Ferromagnetic superconductors

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Outline

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Conventional vs. Unconventional

- **Conventional non-magnetic superconductors:** Cooper pairs have zero spin, zero angular momentum.
- **Unconventional superconductors:** Cooper pairs have non-zero angular momentum and may carry a net spin.

- Bulk or surface effect?
- Uniform coexistence or phase-separated?
- Pairing symmetry of SC order parameter?

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Our model of a ferromagnetic superconductor

[M. Grønsleth, J. L., J.-M. Børven, A. Sudbø, Phys. Rev. Lett. **97**, 147002 (2006)]:

- Thin film with in-plane magnetization→ no orbital pair-breaking effect and no diffraction pattern in tunneling currents.
- Uniform coexistence of both order parameters - no vortex flux lattice (although it has been suggested to exist, see Tewari *et al.*, Phys. Rev. Lett. **93**, 177002).
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Model

$$\hat{H}[c,c^\dagger] = \sum_{\boldsymbol{k},\sigma} \varepsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k},\sigma} c_{\boldsymbol{k},\sigma}$$

$$+ \frac{1}{2} \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} V_{\boldsymbol{k},\boldsymbol{k}'} c^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2,\alpha} c^\dagger_{-\boldsymbol{k}+\boldsymbol{q}/2,\beta} c_{-\boldsymbol{k}'+\boldsymbol{q}/2,\beta} c_{\boldsymbol{k}'+\boldsymbol{q}/2},$$

$$+ \frac{1}{2} \sum_{\boldsymbol{q}} J\gamma(\boldsymbol{q}) \boldsymbol{S}_{\boldsymbol{q}} \cdot \boldsymbol{S}_{-\boldsymbol{q}},$$

Model

$$Z = \mathrm{Tr}\left(e^{-\beta \hat{H}[c,c^{\dagger}]}\right) = e^{-\beta F}$$

Model

$$S_{\mathrm{eff}} = -\frac{1}{2}\int_0^\beta d\tau \sum_{\boldsymbol{k},\sigma} [\xi^\dagger_{\boldsymbol{k},\sigma}(\partial_\tau + \varepsilon_{\boldsymbol{k}})\xi_{\boldsymbol{k},\sigma} + \xi_{\boldsymbol{k},\sigma}(\partial_\tau - \varepsilon_{\boldsymbol{k}})\xi^\dagger_{\boldsymbol{k},\sigma}]$$

$$+ \sum_{\substack{\boldsymbol{k},\boldsymbol{q}\\ \alpha,\beta}} [\xi^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2,\alpha}(\boldsymbol{\sigma}\cdot\boldsymbol{M}_{\boldsymbol{q}})_{\alpha\beta}\xi_{\boldsymbol{k}-\boldsymbol{q}/2,\beta}$$

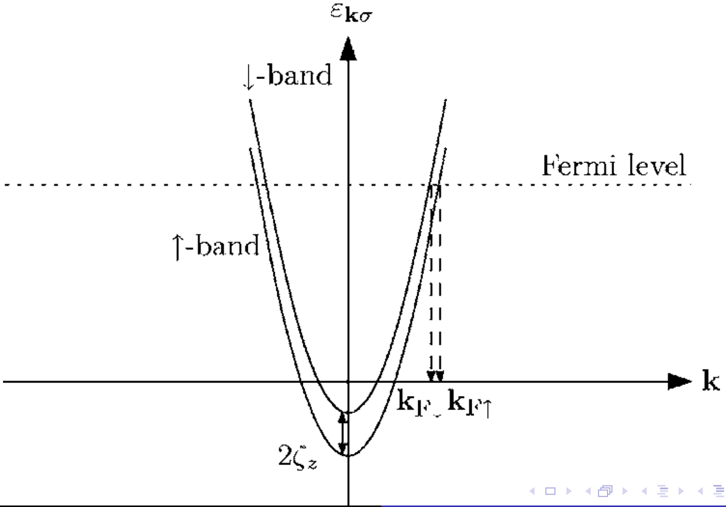
$$- \xi_{-\boldsymbol{k}-\boldsymbol{q}/2\beta}(\boldsymbol{\sigma}\cdot\boldsymbol{M}_{-\boldsymbol{q}})_{\alpha\beta}\xi^\dagger_{-\boldsymbol{k}+\boldsymbol{q}/2\alpha}]$$

$$+ \sum_{\boldsymbol{k},\boldsymbol{q}} [\Delta^\dagger_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q})\xi_{\boldsymbol{k}+\boldsymbol{q}/2,\beta}\xi_{-\boldsymbol{k}+\boldsymbol{q}/2,\alpha}$$

$$+ \Delta_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q})\xi^\dagger_{-\boldsymbol{k}+\boldsymbol{q}/2,\beta}\xi^\dagger_{\boldsymbol{k}+\boldsymbol{q}/2,\alpha}] - \sum_{\boldsymbol{q}} \frac{1}{J\gamma(\boldsymbol{q})}\boldsymbol{M}_{\boldsymbol{q}}\cdot\boldsymbol{M}_{-\boldsymbol{q}}$$

$$- \sum_{\boldsymbol{k},\boldsymbol{k}'} \Delta^\dagger_{\alpha\beta}(\boldsymbol{k}',\boldsymbol{q})V^{-1}_{\boldsymbol{k}',\boldsymbol{k}}\Delta_{\beta\alpha}(\boldsymbol{k},\boldsymbol{q}).$$

