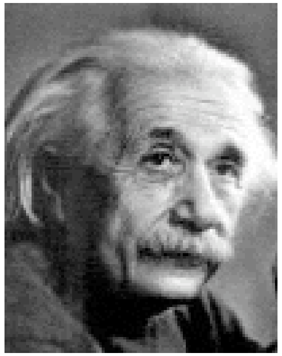
Equilibrium and non-equilibrium physics of low dimensional quantum gases

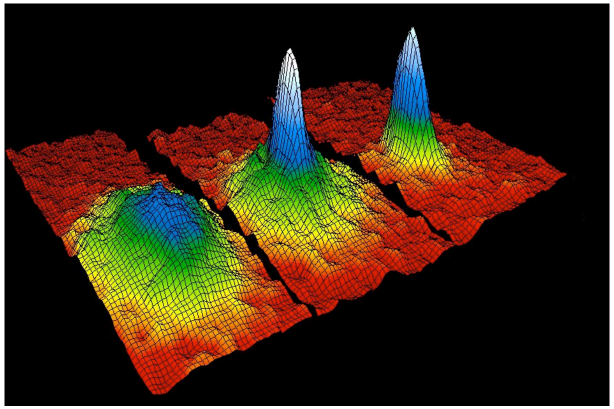
Miguel A. Cazalilla

CFM-CSIC & DIPC
San Sebastian, Spain

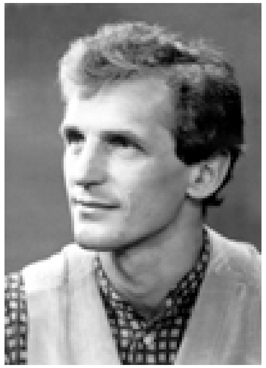
Inhomogeneous superfluids Pisa July 2007

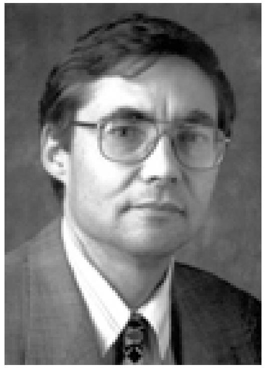
Bose - Einstein   Condensation

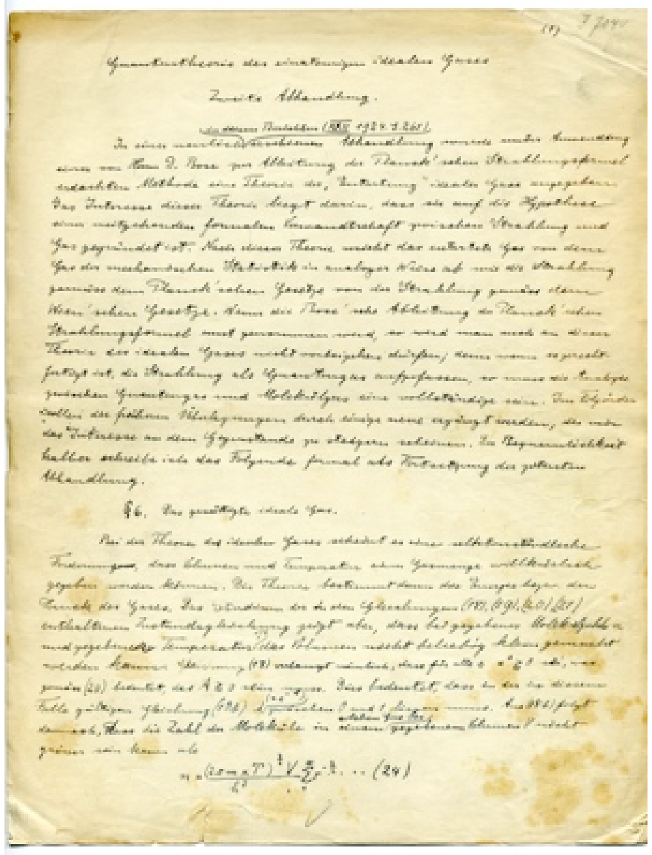




A picture worth a Nobel Prize (2001)...







Quantentheorie des einatomigen idealen Gases

Zweite Abhandlung.

A. Einstein (1924)
(found at Lorenz Institute, Leiden, 2005)

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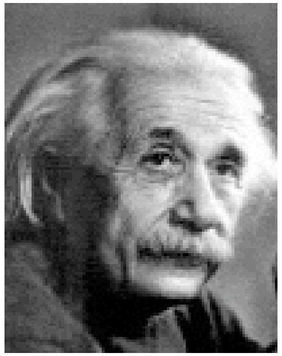

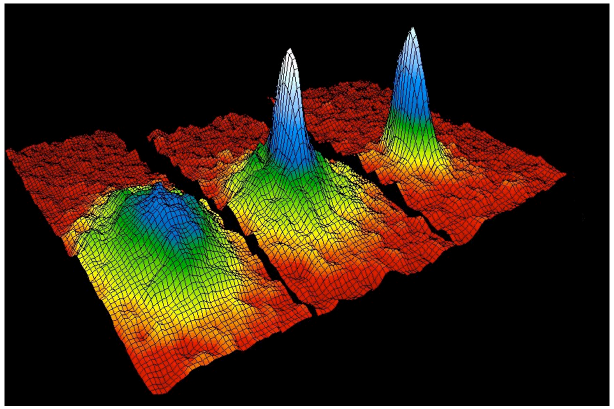

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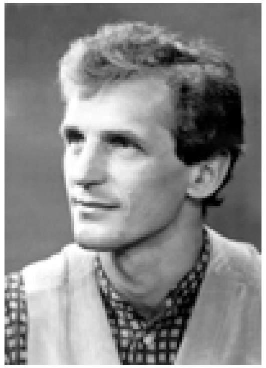

