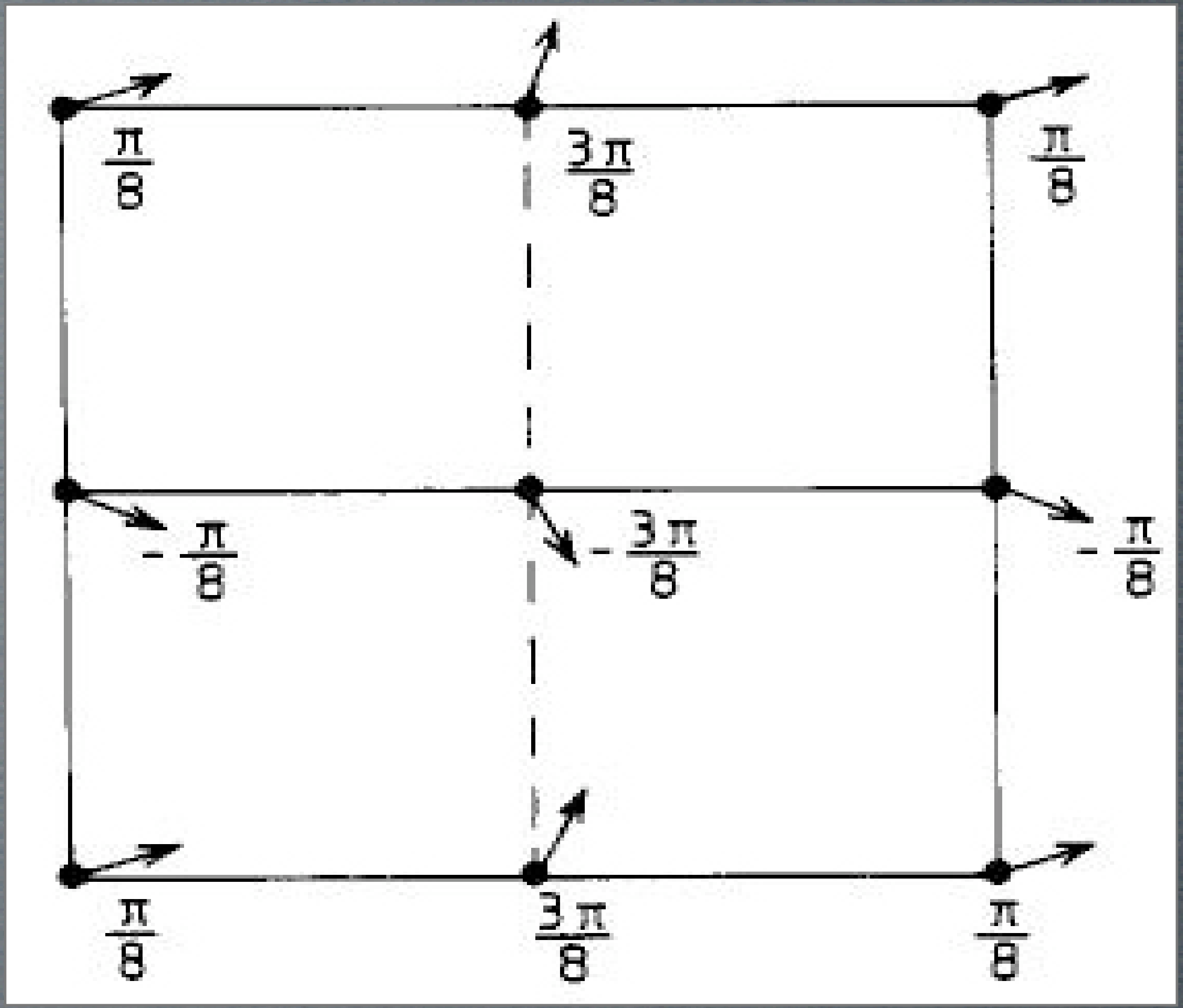
Sorensen, Demler & Lukin (2004)