Superconducting order parameter



Superconducting order parameter

$$\hat{\Delta}_{\mathbf{k}} = \begin{pmatrix} \Delta_{\mathbf{k}\uparrow\uparrow} & \Delta_{\mathbf{k}\uparrow\downarrow} \\ \Delta_{\mathbf{k}\downarrow\uparrow} & \Delta_{\mathbf{k}\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x(\mathbf{k}) + id_y(\mathbf{k}) & d_z(\mathbf{k}) \\ d_z(\mathbf{k}) & d_x(\mathbf{k}) + id_y(\mathbf{k}) \end{pmatrix}$$

$$= i\mathbf{d}_{\mathbf{k}} \cdot \hat{\boldsymbol{\sigma}}\hat{\sigma}_y,$$

Superconducting order parameter

$$\mathbf{d}_{\mathbf{k}} = \left(\frac{\Delta_{\mathbf{k}\downarrow\downarrow} - \Delta_{\mathbf{k}\uparrow\uparrow}}{2}, \quad -i\frac{(\Delta_{\mathbf{k}\downarrow\downarrow} + \Delta_{\mathbf{k}\uparrow\uparrow})}{2}, \Delta_{\mathbf{k}\uparrow\downarrow} \right)$$

Superconducting order parameter

$$\langle \mathbf{S}_{\mathbf{k}} \rangle = i\mathbf{d}_{\mathbf{k}} \times \mathbf{d}^*_{\mathbf{k}}$$

Unitary states: $\langle S_{\mathbf{k}} \rangle = 0$
Non-unitary states: $\langle S_{\mathbf{k}} \rangle \neq 0$
Ferromagnetic superconductors: Non-unitary

Superconducting order parameter

$$\langle \mathbf{S}_{\mathbf{k}} \rangle = (1/2)[|\Delta_{\mathbf{k}\uparrow\uparrow}|^2 - |\Delta_{\mathbf{k}\downarrow\downarrow}|^2]\hat{\mathbf{z}}$$

Superconducting order parameter

$$\Delta_{\mathbf{k},\sigma,\sigma} = |\Delta_{\mathbf{k},\sigma,\sigma}| e^{i(\theta_{\mathbf{k}} + \theta^{R(L)}_{\sigma\sigma})}, \quad \theta_{\mathbf{k}} = \theta_{-\mathbf{k}} + \pi$$

Analog of He^3 A-phase

$$|\Delta_{\mathbf{k}\uparrow\uparrow}| = |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0$$

Analog of He^3 A1-phase

$$|\Delta_{\mathbf{k}\sigma\sigma}| \neq 0; \quad |\Delta_{\mathbf{k}-\sigma-\sigma}| = 0$$

Analog of He^3 A2-phase

$$|\Delta_{\mathbf{k}\uparrow\uparrow}| \neq |\Delta_{\mathbf{k}\downarrow\downarrow}| \neq 0$$

All A-phases feature $|\Delta_{\mathbf{k},\uparrow,\downarrow}| = 0$

Model Hamiltonian

$$H_{\mathrm{FMSC}} = H_0 + \sum_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \hat{\mathcal{A}}_{\mathbf{k}} \psi_{\mathbf{k}},$$

$$H_0 = JN\eta(0)\mathbf{m}^2 + \frac{1}{2}\sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\alpha\beta} \Delta^{\dagger}_{\mathbf{k}\alpha\beta} b_{\mathbf{k}\alpha\beta}.$$

Model Hamiltonian

$$\varepsilon_{\mathbf{k}\sigma} = \varepsilon_{\mathbf{k}} - \sigma\zeta_z, \qquad \sigma = \uparrow, \downarrow = \pm 1$$

Model Hamiltonian

$$\psi_{\mathbf{k}} = (c_{\mathbf{k}\uparrow} c_{\mathbf{k}\downarrow} c^{\dagger}_{-\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow})^{\mathrm{T}}$$

Model Hamiltonian

$$\hat{\mathcal{A}}_{\mathbf{k}} = -\frac{1}{2}\begin{pmatrix} -\varepsilon_{\mathbf{k}\uparrow} & \zeta & \Delta_{\mathbf{k}\uparrow\uparrow} & \Delta_{\mathbf{k}\uparrow\downarrow} \\ \zeta^\dagger & -\varepsilon_{\mathbf{k}\downarrow} & \Delta_{\mathbf{k}\downarrow\uparrow} & \Delta_{\mathbf{k}\downarrow\downarrow} \\ \Delta^\dagger_{\mathbf{k}\uparrow\uparrow} & \Delta^\dagger_{\mathbf{k}\downarrow\uparrow} & \varepsilon_{\mathbf{k}\uparrow} & -\zeta^\dagger \\ \Delta^\dagger_{\mathbf{k}\uparrow\downarrow} & \Delta^\dagger_{\mathbf{k}\downarrow\downarrow} & -\zeta & \varepsilon_{\mathbf{k}\downarrow} \end{pmatrix}$$

Model Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \xi_{\mathbf{k}} + \frac{INM^2}{2} - \frac{1}{2}\sum_{\mathbf{k}\sigma} \Delta^{\dagger}_{\mathbf{k}\sigma\sigma} b_{\mathbf{k}\sigma\sigma}$$
$$+ \frac{1}{2}\sum_{\mathbf{k}\sigma} \begin{pmatrix} \hat{c}^{\dagger}_{\mathbf{k}\sigma} & \hat{c}_{-\mathbf{k}\sigma} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}\sigma} & \Delta_{\mathbf{k}\sigma\sigma} \\ \Delta^{\dagger}_{\mathbf{k}\sigma\sigma} & -\xi_{\mathbf{k}\sigma} \end{pmatrix} \begin{pmatrix} \hat{c}_{\mathbf{k}\sigma} \\ \hat{c}^{\dagger}_{-\mathbf{k}\sigma} \end{pmatrix}$$

