Meromorphic functions with zeros or poles at the Maass spectral parameters

Abstract

The Selberg zeta function $Z(s)$ of a Riemann surface X is the classical example of a meromorphic function which vanishes on the set of Maass spectral parameters, i.e. on the set of complex numbers s such that $s(1-s)$ is an eigenvalue in the discrete spectrum of the Laplacian on X. $Z(s)$ is also in particular given on a right-half plane by a generalized Dirichlet series. For the case of the modular group we give two other examples of meromorphic functions with zeros or poles on this same set. The first is given by an ordinary Dirichlet series on the half-plane $\mathrm{Re}(s) > 0$ with poles at the zeros of $Z(s)$; it is constructed using Mayer's expression for $Z(s)$ as the determinant of the transfer function. The second is given as the limit of a sequence of Dirichlet polynomials, holomorphic on $\mathrm{Re}(s) > 0$, and which vanishes at the zeros of $Z(s)$; it is constructed again using determinants of trace-class operators, here ones acting on square-summable sequences given by the coefficients of the period functions corresponding to the Maass forms on X in a certain series expansion.