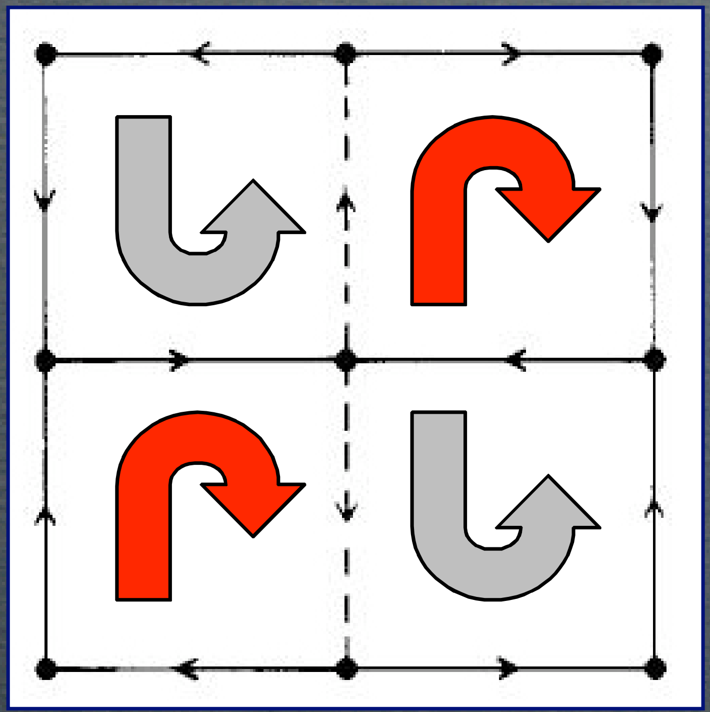
- Atoms with different internal states in different columns

Jaksch & Zoller (2003)