Self-consistent equations + solutions

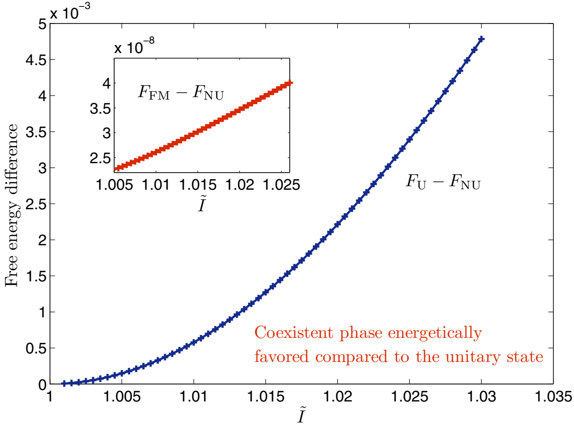
$$M = -\frac{1}{N}\sum_{\mathbf{k}\sigma}\frac{\sigma\xi_{\mathbf{k}\sigma}}{2E_{\mathbf{k}\sigma}}\tanh(\beta E_{\mathbf{k}\sigma}/2)$$

$$\Delta_{\mathbf{k}\sigma\sigma} = -\frac{1}{N}\sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'\sigma\sigma}\frac{\Delta_{\mathbf{k}'\sigma\sigma}}{2E_{\mathbf{k}'\sigma}}\tanh(\beta E_{\mathbf{k}'\sigma}/2)$$

$$\Delta_{\mathbf{k},\sigma,\sigma} = \frac{\Delta_{\sigma,0}}{\sqrt{3/8\pi}} Y^{\sigma}_{l=1}(\theta,\phi)$$

$$Y^{\sigma}_{l=1}(\theta,\phi) = -\sigma\sqrt{3/8\pi}\, e^{i\sigma\theta}\sin(\phi)$$

Self-consistent equations + solutions



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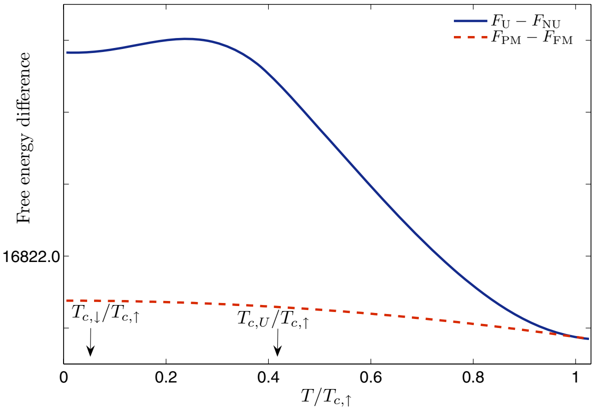

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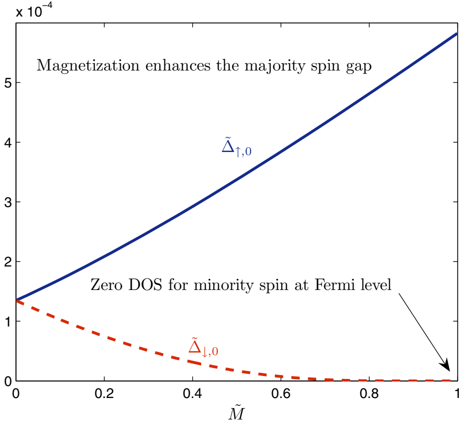

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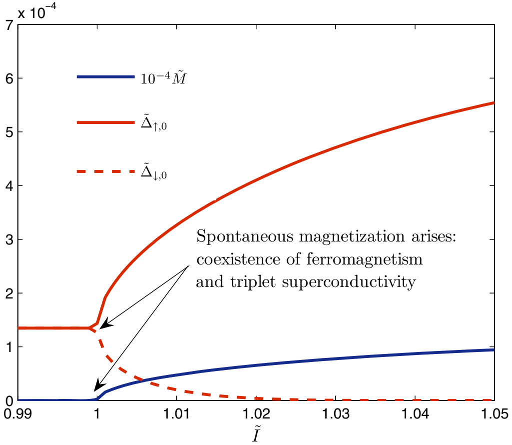

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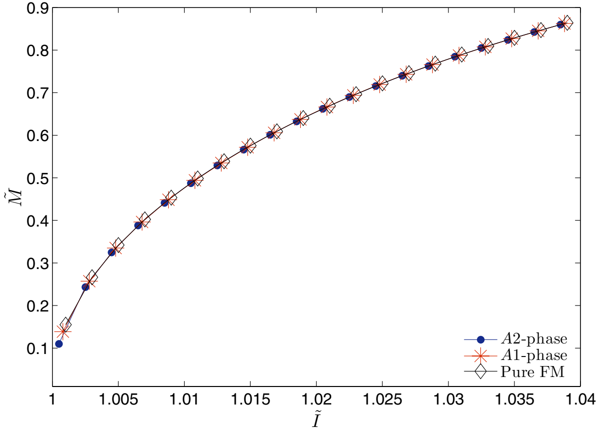

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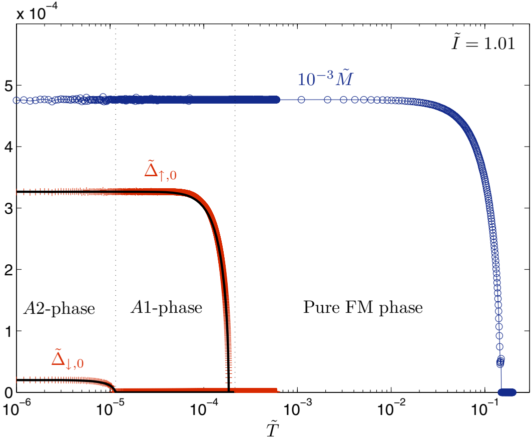

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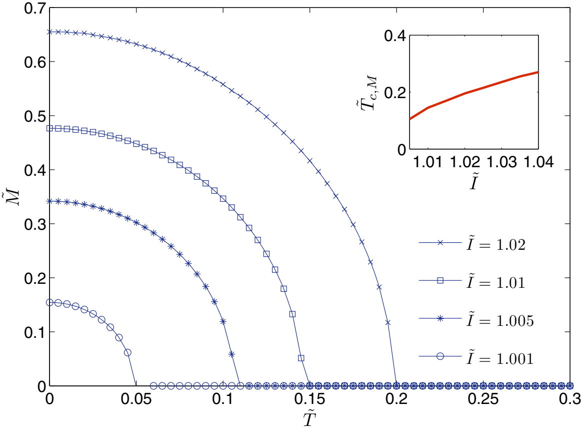