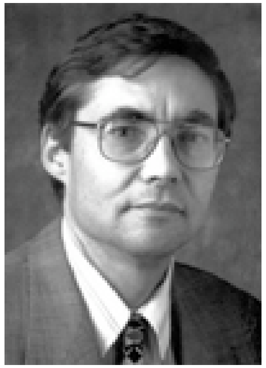

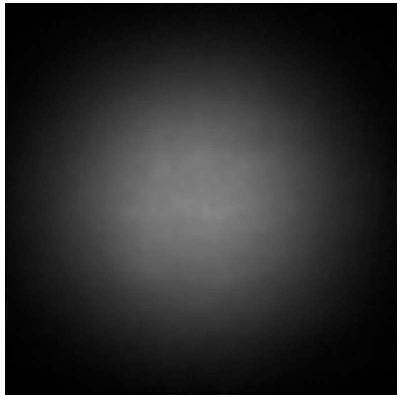

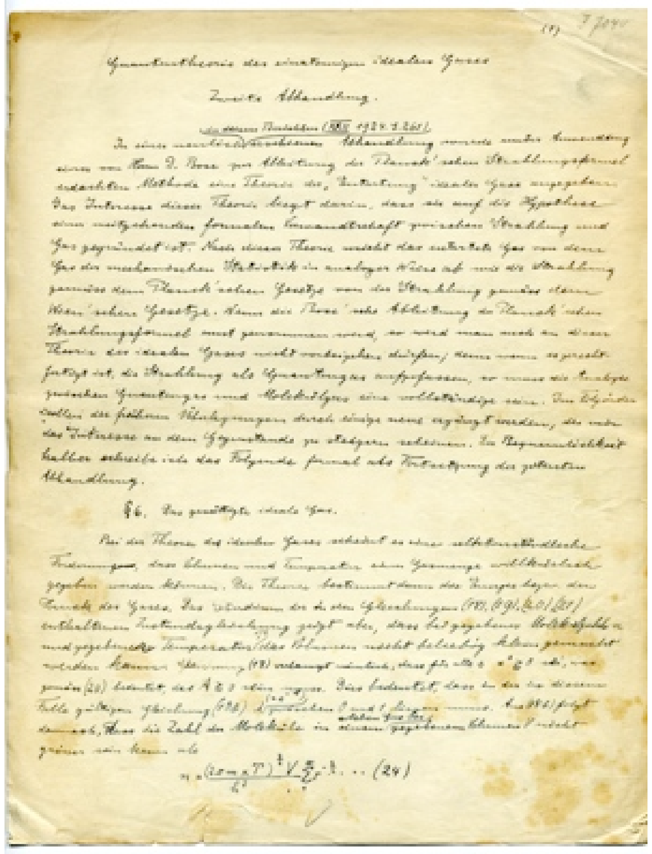

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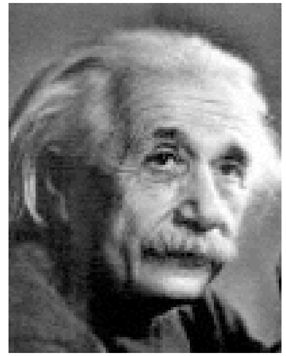

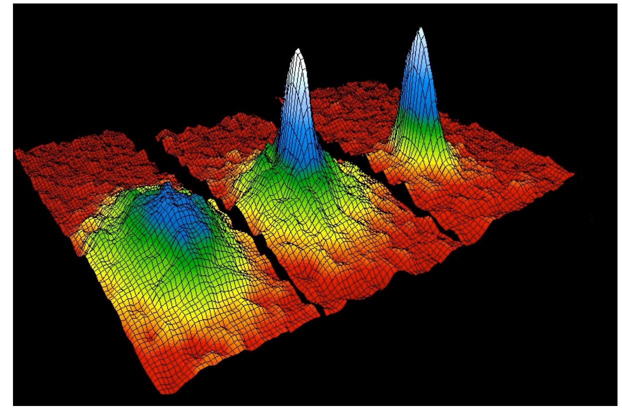

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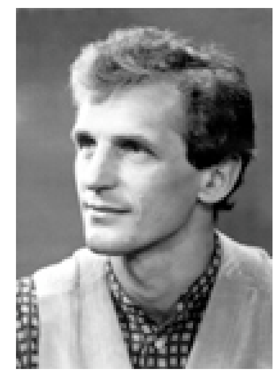