f=1/2



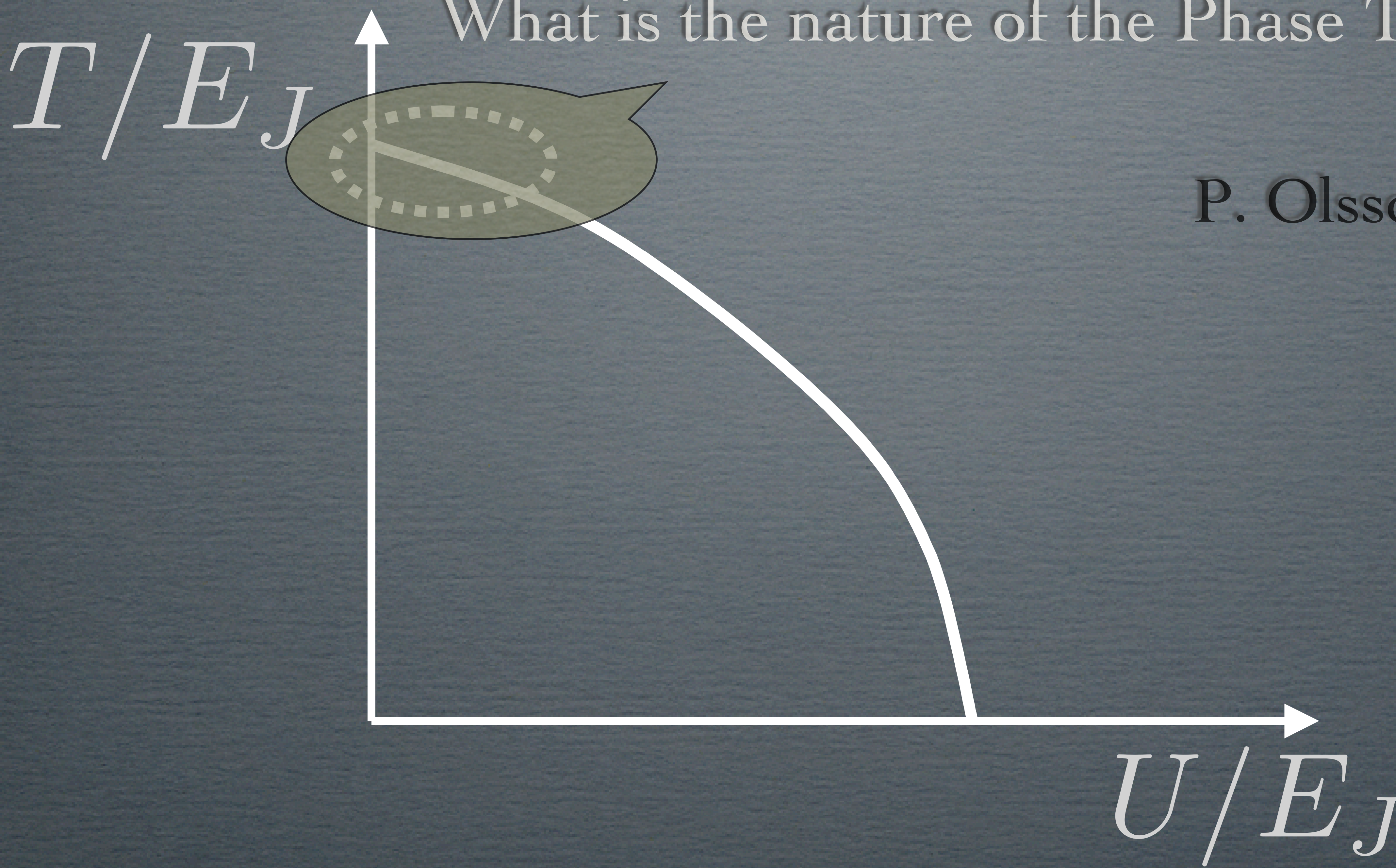




Double degenerate ground state

A LONG STANDING PROBLEM

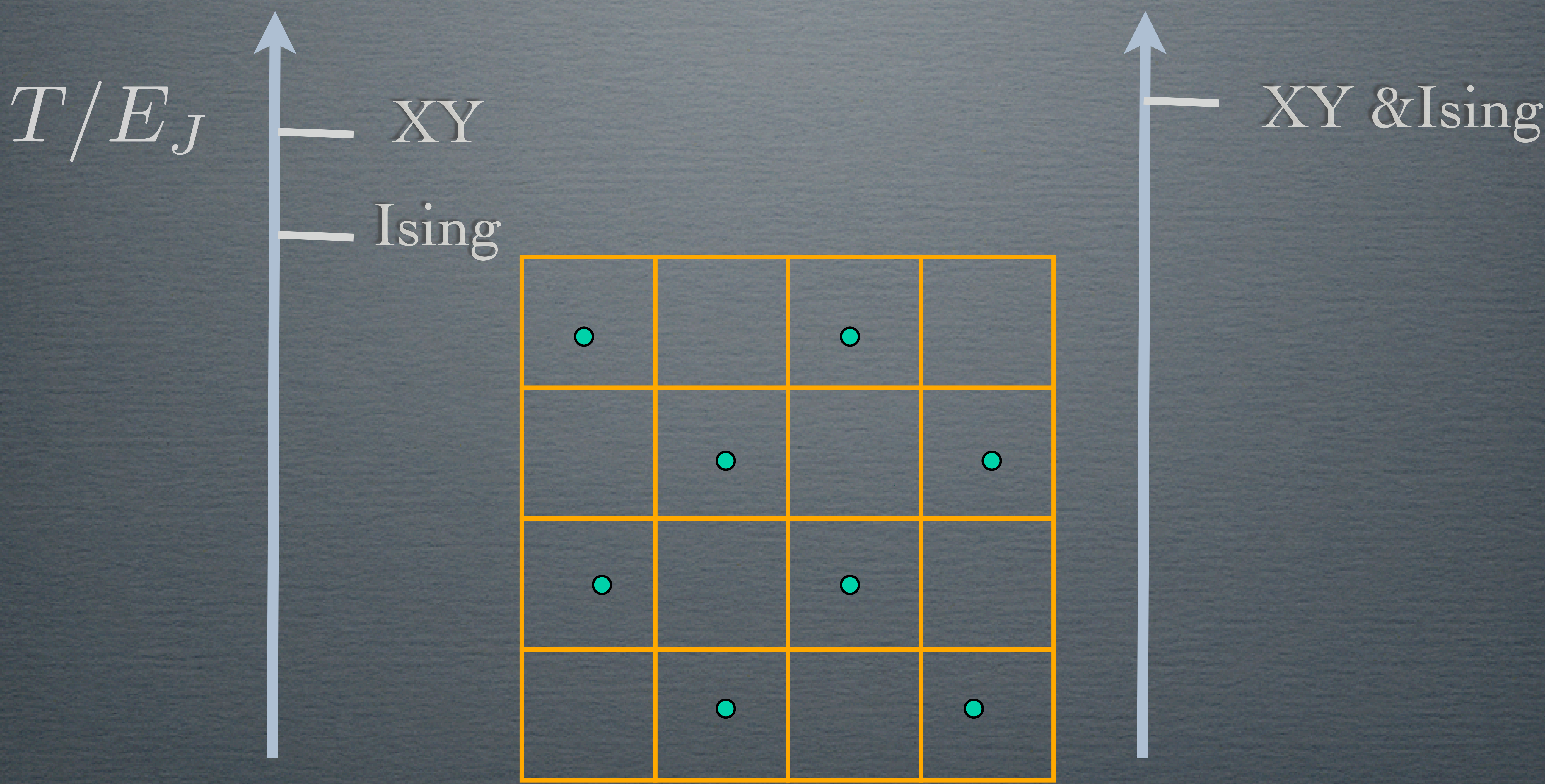
What is the nature of the Phase Transition?