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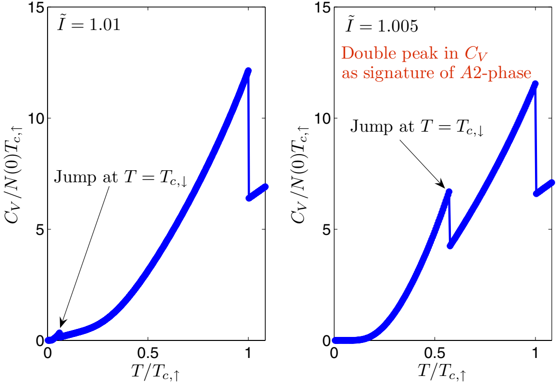
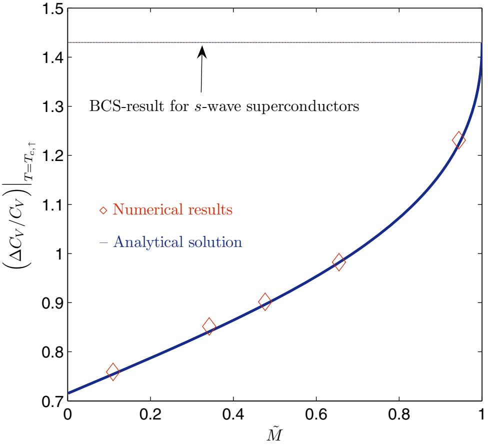


Self-consistent equations + solutions

The ratio $\Delta C_V / C_V|_{T=T_c}$

$$\Delta C_V = \frac{3.03\Delta^2_{\uparrow,0} N^{\uparrow}(0)}{2T_{c,\uparrow}}$$

$$\frac{\Delta C_V}{C_V|_{T=T_c}} = 1.43 \frac{1}{1+\sqrt{\frac{1-\tilde{M}}{1+\tilde{M}}}}$$



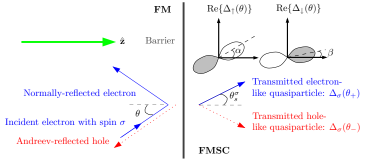
Outline

Calculations: BdG-equations (BTK-formalism)

FM/FMSC-interface in *xy*-plane. BdG-equations describe quasiparticle states in FMSC:

$$\begin{pmatrix} \hat{\mathcal{M}}_{\mathbf{k}} & \hat{\Delta}_{\mathbf{k}} \\ \hat{\Delta}^*_{\mathbf{k}} & -\hat{\mathcal{M}}_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{k}\sigma} \\ v_{\mathbf{k}\sigma} \end{pmatrix} = E_{\mathbf{k}\sigma} \begin{pmatrix} u_{\mathbf{k}\sigma} \\ v_{\mathbf{k}\sigma} \end{pmatrix},$$

with $\hat{\mathcal{M}}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} \hat{1} - \hat{\sigma}_z U_{\mathrm{R}}$ and $\hat{\Delta}_{\mathbf{k}} = \hat{\boldsymbol{\sigma}} \cdot \mathbf{d}_{\mathbf{k}} \mathrm{i}\hat{\sigma}_y$. Barrier described by dimensionless parameter Z.



Spin-generalized BTK formalism [Phys. Rev. B **25**, 4515 (1982)].

Calculations: BdG-equations (BTK-formalism)

$$\Psi^{\sigma}_{\text{tot}}(z) = \Theta(-z)\psi^{\sigma}(z) + \Theta(z)\Psi^{\sigma}(z)$$

Calculations: BdG-equations (BTK-formalism)

$$\psi^\sigma(z) = e^{ik^\sigma \sin\ \theta y}\Bigg[\begin{pmatrix}1\\0\end{pmatrix}e^{ik^\sigma \cos\ \theta z} + r_e^\sigma(E,\theta)\begin{pmatrix}1\\0\end{pmatrix}e^{-ik^\sigma \cos\ \theta z}$$

$$+ r_h^\sigma(E,\theta)\begin{pmatrix}0\\1\end{pmatrix}e^{ik^\sigma \cos\ \theta z}\Bigg],$$

$$\Psi^\sigma(z) = e^{iq^\sigma \sin\ \theta y}\Bigg[t_e^\sigma(E,\theta)\begin{pmatrix}u_\sigma(\theta^\sigma_{s+})\\ v_\sigma(\theta^\sigma_{s+})\gamma^*_\sigma(\theta^\sigma_{s+})\end{pmatrix}e^{iq^\sigma \cos\ \theta^\sigma_s z}$$

$$+ t_h^\sigma(E,\theta)\begin{pmatrix}v_\sigma(\theta^\sigma_{s-})\gamma_\sigma(\theta^\sigma_{s-})\\ u_\sigma(\theta^\sigma_{s-})\end{pmatrix}e^{-iq^\sigma \cos\ \theta^\sigma_s z}\Bigg],$$

Calculations: BdG-equations (BTK-formalism)

$$k^\sigma = [2m(E_F + \sigma U_L)]^{1/2}$$

$$q^\sigma = [2m(E_F + \sigma U_R)]^{1/2}$$

$$u_\sigma(\theta_{s\pm}^\sigma) = \frac{1}{4}\{1 + \sqrt{1 - (|\Delta_\sigma(\theta_{s\pm}^\sigma)|/E)^2}\}^{1/2}$$

$$v_\sigma(\theta_{s\pm}^\sigma) = \frac{1}{4}\{1 - \sqrt{1 - (|\Delta_\sigma(\theta_{s\pm}^\sigma)|/E)^2}\}^{1/2}$$

Calculations: BdG-equations (BTK-formalism)

$$G^\sigma(E) = \int_{-\pi/2}^{\pi/2} d\theta \cos\,\theta g^\sigma(E,\theta) P^L_\sigma P^R_\sigma,$$

