Automorphic forms, cohomology groups, transfer operator

Abstract

I'll report on joint work with Tobias Mühlenbruch.

The transfer operator $\mathcal{L}_s^M$ of Mayer has eigenvalue 1 or -1 if the spectral parameter s is in the automorphic spectrum. For $0 < \mathrm{Re}s < 1$, $s \neq \frac{1}{2}$ this means that $s(1-s)$ is the eigenvalue of the hyperbolic Laplace operator on a Maass cusp form for the full modular group, or on a residue of the Eisenstein series for a value for which $\zeta(2s) = 0$.

The transfer operator of Mayer is derived from the geodesic flow on the quotient of the upper half plane by the modular group. Other reductions are possible. The nearest integer continued fraction algorithm leads to a transfer operator $\mathcal{L}_s$. Since both operators ultimately come from the same geodesic flow, one may expect that they are related, although they look quite different.

We show that under the assumption $0 < \mathrm{Re}s < 1$, $s \neq \frac{1}{2}$, there is an explicit correspondence between

$$\ker(\mathcal{L}_s^M - 1) \oplus \ker(\mathcal{L}_s^M + 1) \quad \text{and} \quad \ker(\mathcal{L}_s - 1)\,.$$

To prove this, we do not use that both transfer operators originate from the geodesic flow.

We establish an explicit bijection between both spaces via a parabolic cohomology group. Work of Lewis and Zagier shows that this parabolic cohomology group is in explicit bijection with the space of Maass form with spectral parameter s for which there is no multiple of y^s in the Fourier term of order zero.

Roelof Bruggeman