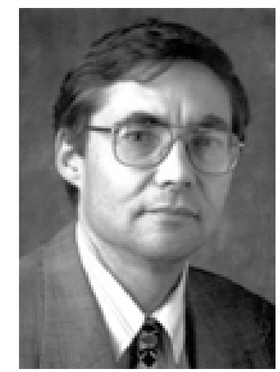

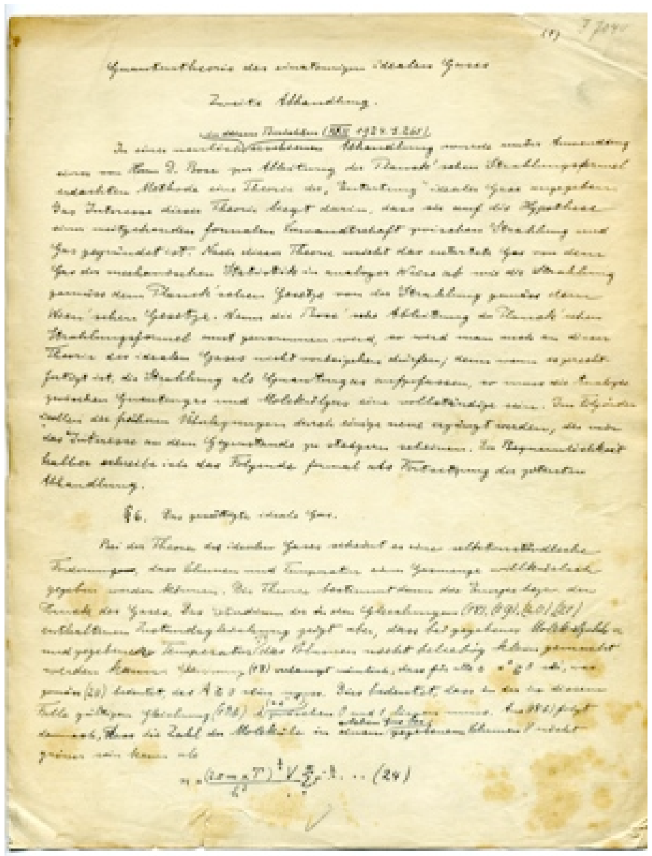
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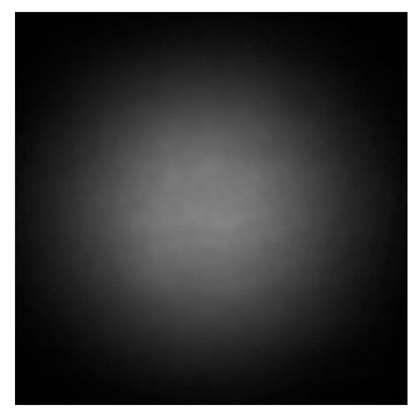
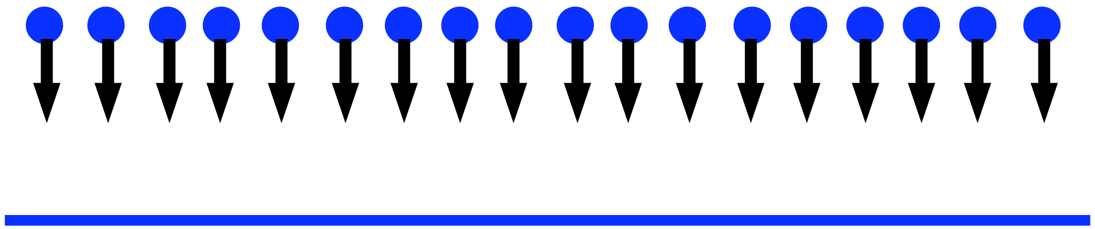


ETH (Zurich):
N ~ 10^6 ^{87}Rb atoms (T ~ 100 nK)

Routes to break up the BEC

Bose-Einstein Condensation

[T L Ho]



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$\# \, \text{states} = G = 1$

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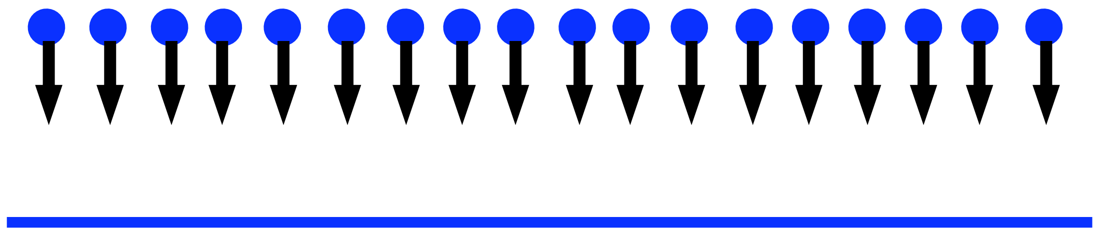

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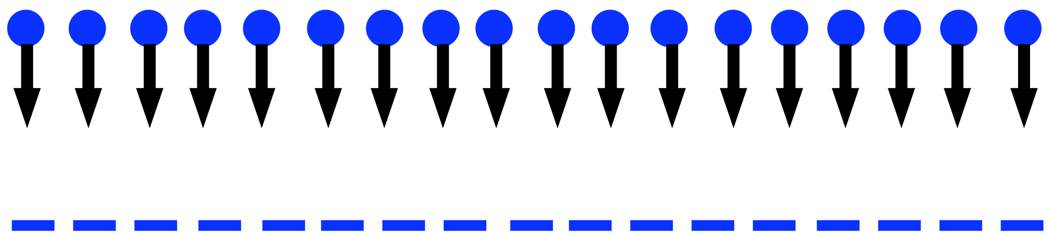
Optical Lattices (0D, 1D, 2D)



$N \gg 1$

$G \sim N$

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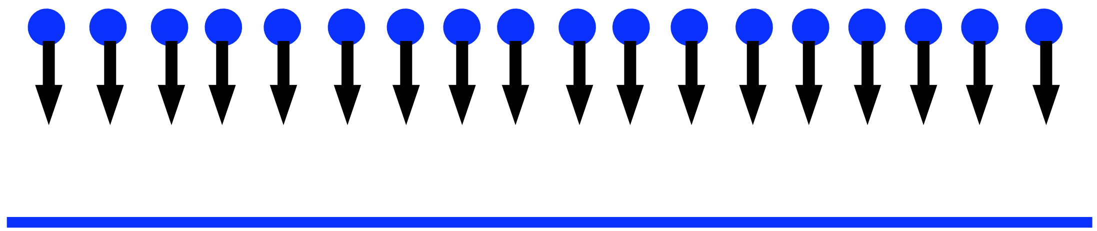