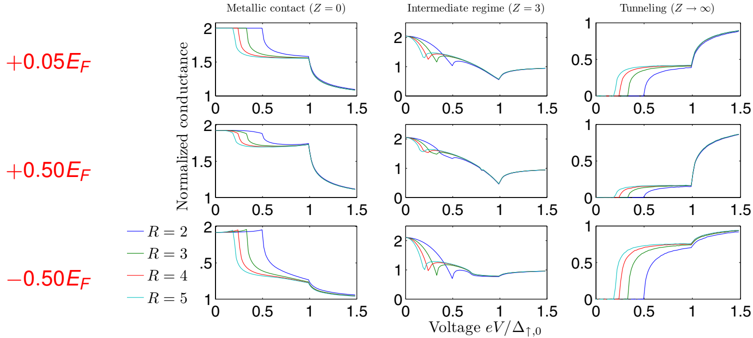
$$g^\sigma(E,\theta) = 1 + |r^\sigma_h(E,\theta)|^2 - |r^\sigma_e(E,\theta)|^2,$$

$$F^\sigma = \int_{-\pi/2}^{\pi/2} d\theta \cos\,\theta f^\sigma(\theta) P^L_\sigma P^R_\sigma,$$

$$f^\sigma(\theta) = 1 - |1 - 2k^\sigma \cos\,\theta/\Upsilon^\sigma_+|^2.$$

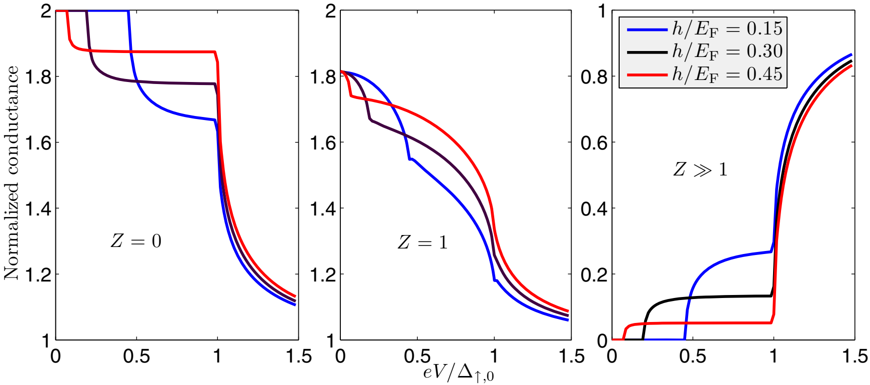
$$P^{L(R)}_\sigma = (E_F + \sigma U_{L(R)})/2E_F$$

Conductance spectrum: Analogue of $A2$-phase

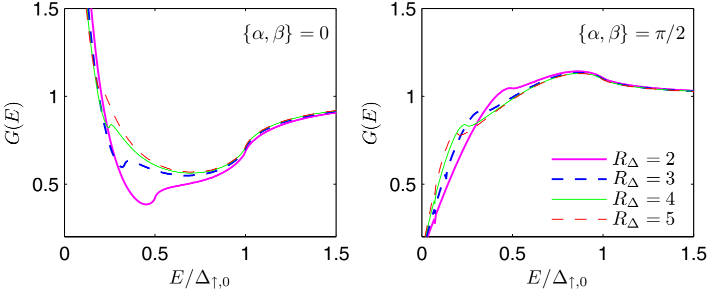


FM exchange U_L. R = $\Delta_{\uparrow,0}/\Delta_{\downarrow,0}$. $\Delta_\sigma = -\sigma\Delta_{\sigma,0}e^{i[\theta+\phi_\sigma-\alpha(\beta)]}$.

Conductance spectrum: Analogue of $A2$-phase

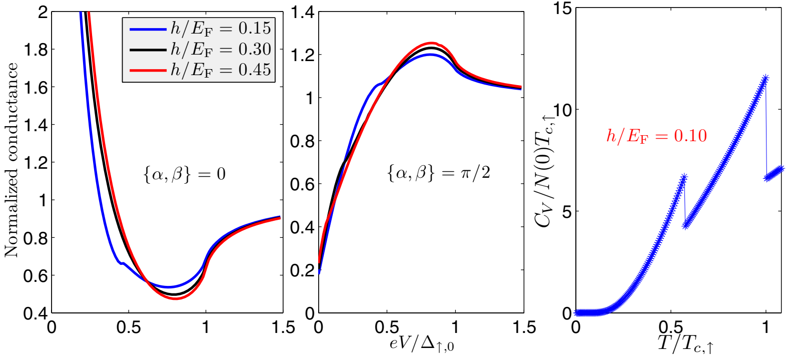


Conductance spectrum: Simple odd-parity model



$$\Delta_\uparrow = \Delta_{\uparrow,0} \cos(\theta - \alpha),\ \Delta_\downarrow = \Delta_{\downarrow,0} \cos(\theta - \beta)\ \ (Z = 3).$$

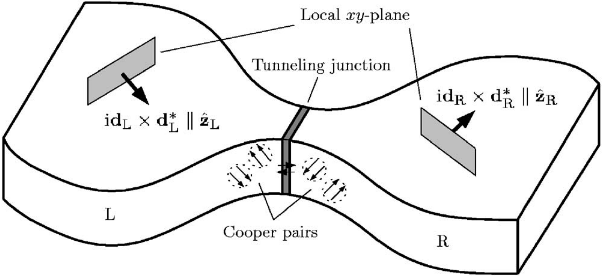
Conductance spectrum: Simple odd-parity model



$$\Delta_\uparrow = \Delta_{\uparrow,0}\cos(\theta-\alpha),\ \Delta_\downarrow = \Delta_{\downarrow,0}\cos(\theta-\beta)\ \ (Z=3).$$