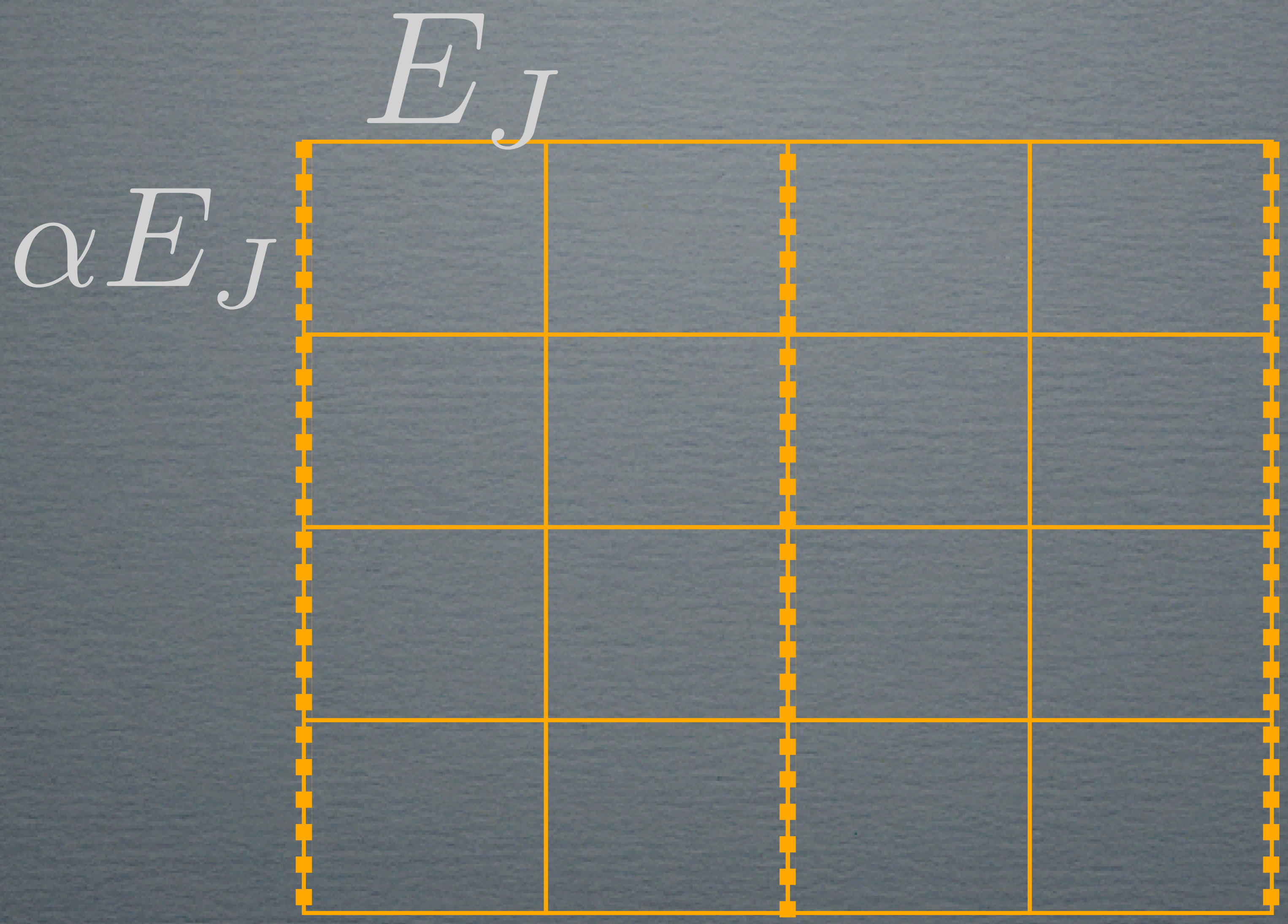
P. Olsson (1997)