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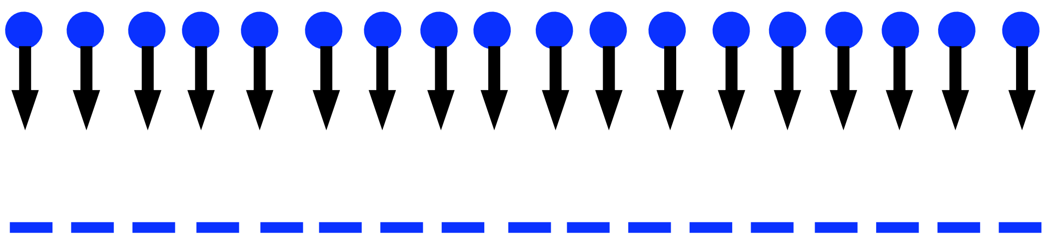

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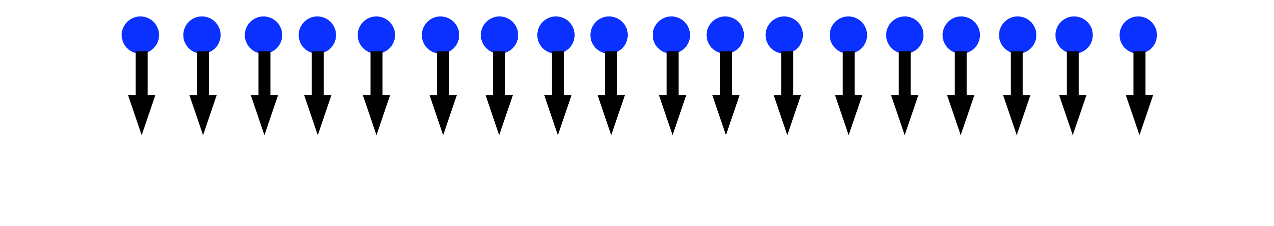
Rapidly Rotating Bose Gas



$N \gg 1$

$G \gg N$

First lecture

1. Many-Body physics with cold atoms.
   Optical lattices.
2. Fermions: lattice and continuum.
   Tomonaga-Luttinger liquids.
3. Hubbard models of cold atoms in low-D.
4. Non-equilibrium phenomena in
   1D quantum gases.

Second lecture

1. Competing phases in optical lattices: quasi-1D lattices.
2. 2D Bose gas: BKT phenomena. BKT in the presence of Josephson coupling.
3. Fast rotation: quantum Hall regime. Edge excitations and Topological order in vortex liquids.

Quantum Many-body Physics with Gases

$$\lambda_{dB}(T_{qD}) = \frac{\hbar}{\sqrt{3MkT_{qD}}} \sim \rho_0^{-1/3} \Rightarrow T_{qD} \propto \frac{\rho_0^{2/3}}{M}$$

$\boldsymbol{T_{qD}}$

Cold atomic gases
$T_{qD} \sim 10^{-6}$ - 10^{-8} K
$\rho_0 \sim 10^{13}$ - 10^{15} cm^{-3}

Metals
$T_{qD} \sim 10^{4}$ K
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Interaction vs. kinetic energy

$$v_{\mathrm{at-at}}(\mathbf{r}) = \frac{4\pi\hbar^2}{M} a_s(B)\,\delta(\mathbf{r})$$

$$a_s \sim 10^2 - 10^3\,\text{Å}$$
$$\rho_0^{-1/3} \sim 10^3 - 10^4\,\text{Å}$$

$$E_{\mathrm{kin}} \sim \frac{\hbar^2 \rho_0^{2/3}}{2M} \gg E_{\mathrm{int}} \sim \frac{\hbar^2 a_s^{-2}}{2M}$$

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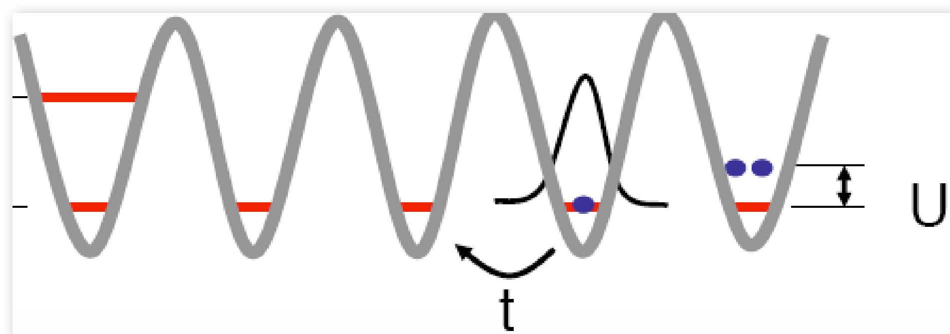
Hopping on a lattice



$$t \lesssim U \approx E_{\text{int}}$$

Optical lattices (or how to make a perfect crystal)

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$$V(\mathbf{r}) = E(\mathbf{r},\omega) \cdot \alpha(\omega) \cdot E(\mathbf{r},\omega)$$

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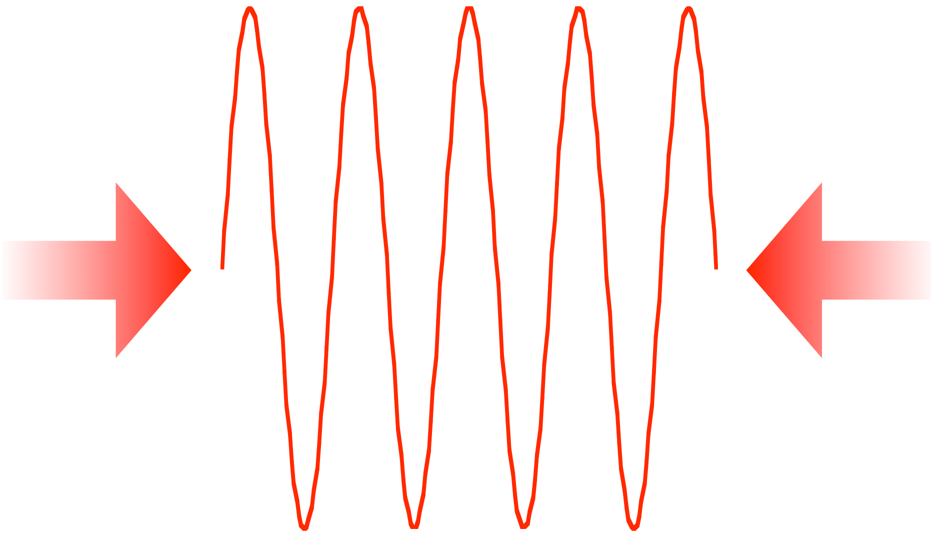