Outline

Tunneling in thin-film structure



Tunneling in thin-film structure

$$\dot{N}_{\alpha\beta} = i[H_{\mathrm{T}}, N_{\alpha\beta}]$$

$$= -i \sum_{\mathbf{kp}\sigma} [\hat{\mathcal{D}}^{(1/2)}_{\sigma\beta}(\vartheta) T_{\mathbf{kp}} c^{\dagger}_{\mathbf{k}\alpha} d_{\mathbf{p}\sigma} - \hat{\mathcal{D}}^{(1/2)}_{\sigma\alpha}(\vartheta) T^{*}_{\mathbf{kp}} d^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{k}\beta}]$$

Tunneling in thin-film structure

Charge currents, 1- and 2-particle channels

$$I^C(t) = I^C_{1p}(t) + I^C_{2p}(t) = -e \sum_{\alpha} \langle \dot{N}_{\alpha\alpha} \rangle$$

Spin currents, 1- and 2-particle channels

$$\mathbf{I}^S(t) = \mathbf{I}^S_{\mathrm{sp}}(t) + \mathbf{I}^S_{\mathrm{tp}}(t) = \sum_{\alpha\beta} \boldsymbol{\sigma}_{\alpha\beta} \langle \dot{N}_{\alpha\beta}(t) \rangle$$

Tunneling in thin-film structure

DC Charge/Spin Josephson current, A1-phase

$$I_{\mathrm{tp}}^{\mathrm{C}} = e \cos^2(\vartheta/2) X_\alpha \qquad \alpha \in \{\uparrow,\downarrow\},$$
$$I_{\mathrm{tp},z}^{\mathrm{S}} = -\,\alpha \cos^2(\vartheta/2) X_\alpha$$

where we have defined the quantity

$$X_\alpha = \sum_{\mathbf{kp}} |T_{\mathbf{kp}}|^2 \frac{|\Delta_{\mathbf{k}\alpha\alpha}\Delta_{\mathbf{p}\alpha}|}{E_{\mathbf{k}\alpha}E_{\mathbf{p}\alpha}} F_{\mathbf{kp}\alpha\alpha}[\sin\Delta\theta_{\alpha\alpha}\cos\Delta\theta_{\mathbf{pk}} + \cos\Delta\theta_{\alpha\alpha}\sin\Delta\theta_{\mathbf{pk}}],$$

Tunneling in thin-film structure

DC Charge/Spin Josephson current, A2-phase

$$I_{J(,z)}^{C(S)} = \sum_{\mathbf{k},\mathbf{k},\sigma,\alpha} I^{C(S)}(\Delta\theta_{\mathbf{p},\mathbf{k}}, \theta_{\sigma\sigma}^{L} - \theta_{\alpha\alpha}^{R})$$

$$I^{C}(\phi_1,\phi_2) = \frac{e}{2}[1+\sigma\alpha\cos(\vartheta)]|T_{\mathbf{k},\mathbf{p}}|^2\frac{|\Delta_{\mathbf{k}}\alpha\alpha||\Delta_{\mathbf{p}\sigma\sigma}|}{E_{\mathbf{k}\alpha}E_{\mathbf{p}\sigma}} \cos(\phi_1)\sin(\phi_2)F_{\mathbf{kp}\alpha\sigma}$$

$$I^{S}(\phi_1,\phi_2) = -\frac{1}{2}\alpha[1+\sigma\alpha\cos(\vartheta)]|T_{\mathbf{k},\mathbf{p}}|^2\frac{|\Delta_{\mathbf{k}}\alpha\alpha||\Delta_{\mathbf{p}\sigma\sigma}|}{E_{\mathbf{k}\alpha}E_{\mathbf{p}\sigma}} \cos(\phi_1)sin(\phi_2)F_{\mathbf{kp}\alpha\sigma}$$

$$F_{\mathbf{kp}\alpha\sigma} = \sum_{\pm}\frac{f(\pm E_{\mathbf{k}\alpha}) - f(E_{\mathbf{p}\sigma})}{E_{\mathbf{k}\alpha}\mp E_{\mathbf{p}\sigma}}$$

Outline

Preliminaries

$$S_{\text{eff}} = -\frac{1}{2}\int_0^\beta d\tau \Bigg\{ \sum_{\boldsymbol{k},\boldsymbol{k}'} \phi^\dagger_{\boldsymbol{k}'} \mathcal{G}^{-1} \phi_{\boldsymbol{k}} - \sum_{\boldsymbol{q}} \frac{1}{J\gamma(\boldsymbol{q})} \boldsymbol{M}_{\boldsymbol{q}} \cdot \boldsymbol{M}_{-\boldsymbol{q}} - \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \text{tr}\Delta^\dagger(\boldsymbol{k}',\boldsymbol{q}) V^{-1}_{\boldsymbol{k},\boldsymbol{k}'} \Delta(\boldsymbol{k},\boldsymbol{q}) \Bigg\}.$$

Preliminaries

$$\mathcal{G}^{-1} = \mathcal{G}_0^{-1} - \Sigma$$

Preliminaries

$$\Sigma = \begin{bmatrix} \mathcal{M} & \mathcal{D} \\ \mathcal{D}^\dagger & \mathcal{M} \end{bmatrix}$$

Preliminaries

$$\mathcal{D} = d_0 \cdot 1 + \boldsymbol{d} \cdot \sigma = -i\Delta\sigma_y$$

Preliminaries

$$\beta F_{\mathrm{GL}} = -\,\mathrm{Tr}\ln \mathcal{G}^{-1} - \frac{1}{2}\int_0^\beta d\tau \Bigg[\sum_{\boldsymbol{q}} \frac{1}{J\gamma(\boldsymbol{q})} \boldsymbol{M}_{\boldsymbol{q}} \cdot \boldsymbol{M}_{-\boldsymbol{q}} + \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \mathrm{tr}\Delta^\dagger(\boldsymbol{k}',\boldsymbol{q}) V^{-1}_{\boldsymbol{k},\boldsymbol{k}'} \Delta(\boldsymbol{k},\boldsymbol{q}) \Bigg],$$

