Equilibrium and non-equilibrium physics of low dimensional quantum gases

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Inhomogeneous superfluids Pisa July 2007

Bose - Einstein Condensation





in

Guanturtheorie des einstennigen idealers Guass

Zweite Albendhing

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n. (25- xT) + V = 1 ... (24)

A. Einstein (1924) (found at Lorenz Institute, Leiden, 2005)



A picture worth a Nobel Prize (2001)...







Bose - Einstein Condensation





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A picture worth a Nobel Prize (2001)...









ETH (Zurich): $N \sim 10^{6} {}^{87}\text{Rb}$ atoms (T ~ 100 nK)

Routes to break up the BECBose-Einstein Condensation[T L Ho]Image: transform to the transformation# particles = $N \gg 1$ Image: transform to the transform to the transformation to the transformat





First lecture

- 1. Many-Body physics with cold atoms. Optical lattices.
- 2. Fermions: lattice and continuum. Tomonaga-Luttinger liquids.
- 3. Hubbard models of cold atoms in low-D.
- Non-equilibrium phenomena in 1D quantum gases.

Second lecture

- 1. Competing phases in optical lattices: quasi-1D lattices.
- 2. 2D Bose gas: BKT phenomena. BKT in the presence of Josephson coupling.
- 3. Fast rotation: quantum Hall regime. Edge excitations and Topological order in vortex liquids.

Quantum Many-body Physics with Gases



Quantum Many-body Physics with Gases



Interaction vs. kinetic energy

 $\begin{aligned} v_{\rm at-at}(\mathbf{r}) &= \frac{4\pi\hbar^2}{M} a_s(\mathbf{B}) \,\delta(\mathbf{r}) \\ a_s &\sim 10^2 - 10^3 \text{\AA} \\ \rho_0^{-1/3} &\sim 10^3 - 10^4 \text{\AA} \\ E_{\rm kin} &\sim \frac{\hbar^2 \rho_0^{2/3}}{2M} \gg E_{\rm int} \sim \frac{\hbar^2 a_s^{-2}}{2M} \end{aligned}$

Quantum Many-body Physics with Gases



$$V(\mathbf{r}) = E(\mathbf{r}, \omega) \cdot \boldsymbol{\alpha}(\omega) \cdot E(\mathbf{r}, \omega)$$

Atomic Polarizability (two-level atom)
$$\alpha(\omega) \propto rac{\omega - \omega_0}{(\omega - \omega_0)^2 + \Gamma^2}$$

$$V(\mathbf{r}) = E(\mathbf{r},\omega) \cdot \boldsymbol{\alpha}(\omega) \cdot E(\mathbf{r},\omega)$$

Red detuned: $\omega < \omega_0 \Rightarrow \alpha(\omega) < 0$

Atomic Polarizability (two-level atom) $\alpha(\omega) \propto \frac{\omega - \omega_0}{(\omega - \omega_0)^2 + \Gamma^2}$

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$$\overset{}{\longrightarrow} \qquad \overset{}{\longrightarrow} \qquad} \qquad \overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad \overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad} \overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad\overset{}{\longrightarrow} \qquad} \qquad\overset{}{\longrightarrow} \qquad \overset{}{\longrightarrow}$$

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2 Lasers \rightarrow 1D optical lattice

Optical lattices (OL's)

More lasers = more complex geometries





6 lasers \rightarrow 3D



Optical lattices (OL's)

More lasers = more complex geometries



6 lasers \rightarrow 3D





Fermions: lattice and continuum limit **1st Bloch band only** $H = -t \sum f_m^{\dagger} f_n = \sum \epsilon(k) f^{\dagger}(k) f(k)$ $\langle n,m\rangle$ $t_{\perp} \ll \frac{\hbar}{t_{\text{decoh}}} \begin{bmatrix} 32 \\ 800 \\ 72 \\ 100 \end{bmatrix}$ = -0.5 (k) $\overset{\mathbf{0}}{k}$ -0.5 0.5 -1 1 At low energies: linearized spectrum $H \simeq \sum \hbar v_F p \left| : \psi_r^{\dagger}(p) \psi_r(p) + \psi_l^{\dagger}(p) \psi_l(p) : \right|$ p

The Luttinger model (LM)