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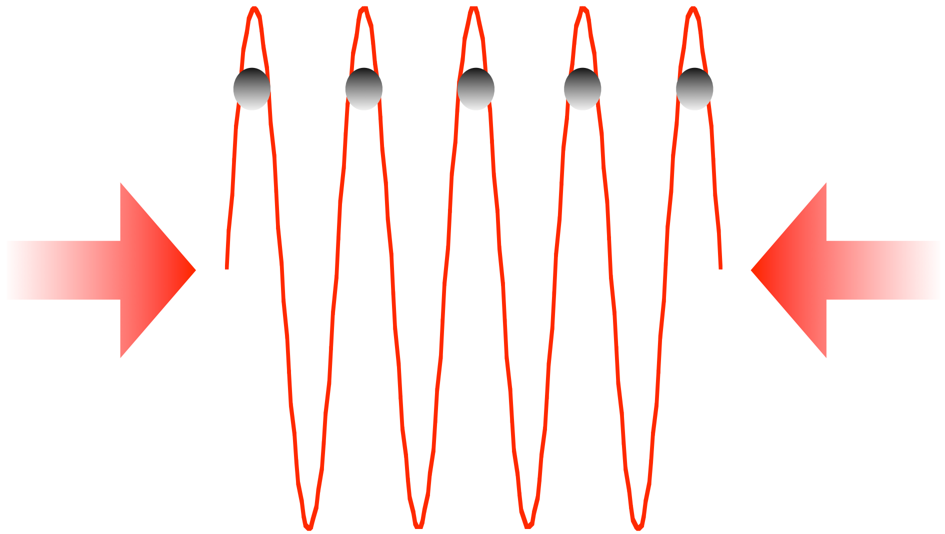

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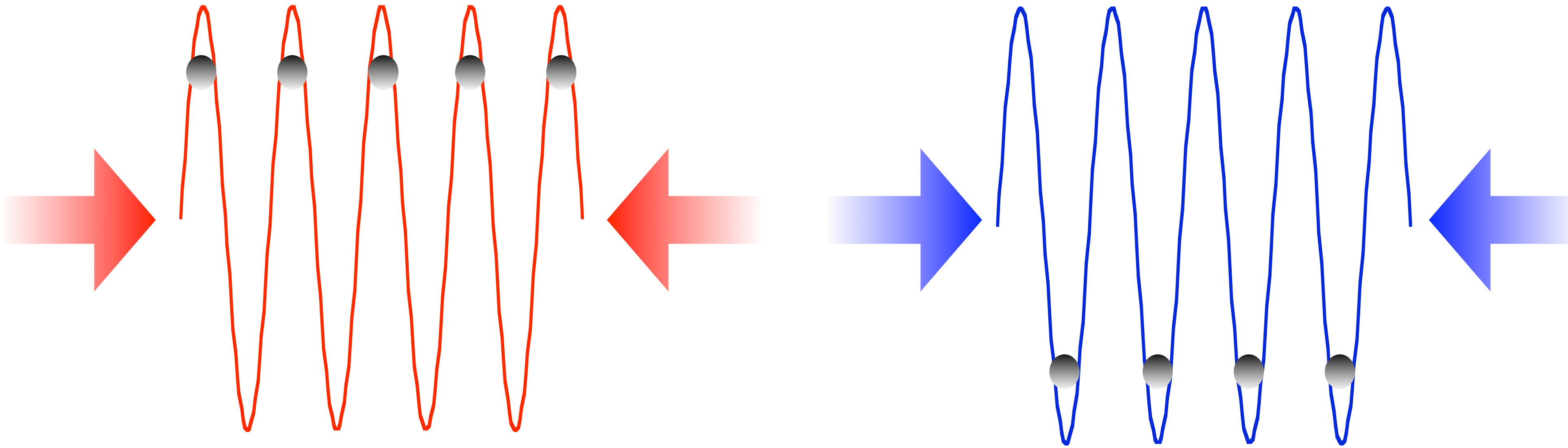