Magnetic and superconducting contributions

$$E_2 = \frac{1}{2}\mathrm{Tr}\mathcal{G}_0\Sigma\mathcal{G}_0\Sigma = \frac{1}{2}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2} \mathcal{G}_{0\boldsymbol{k}_1}\Sigma_{\boldsymbol{k}_1,\boldsymbol{k}_2}\mathcal{G}_{0\boldsymbol{k}_2}\Sigma_{\boldsymbol{k}_2,\boldsymbol{k}_1}$$

Magnetic and superconducting contributions

$$F_2 = \sum_{\substack{\boldsymbol{k},\boldsymbol{q}\\ \omega_n}} \Bigg\{ - \left[\frac{\delta_{\boldsymbol{k},0}}{J\gamma(\boldsymbol{q})} - \frac{1}{\beta}(g_{0,\boldsymbol{k}+\boldsymbol{q}/2}\, g_{0,\boldsymbol{k}-\boldsymbol{q}/2} + \text{c.c.}) \right] \boldsymbol{M}_{\boldsymbol{q}} \cdot \boldsymbol{M}_{-\boldsymbol{q}} - \left[\frac{1}{\beta}(g_{0,\boldsymbol{k}+\boldsymbol{q}/2}\, g^{*}_{0,\boldsymbol{k}-\boldsymbol{q}/2} + \text{c.c.}) + 2/V \right] |\boldsymbol{d}_{\boldsymbol{k},\boldsymbol{q}}|^2 \Bigg\}. \tag{16}$$

Magnetic and superconducting contributions

$$F^{c}_{2,m} = \frac{\beta}{2}\sum_{\boldsymbol{q}}\left[4N_0\tanh\left(\frac{\epsilon_m\beta}{2}\right) + \frac{1}{6J}\right]\boldsymbol{M}_{\boldsymbol{q}}\cdot\boldsymbol{M}_{-\boldsymbol{q}}.$$

Magnetic and superconducting contributions

$$F_{2,m}^{d} = -\frac{\beta}{2}\sum_{\boldsymbol{q}} \left\{ \frac{N_0(v_F\beta)^2\epsilon_m^3}{72} \tanh\left(\frac{\epsilon_m\beta}{2}\right)\left[1 - \tanh^2\left(\frac{\epsilon_m\beta}{2}\right)^2\right] + \frac{1}{36J} \right\} \boldsymbol{q}^2 \boldsymbol{M}_{\boldsymbol{q}} \boldsymbol{M}_{-\boldsymbol{q}}. \quad (19)$$

Magnetic and superconducting contributions

$$d_{\mu}(\boldsymbol{k},\boldsymbol{q}) = A_{\mu i}(\boldsymbol{q})\hat{k}_i$$

Magnetic and superconducting contributions

$$F^{c}_{2,S} = \frac{1}{2}\sum_{q}\frac{4N_0}{3}\frac{T-T_c}{T_c}\mathrm{tr}(AA^{\dagger})$$

Magnetic and superconducting contributions

$$F^d_{2,S} = -\frac{7v_F^2 N_0 \beta^3 \zeta(3)}{120\pi^2}[q^2 \mathrm{tr}\mathcal{A}\mathcal{A}^\dagger + q_i\mathcal{A}_{\mu i}q_j\mathcal{A}^\dagger_{j\mu} + q_j\mathcal{A}_{\mu i}q_i\mathcal{A}^\dagger_{j\mu}]$$

Cubic coupling of magnetic and superconducting contributions

$$E_3 = \frac{1}{3} \sum_{\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3} \mathcal{G}_{0\boldsymbol{k}_1} \Sigma_{\boldsymbol{k}_1,\boldsymbol{k}_2} \mathcal{G}_{0\boldsymbol{k}_2} \Sigma_{\boldsymbol{k}_2,\boldsymbol{k}_3} \mathcal{G}_{0\boldsymbol{k}_3} \Sigma_{\boldsymbol{k}_3,\boldsymbol{k}_1}$$

Cubic coupling of magnetic and superconducting contributions

$$E_3 = 4 \sum_{k,\{q_i\}}{}' (g_{0,k} g^*_{0,k} g_{0,k} + \mathrm{c.c.}) i \boldsymbol{d}^*_{k,-\boldsymbol{q}_1} \times \boldsymbol{d}_{k,\boldsymbol{q}_3} \cdot \boldsymbol{M}_{\boldsymbol{q}_2}$$

Cubic coupling of magnetic and superconducting contributions

$$F_3 = - \sum_{q_1,q_2,q_3}{}' \frac{\alpha_3}{3!} i\varepsilon_{\mu\nu\lambda} A_{\mu i} A^*_{\nu i} M_\lambda$$

Cubic coupling of magnetic and superconducting contributions

$$\tilde{\alpha}_3/3! = 4\,\mathrm{Re} \sum_{\omega_n,\boldsymbol{k}} \frac{1}{\omega_n^2+\varepsilon_{\boldsymbol{k}}^2} \frac{\hat{k}_i \hat{k}_j}{-i\omega_n+\varepsilon_{\boldsymbol{k}}} = N_0' \beta \delta_{ij}/3 \sum_{n\geqslant 0} \frac{1}{n+1/2}$$

Quartic coupling of magnetic and superconducting contributions

$$E_4 = \frac{1}{4} \sum_{\substack{k_1,k_2 \\ k_3,k_4}} \mathcal{G}_{0k_1} \Sigma_{k_1,k_2} \mathcal{G}_{0k_2} \Sigma_{k_2,k_3} \mathcal{G}_{0k_3} \Sigma_{k_3,k_4} \mathcal{G}_{0k_4} \Sigma_{k_4,k_1}.$$

Quartic coupling of magnetic and superconducting contributions

$$\beta F_{4,m} = \sum_{\{q_i\}}{}' \tilde{\alpha}_{4m} (\boldsymbol{M}_{q_1} \cdot \boldsymbol{M}_{-q_2})(\boldsymbol{M}_{q_3} \cdot \boldsymbol{M}_{-q_4}) . \qquad (31)$$