Joaquin M. Luttinger





The Luttinger model (LM) $\epsilon_{\rm kin}(p) = v_F p$ Joaquin M. Luttinger Dirac sea $|0 angle_{ m Dirac}$ r 朝永振一郎 "Infinite storey hotel" $[ho_{lpha}(q), ho_{eta}(-q')]=rac{qL}{2\pi}\delta_{q,q'}\delta_{lpha,eta}\quad (lpha,eta=r,l)$ $\rho_{\alpha}(q) = \sum : \psi_{\alpha}^{\dagger}(p+q)\psi_{\alpha}(p) :$ Daniel C. Mattis & Elliot H. Lieb

[J. Math. Phys. (N.Y.) <u>6</u> (1965)] [F. D. M. Haldane, J. Phys. C <u>14</u> (1981)]

Solution of the LM

$$\begin{split} H_{\rm LM} &= H_{\rm kin} + H_{\rm int}, \\ H_{\rm kin} &= \sum_{p,\alpha=r,l} \hbar v_F p : \psi_{\alpha}^{\dagger}(p)\psi_{\alpha}(p) :, \\ FDM \, {\rm Haldane, J. Phys. C} \, \underline{14} \, (1981)] \\ H_{\rm kin} &= \sum_{p,\alpha=r,l} \hbar v_F p : \psi_{\alpha}^{\dagger}(p)\psi_{\alpha}(p) :, \\ H_{\rm int} &= \frac{\hbar \pi}{2L} \sum_{q,\alpha=r,l} g_4(q) : \rho_{\alpha}(q)\rho_{\alpha}(-q) : + \frac{\hbar \pi}{2L} \sum_{q} g_2(q)\rho_r(q)\rho_l(q), \\ a(q) &= -i\sqrt{\frac{2\pi}{|q|L}} \left[\theta(q)\rho_r(-q) - \theta(-q)\rho_l(q)\right], \\ b(q) &= \cosh \varphi(q) \, a(q) + \sinh \varphi(q) \, a^{\dagger}(-q), \\ \tanh \varphi(q) &= \frac{g_2(q)}{v_F + g_4(q)} \\ H_{\rm LM} &= \sum_{q \neq 0} \hbar v(q)|q| \, b^{\dagger}(q)b(q) + \frac{\hbar \pi}{2L} (v_N N^2 + v_J J^2) \end{split}$$

Solution of the LM

$$\begin{split} H_{\mathrm{LM}} &= H_{\mathrm{kin}} + H_{\mathrm{int}}, & [\mathrm{EH\ Lieb\ \&\ DC\ Mattis\ J.\ Math.\ Phys.\ (\mathrm{N.Y.})\ \underline{6}\ (1965)]} \\ H_{\mathrm{kin}} &= \sum_{p,\alpha=r,l} hv_F p: \psi_{\alpha}^{\dagger}(p)\psi_{\alpha}(p):, & [\mathrm{T\ Giamarchi,\ Quantum\ Physics\ in\ One\ Dimension,\ Clarendon\ Press,\ 2004]} \\ H_{\mathrm{int}} &= \frac{h\pi}{2L}\sum_{q,\alpha=r,l} g_4(q): \rho_{\alpha}(q)\rho_{\alpha}(-q): + \frac{h\pi}{2L}\sum_{q} g_2(q)\rho_r(q)\rho_l(q), \\ a(q) &= -i\sqrt{\frac{2\pi}{|q|L}}\left[\theta(q)\rho_r(-q) - \theta(-q)\rho_l(q)\right], \\ b(q) &= \cosh\varphi(q)\ a(q) + \sinh\varphi(q)\ a^{\dagger}(-q), \tanh\varphi(q) = \frac{g_2(q)}{v_F + g_4(q)} \\ H_{\mathrm{LM}} &= \sum_{q\neq 0} hv(q)|q|\ b^{\dagger}(q)b(q) + \frac{h\pi}{2L}(v_N N^2 + v_J J^2) \\ a(q)\psi_r(x)|0\rangle_{\mathrm{Dirac}} &= \left[i\left(\frac{2\pi}{qL}\right)^{1/2}e^{-iqx}\right]\psi_r(x)|0\rangle_{\mathrm{Dirac}}, & \text{Bosonization} \\ [DC\ Mattis,,\ Schotte\ \&\ Schotte,\ A\ Luther,\ ...]} \\ \psi_R(x) &= U_r \ \frac{e^{2\pi i(N+\frac{1}{2})x/L}}{\sqrt{L}}\ e^{i\Phi_r^{\dagger}(x)}e^{i\Phi_r(x)}, \quad \Phi_r(x) = \sum_{q>0} \left(\frac{2\pi}{qL}\right)^{1/2}e^{iqx}\ a(q) \end{split}$$