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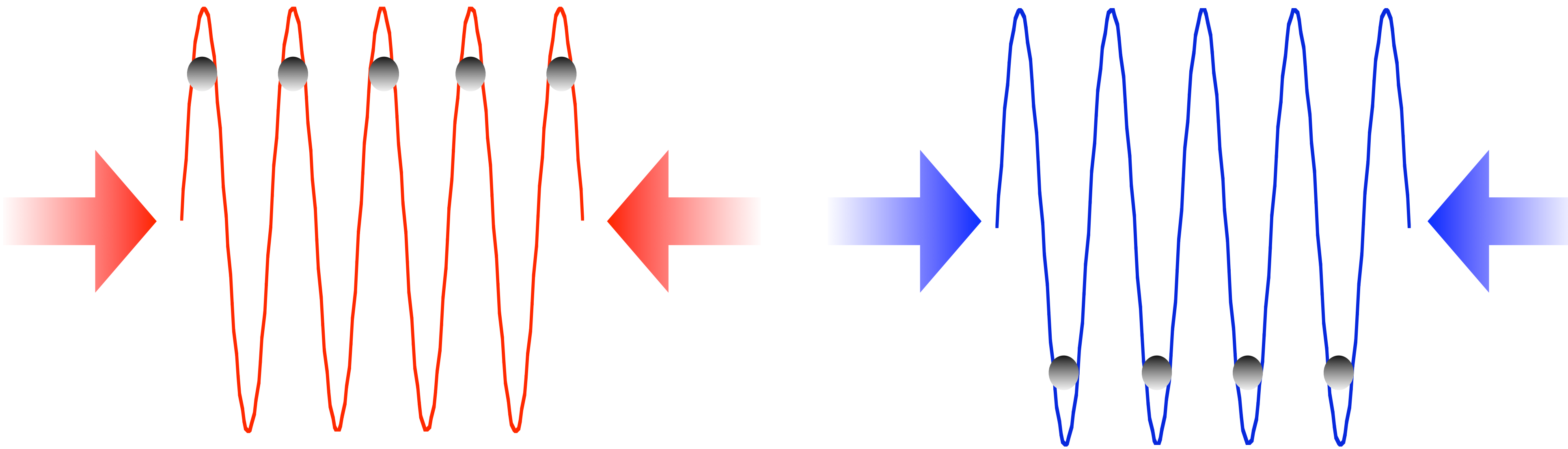

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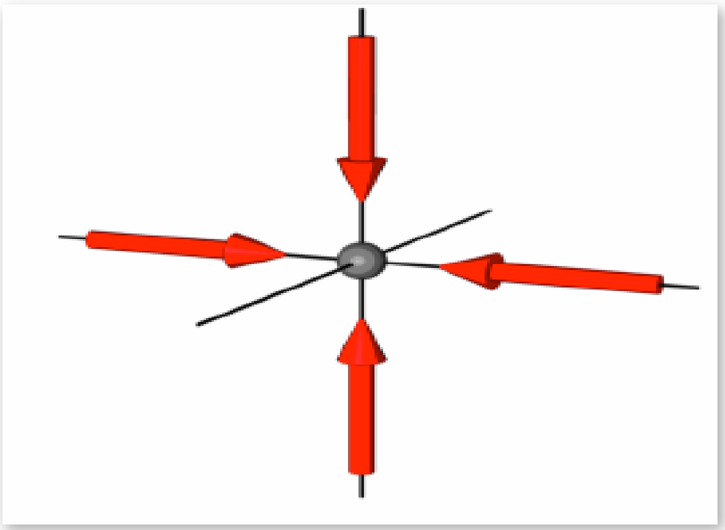
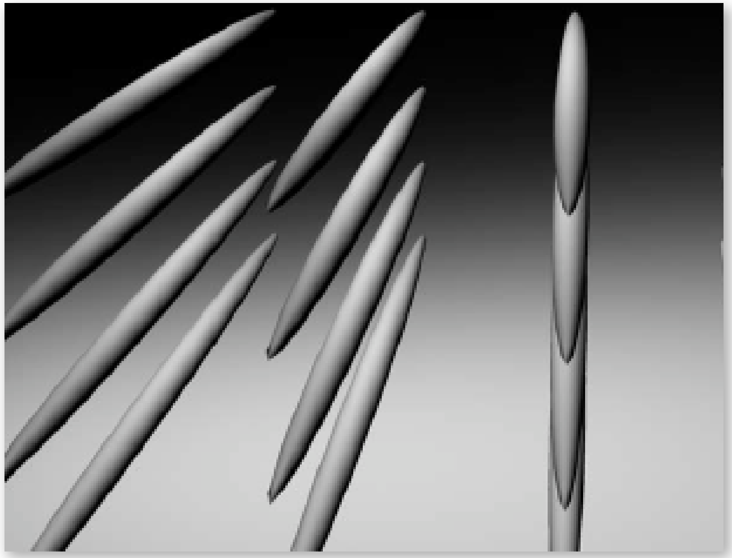
2 Lasers ➔ 1D optical lattice

Optical lattices (OL's)