Classical limit - U << 1

Two possible scenarios



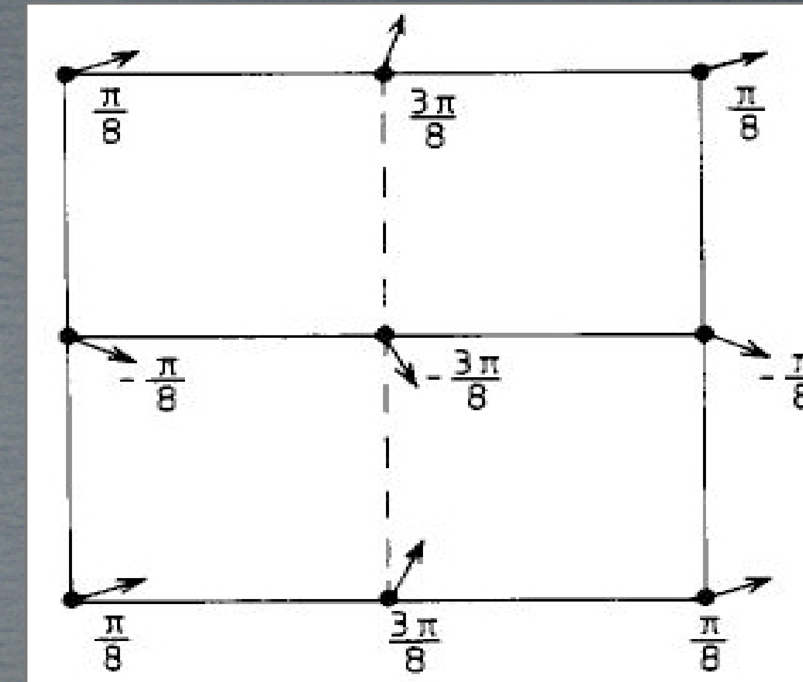
Modulated Array



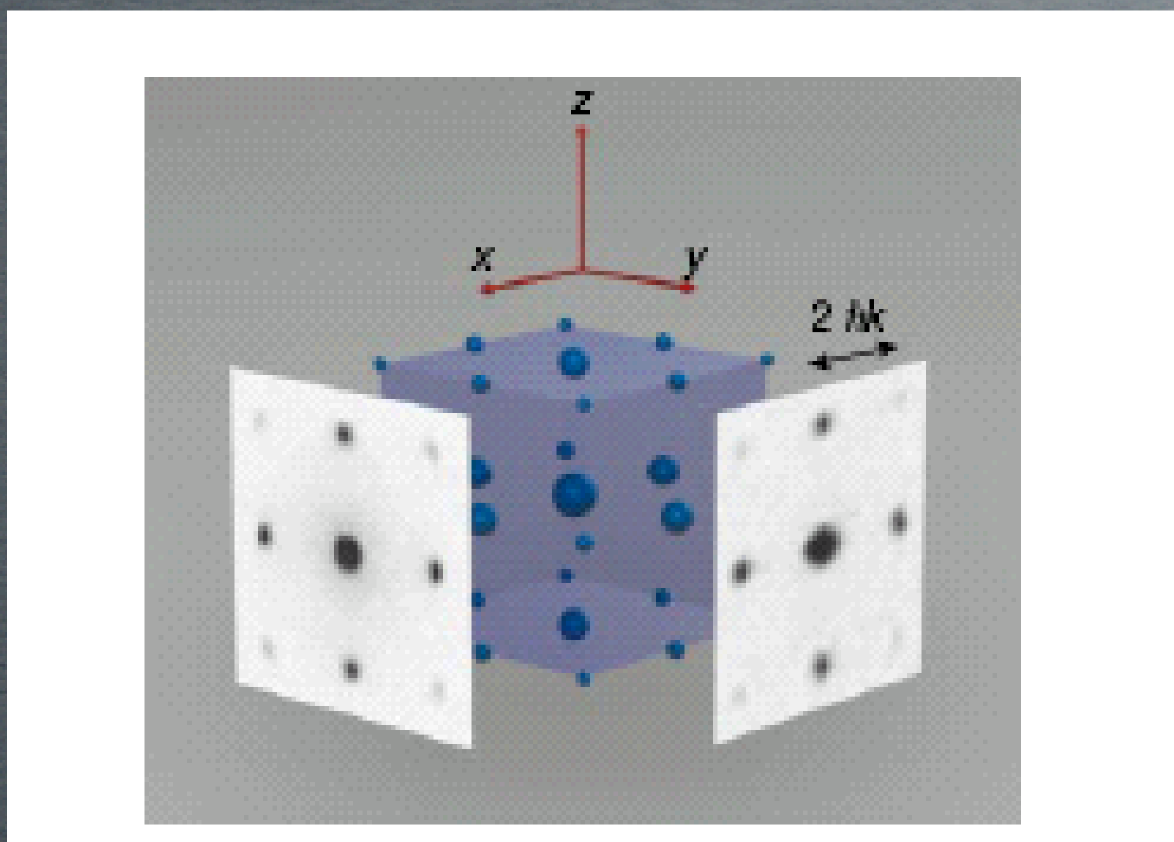
In modulated arrays the XY and Ising transition are separated