Particle-hole spectrum in the LM and in other 1D models







(Same spectral degeneracies)

 $Z = \operatorname{Tr} e^{-H/T} \simeq \operatorname{Tr} e^{-H_{\rm LM}/T}$

80

Particle-hole spectrum in the LM and in other 1D models



At low temperature and frequency... (Same spectral degeneracies)

$$\langle O(x)O(0)\rangle \sim x^{-\alpha}$$



Tomonaga-Luttinger liquids



What is it made of? Fermions? Bosons?

"In 1D [...] the symmetry of the wave function cannot be tested by a continuous change of coordinates that exchanges particles without close approach (collision). Thus interaction and statistics effects cannot be separated."

[FDM Haldane, PRL <u>47</u> (1981)]

Tomonaga-Luttinger liquids





"In 1D [...] the symmetry of the wave function cannot be tested by a continuous change of coordinates that exchanges particles without close approach (collision). Thus interaction and statistics effects cannot be separated."

[FDM Haldane, PRL <u>47</u> (1981)]

space

Collective modes exhaust the low-energy spectrum

$$H = \frac{\hbar}{2\pi} \int dx \begin{bmatrix} v_J (\partial_x \phi)^2 + v_N (\partial_x \theta)^2 \end{bmatrix} \begin{bmatrix} \rho &= \rho_r + \rho_l = \frac{1}{\pi} \partial_x \phi, \\ j &= \rho_r - \rho_l = \frac{1}{\pi} \partial_x \theta \end{bmatrix}$$

$$v_s = \sqrt{v_J v_N} \qquad K = \sqrt{\frac{v_J}{v_N}}$$

Hubbard models of cold atoms in low-D

Bose-Hubbard and Lieb-Liniger

Bose Hubbard model (not integrable)
$$\gamma = \frac{E_{\text{int}}}{E_{\text{kin}}} = \frac{U}{J}$$

 $H_{\text{BH}} = -\frac{J}{2} \sum_{m} \left(b_{m+1}^{\dagger} b_m + \text{H.c.} \right) + \frac{U}{2} \sum_{m} (b_m^{\dagger})^2 (b_m)^2$

[T Stöferle *et al*, PRL <u>92</u> (2004), B. Paredes et al. Nature <u>429</u> (2004)]

Low filling limit: Lieb-Liniger model (BA integrable, PR 63 (19)

$$H_{\rm LL} = \int_0^L dx \, \frac{\hbar^2}{2M} \left| \partial_x \Psi(x) \right|^2 + \frac{g}{2} (\Psi^{\dagger}(x))^2 (\Psi(x))^2$$

[T. Kinoshita et al. Science 305 (2004)]

Dimensionless parameter
$$\gamma = \frac{E_{\text{int}}}{E_{\text{kin}}} = \frac{Mg}{\hbar^2 \rho_0}$$

Low density ($\rho_0 \rightarrow 0$) means strong interaction $\gamma \rightarrow +\infty$

One-component 1D Bose gas: phase diagram Lieb-Liniger model Bose-Hubbard model K = 1 (Tonks gas) $U/J = +\infty$ $\gamma = +\infty$ $\lim_{U \to \infty} H_{\rm BH} = -\frac{J}{2} \sum f_m^{\dagger} f_n, \quad f_m = e^{i\pi \sum_{l < m} b_l^{\dagger} b_l} b_m$ **Bethe-Ansatz results** $\langle n.m \rangle$ Tonks regime $\gamma = \frac{U}{J} \gg 1$ $f_0 = \frac{N_0}{M_0}$ Luttinger parameters $K \simeq 1 + 4\gamma^{-1} \sin \pi f_0 / \pi$ 3 $v_s/v_F \simeq 1 - 4\gamma^{-1} \left(f_0 \cos \pi f_0 \right)$ 2 Κ [MAC PRA 67 (2003), PRA <u>70</u>R (2004)] v_s/v_e Weakly interacting regime 10 100 0.1 $K \simeq \sqrt{\frac{Jf_0}{II}},$ $v_F = \frac{\hbar k_F}{2M}, \ k_F = \pi \rho_0$ $v_s \simeq \frac{\pi a}{\hbar} \sqrt{JUf_0}$ [MAC J Phys B <u>37</u> (2004)] K >> 1 (Quasi-condensate) $U/J \rightarrow 0$