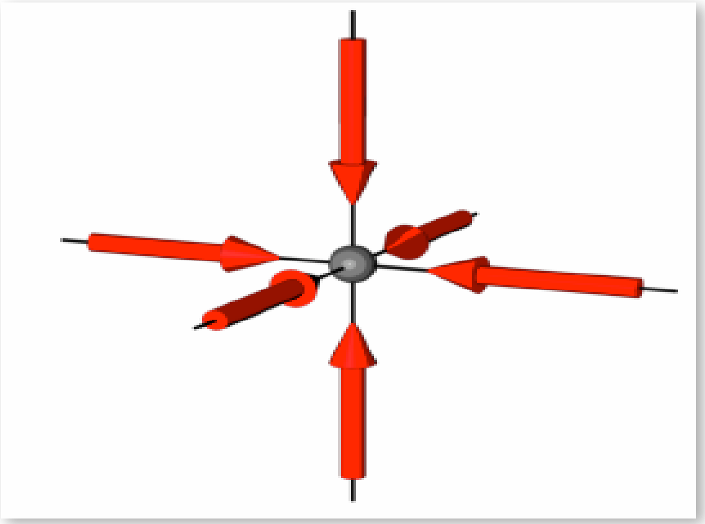
More lasers = more complex geometries

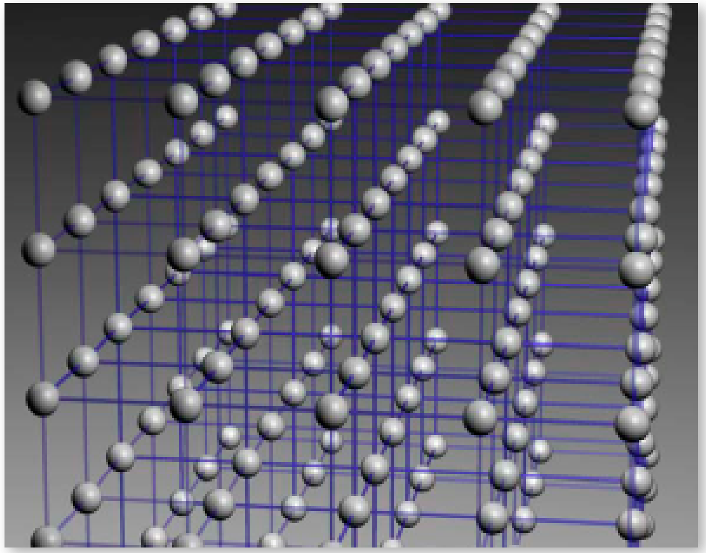
4 lasers ➔ 2D





6 lasers ➔ 3D





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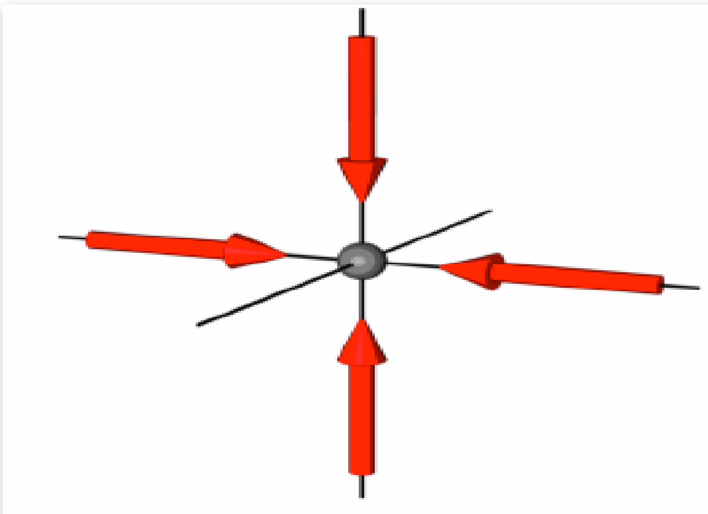
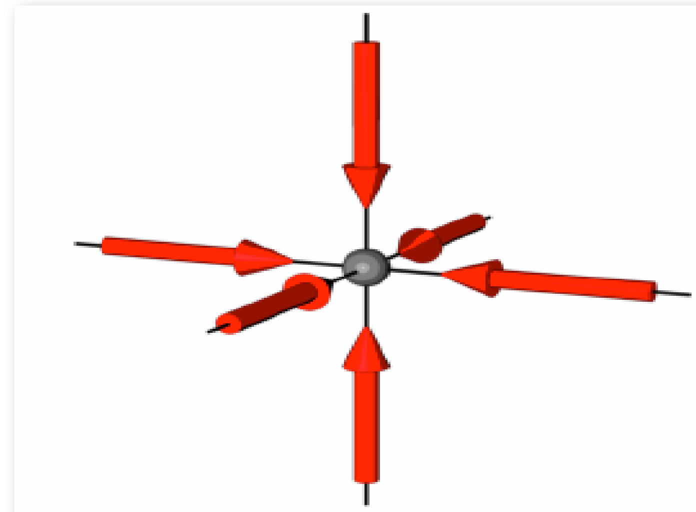
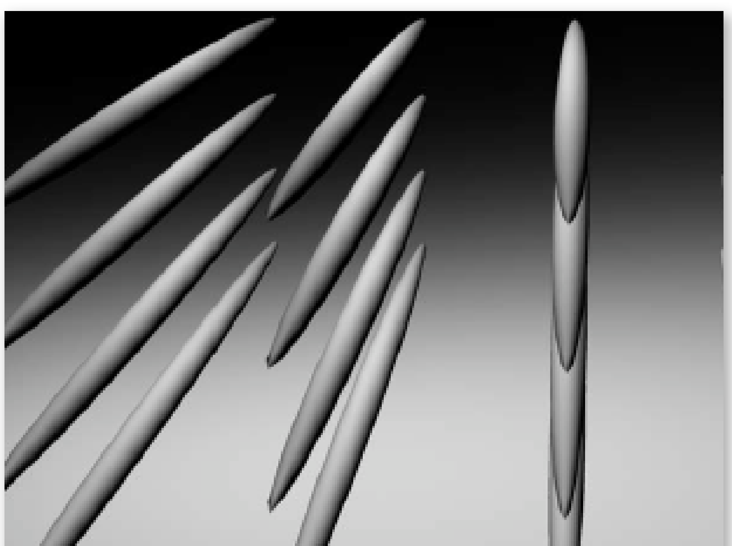
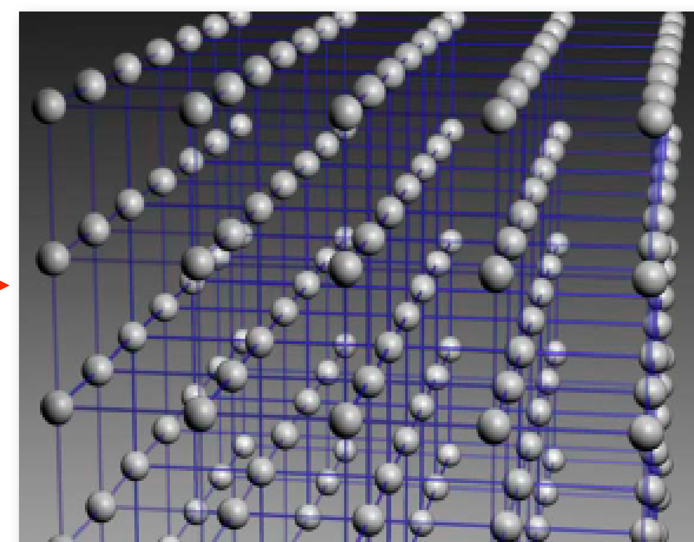