Berge *et al* 1976

Optical lattices



Ideal systems to study superlattices

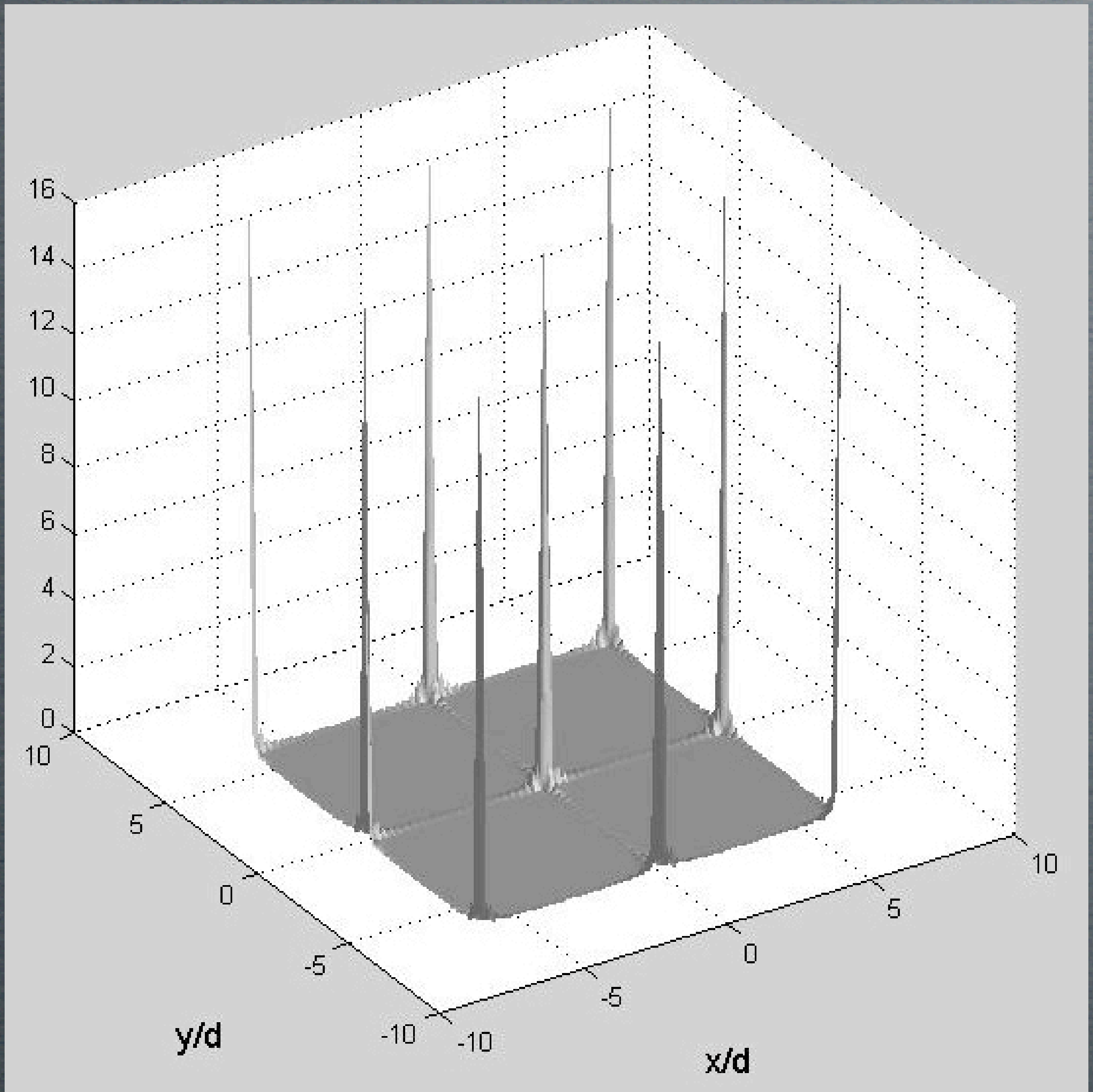


Momentum distribution