Fermions: Asymetric 1D Hubbard model


Fermions: Asymetric 1D Hubbard model



[T Stöferle, H Moritz, C Schori M Köhl, and T Esslinger, PRL <u>92</u> (2004)]



"Almost crystaline" order in 1D

It looks like a crystal on a short distance scale, but it is disordered on a large distance scale. Periodic regions of all sizes.

Adiabatic approximation $(t_{\uparrow} \gg t_{\downarrow})$

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Adiabatic approximation $(t_{\uparrow} \gg t_{\perp})$ U < 0 is a solution of the second $U > 0 \land \land \land \land \land$

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What about d > 1?

[TL Dao, A Georges, and M Capone, arxiv/0407.2260]



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Reading out: Detecting the spin gap

[MAC, AF Ho & T Giamarchi, PRL<u>95</u> (2005)]

A Raman laser induces transitions between hyperfine states [HP Buchler *et al* PRL <u>93</u> (2004)]





Non-equilibrium phenomena in 1D quantum gases

Non-equilibrium steady states



Equilibrium phase diagram of H₂O

Non-equilibrium steady states

Rayleigh-Bénard convection cells



Non-equilibrium steady states

Rayleigh-Bénard convection cells



Quantum Fluids out of equilibrium

Problems with solid state/liquid He quantum fluids:

- Not easily tunable
- Quantum decoherence is a killer

Cold atoms in an optical lattice [D. Jaksch et al. PRL <u>81</u> (1998)] [M Greiner et al. Nature, <u>415</u> (2002)]

Interacting bosons on a lattice [MPA Fisher et al. PRB <u>40</u> (1989)]

$$H_{\rm BH} = -J \sum_{\langle \mathbf{R}, \mathbf{R}' \rangle} b_{\mathbf{R}}^{\dagger} b_{\mathbf{R}'} + U \sum_{\mathbf{R}} n_{\mathbf{R}} (n_{\mathbf{R}} - 1)$$







Quench at t = 0 (sudden approximation):

$$|\Phi(t>0)\rangle = e^{-iH_f t/\hbar} |\Phi(0)\rangle = e^{-iH_f t/\hbar} |\Phi_0\rangle$$

Not an eigenstate of Hf !!

Quench at t = 0 (sudden approximation): $|\Phi(t > 0)\rangle = e^{-iH_f t/\hbar} |\Phi(0)\rangle = e^{-iH_f t/\hbar} |\Phi_0\rangle$ Not an eigenstate of Hf !!

Operators after the quench:

$$O(t>0) = \langle \Phi(t) | \hat{O} | \Phi(t) \rangle = \langle \Phi_0 | e^{iH_f t/\hbar} \hat{O} e^{-iH_f t/\hbar} | \Phi_0 \rangle$$

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Does the system reach a (quasi-) stationary state? If so,

$$\bar{O} = \lim_{T \to +\infty} \lim_{t_0 \to +\infty} \frac{1}{T} \int_{t_0}^{T+t_0} dt O(t)$$

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Is there any statistical ensemble such that $O = \text{Tr}\hat{\rho}_{\text{quench}} O$?