Here, the coefficient of the $\boldsymbol{M}_q$ factors is given by the trace over the electron propagators

$$\tilde{\alpha}_{4m} = \mathrm{Re} \sum_{\boldsymbol{k},\omega_n} (g_{0,\boldsymbol{k}})^4 = N_0 \,\mathrm{Re} \sum_{\omega_n} \int_0^{\epsilon_m} d\xi \left(\frac{1}{-i\omega_n + \xi} \right)^4$$

$$= \frac{\beta^3 2}{4!} \tanh\left(\frac{\epsilon_m \beta}{2}\right) \left[1 - \tanh^2\left(\frac{\epsilon_m \beta}{2}\right) \right] . \qquad (32)$$

Quartic coupling of magnetic and superconducting contributions

$$F_{4,S} = \frac{\alpha_{4S}}{4!} \sum_{\{q_i\}}{}' \{ - |\mathrm{tr} \mathcal{A}\mathcal{A}^T|^2 + 2(\mathrm{tr}\mathcal{A}\mathcal{A}^\dagger)^2 + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^T)(\mathcal{A}\mathcal{A}^T)^* + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)^2 - 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)(\mathcal{A}\mathcal{A}^\dagger)^* \}, \tag{37}$$

Quartic coupling of magnetic and superconducting contributions

$$F_{4,Sm} = \sum_{\boldsymbol{k},\{\boldsymbol{q}_i\},\omega_n}{}' \{8 g_{0,\boldsymbol{k}} g_{0,\boldsymbol{k}} g^*_{0,\boldsymbol{k}} g^*_{0,\boldsymbol{k}} [2(\boldsymbol{M}_{\boldsymbol{q}_1} \cdot \boldsymbol{d}_{-\boldsymbol{q}_2})(\boldsymbol{M}_{\boldsymbol{q}_3} \cdot \boldsymbol{d}^*_{-\boldsymbol{q}_4})$$

$$- (\boldsymbol{M}_{\boldsymbol{q}_1} \cdot \boldsymbol{M}_{\boldsymbol{q}_3})(\boldsymbol{d}_{-\boldsymbol{q}_2} \boldsymbol{d}^*_{-\boldsymbol{q}_4})] - 16 g_{0,\boldsymbol{k}} g_{0,\boldsymbol{k}} g_{0,\boldsymbol{k}} g^*_{0,\boldsymbol{k}} (\boldsymbol{d}_{\boldsymbol{q}_3} \boldsymbol{d}^*_{-\boldsymbol{q}_4})$$

$$\times (\boldsymbol{M}_{\boldsymbol{q}_1} \cdot \boldsymbol{M}_{-\boldsymbol{q}_2})\} = \sum_{\{\boldsymbol{q}_i\}}{}' \frac{7 N_0 \zeta(3) \beta^3}{3\pi^2} (M_\mu \mathcal{A}_{\mu j})(M_\nu \mathcal{A}^*_{\nu j}),$$

Quartic coupling of magnetic and superconducting contributions

$$F_4 = 1/4! \sum_{\{q_i\}}' \{ \alpha_{4m} (\boldsymbol{M}_{\boldsymbol{q}_1} \cdot \boldsymbol{M}_{-\boldsymbol{q}_2})(\boldsymbol{M}_{\boldsymbol{q}_3} \cdot \boldsymbol{M}_{-\boldsymbol{q}_4}) + \alpha_{4Sm} (M_\mu \mathcal{A}_{\mu j}) \times (M_\nu \mathcal{A}^*_{\nu j}) + \alpha_{4S} [- |\mathrm{tr} \mathcal{A}\mathcal{A}^T|^2 + 2(\mathrm{tr}\mathcal{A}\mathcal{A}^\dagger)^2 + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^T) \times (\mathcal{A}\mathcal{A}^T)^* + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)^2 - 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)(\mathcal{A}\mathcal{A}^\dagger)^*] \}. \qquad (39)$$

Complete Ginzburg-Landau model

$$
\begin{aligned}
F_{\mathrm{GL}} = \int d^3\boldsymbol{r} \Bigg\{ & \frac{\alpha_S(T)}{2} \mathrm{tr} \mathcal{A}\mathcal{A}^\dagger + \frac{\beta_S}{2} (D^2 \mathrm{tr} \mathcal{A}\mathcal{A}^\dagger + D_i \mathcal{A}_{\mu i} D_j \mathcal{A}^\dagger_{j\mu} \\
& + D_j \mathcal{A}_{\mu i} D_i \mathcal{A}_{j\mu}) + \frac{\alpha_m(T)}{2} \boldsymbol{M}\cdot\boldsymbol{M} + \frac{\beta_m}{2} \nabla^2 \boldsymbol{M}\cdot\boldsymbol{M} \\
& + \frac{\alpha_3}{3!} i\varepsilon_{\mu\nu\lambda} \mathcal{A}^*_{\mu i} \mathcal{A}_{\nu i} M_\lambda + \frac{\alpha_{4S}}{4!} [2(\mathrm{tr}\mathcal{A}\mathcal{A}^\dagger)^2 - |\mathrm{tr}\mathcal{A}\mathcal{A}^\dagger|^2 \\
& + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^T)(\mathcal{A}\mathcal{A}^T)^* + 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)^2 - 2\mathrm{tr}(\mathcal{A}\mathcal{A}^\dagger)(\mathcal{A}\mathcal{A}^\dagger)^*] \\
& + \frac{\alpha_{4m}}{4!} (\boldsymbol{M}\cdot\boldsymbol{M})^2 + \frac{\alpha_{4ms}}{4!} (M_\mu \mathcal{A}_{\mu i})(M_\nu \mathcal{A}^*_{\nu i}) \Bigg\}, \qquad (40)
\end{aligned}
$$

Outline

Information obtained through spectroscopy

The conductance spectrum of a FM/FMSC junction may reveal information about:

- The magnitude of the SC gap(s) [A1- or A2-phase].
- The relative orientation of the gaps with respect to the crystalline axis.

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Very useful in terms of determining the correct pairing symmetry in the FMSC.