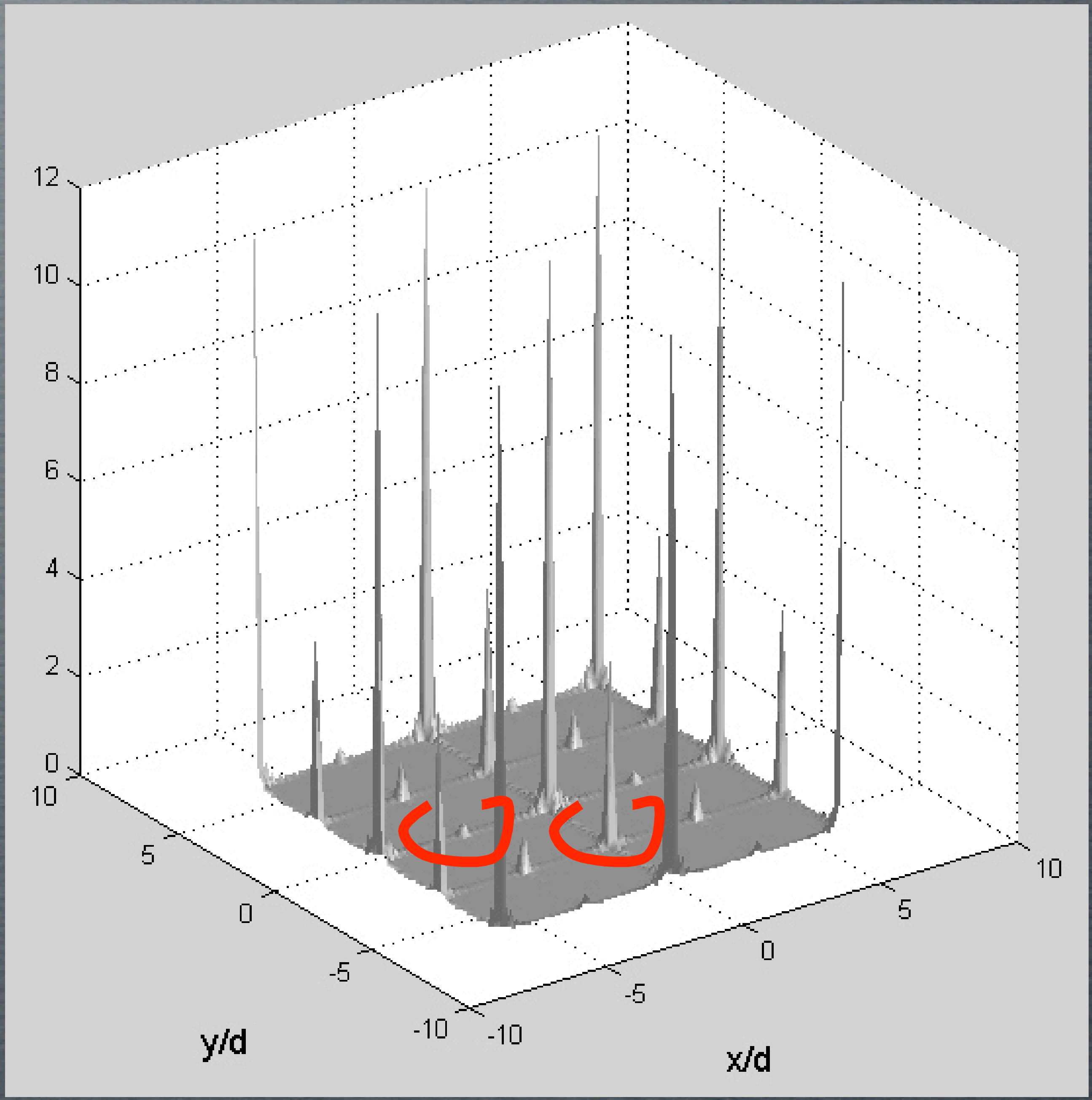
$$n(\vec{k}) = n_0(k) \sum_{ij} < b_i^+ b_j > e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)}$$