Is $\hat{\rho}_{\text{quench}} = \rho_{\text{eq}} = e^{-(H_f - \mu N)/T_{\text{eff}}}$? (ergodic hypotesis)

Won't be looking at the creation of topological defects

Absence of thermalization in 1DBG

nature

LETTERS

[T Kinoshita, T Wenger & D Weiss, Nature (2006)]

A quantum Newton's cradle

Toshiya Kinoshita¹, Trevor Wenger¹ & David S. Weiss¹

 $p_1 + p_2 = p_1' + p_2',$ $\frac{p_1^2}{2M} + \frac{p_2^2}{2M} = \frac{p_1'^2}{2M} + \frac{p_2'^2}{2M},$ $p_1 = p'_1, p_2 = p'_2, \qquad p_1 = p'_2, p_2 = p'_1$ b 37/8





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Free expansion of a Tonks gas

[MRigol, B Dunjko, V Yurovsky, and M Olshanii PRL 98 (2007)]

Integrable XY model

$$H = -J \sum_{\langle n,m \rangle} \sigma_m^+ \sigma_n^- = -\sum_p \left(2J \cos p \right) \, f^{\dagger}(p) f(p)$$



 \square





Quantum quench in the LM

$$H_{\rm kin} = \sum_{q \neq 0} \hbar v_F |q| a^{\dagger}(q) a(q) \quad H_{\rm LM} = \sum_{q \neq 0} \hbar v |q| b^{\dagger}(q) b(q)$$

Non-interacting fermions (t < 0) Interacting fermions (t > 0)

Quantum quench in the LM

$$H_{\rm kin} = \sum_{q \neq 0} \hbar v_F |q| \ a^{\dagger}(q) a(q) \quad H_{\rm LM} = \sum_{q \neq 0} \hbar v |q| \ b^{\dagger}(q) b(q)$$

Non-interacting fermions (t < 0) Interacting fermions (t > 0)

Equilibrium solution $b(q) = \cosh \varphi(q) a(q) + \sinh \varphi(q) a^{\dagger}(-q)$

Quantum quench in the LM

$$H_{\rm kin} = \sum_{q \neq 0} \hbar v_F |q| a^{\dagger}(q) a(q) \quad H_{\rm LM} = \sum_{q \neq 0} \hbar v |q| b^{\dagger}(q) b(q)$$

Non-interacting fermions (t < 0) Interacting fermions (t > 0)

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Non-equilibrium (quench) solution

 $a(q,t) = e^{iH_{LM}t/\hbar}a(q)e^{-iH_{LM}t/\hbar} = f(q,t)a(q) + g^*(q,t)a^{\dagger}(-q),$ $f(q,t) = \cos v|q|t - i\sin v|q|t \cosh 2\varphi(q),$ $g(q,t) = i\sin v|q|t \sinh 2\varphi(q) \qquad [MAC, PRL <u>97</u> (2006)]$
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One-particle density matrix

$$C_{\psi_r}(x,t>0) = \langle 0|e^{iH_{LM}t/\hbar}\psi_r^{\dagger}(x)\psi_r(0)e^{-iH_{LM}t/\hbar}|0\rangle_{\text{Dirac}}$$

Thermodynamic limit

rmodynamic limit

$$C_{\psi_r}(x,t>0) = C_{\psi_r}^{\text{free}}(x) \quad \left| \frac{R}{x} \right|^{\gamma^2} \left| \frac{x^2 - (2vt)^2}{(2vt)^2} \right|^{\gamma^2/2}$$









1D dipolar gas of (spin polarized) fermionic atoms/molecules

E

(not the LM but a Tomonaga-Luttinger liquid)

$$\int \theta V_{\rm dip}(x) \simeq \frac{1}{4\pi\epsilon_0} \frac{D^2(1-3\cos\theta)}{[x^2+R^2]^{3/2}}$$

'Magic' angle: $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\rm dip} = 0$

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Summary

- Ultracold atomic gases are highly controllable quantum systems with long coherence times.
- True one-dimensional models can be realized using e.g. optical lattices.
- New situations (non-existent in the solid state) can be created: conserved magnetization, non-equilibrium states.
- We can address old and new problems: phase diagrams and phase properties and thermalization in absence of reservoirs.

$$n_{m} \leq 1 \qquad \gamma = \frac{U}{J} \gg 1$$

$$H_{F} = -\frac{J}{2} \sum_{\langle m,n \rangle} f_{m}^{\dagger} f_{n} + H_{\text{int}}^{1} + H_{\text{int}}^{2},$$

$$H_{\text{int}}^{1} = \frac{J^{2}}{2U} \sum_{m} \left[f_{m+1}^{\dagger} f_{m}^{\dagger} f_{m} f_{m-1} + \text{H.c.} \right],$$

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