Superfluid Phase

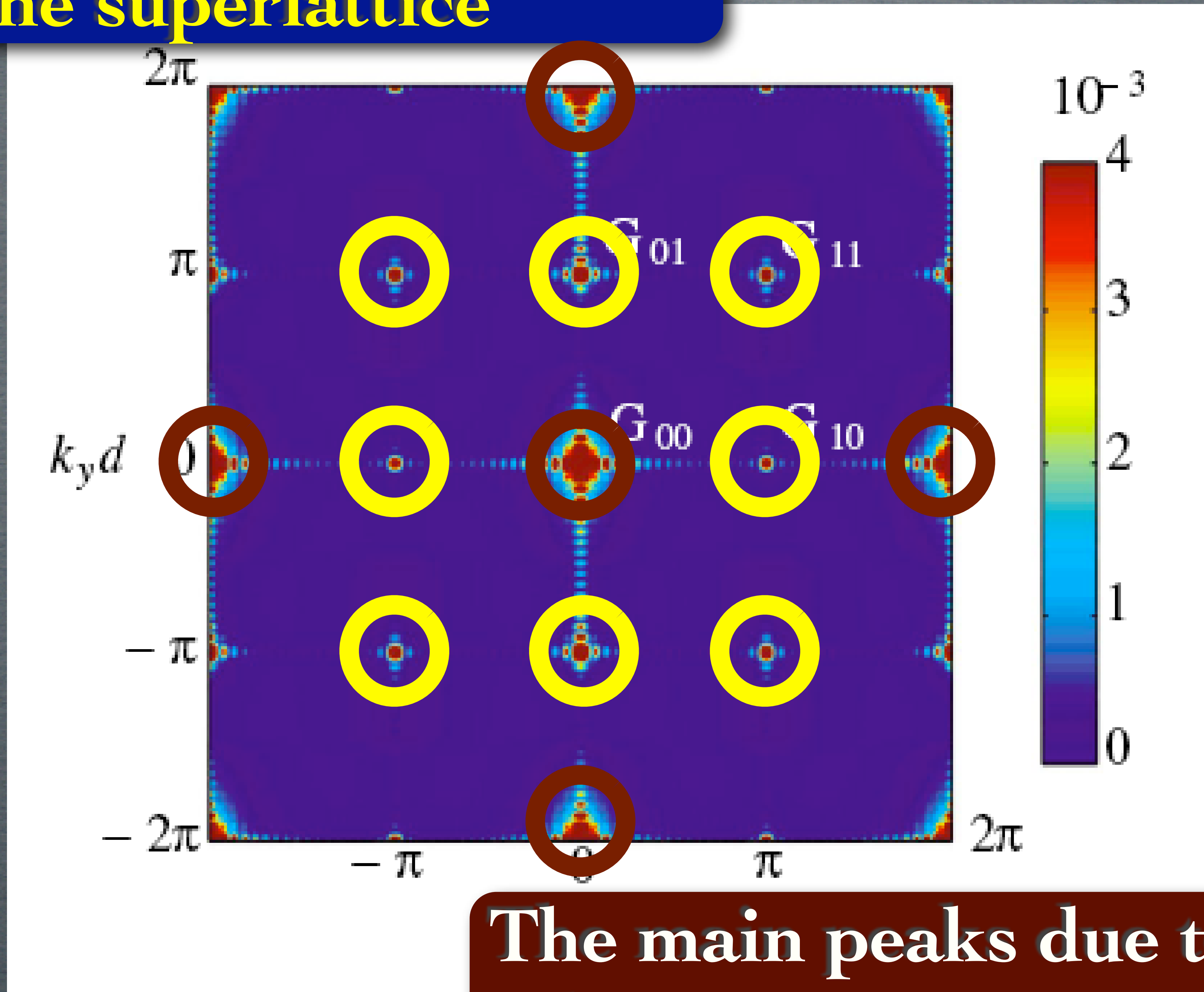
f=0



f=1/2



Additional peaks are present due to the superlattice



The main peaks due to phase coherence

Conclusions

- Ballistic and Quantum behaviour of vortices possible in optical lattices

- Bragg spectroscopy to measure vortex properties

Conclusions

- Optical lattices may help in settle down questions related to the nature of the transition in frustrated systems

- Clear signatures of vortex ordering in the momentum distribution