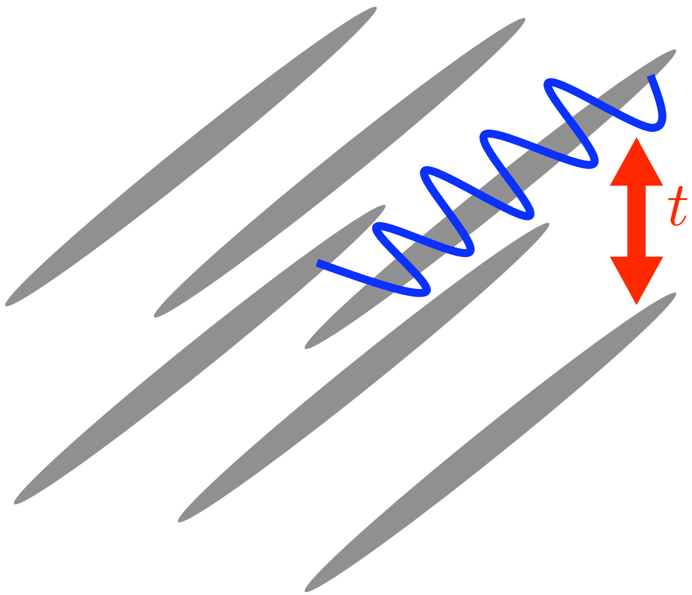


quasi-1D lattice

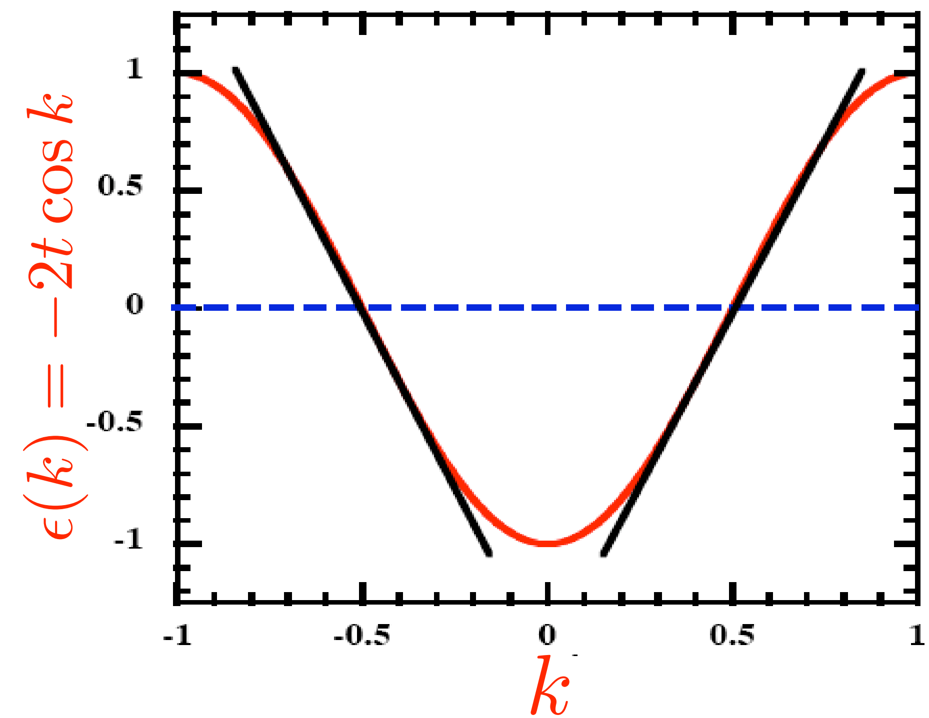


Fermions: lattice and continuum limit

1st Bloch band only $\quad H = -t \sum_{\langle n,m \rangle} f_m^\dagger f_n = \sum_k \epsilon(k)\, f^\dagger(k) f(k)$



$$t_\perp \ll \frac{\hbar}{t_{\mathrm{decoh}}}$$



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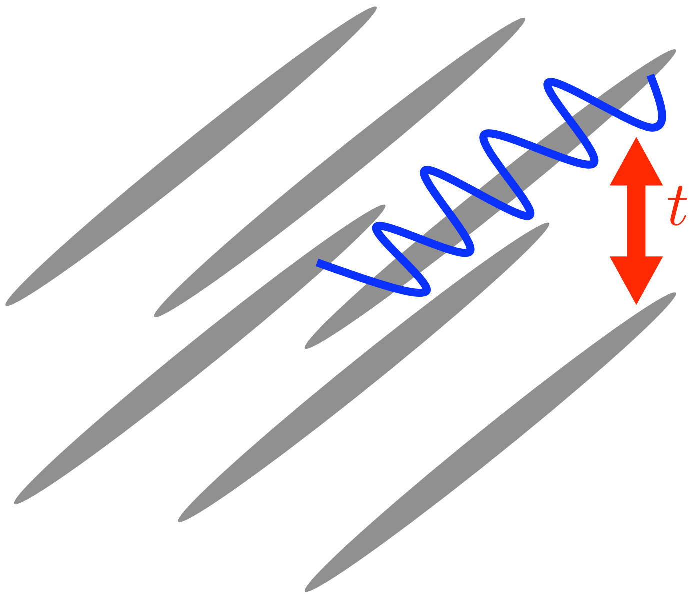

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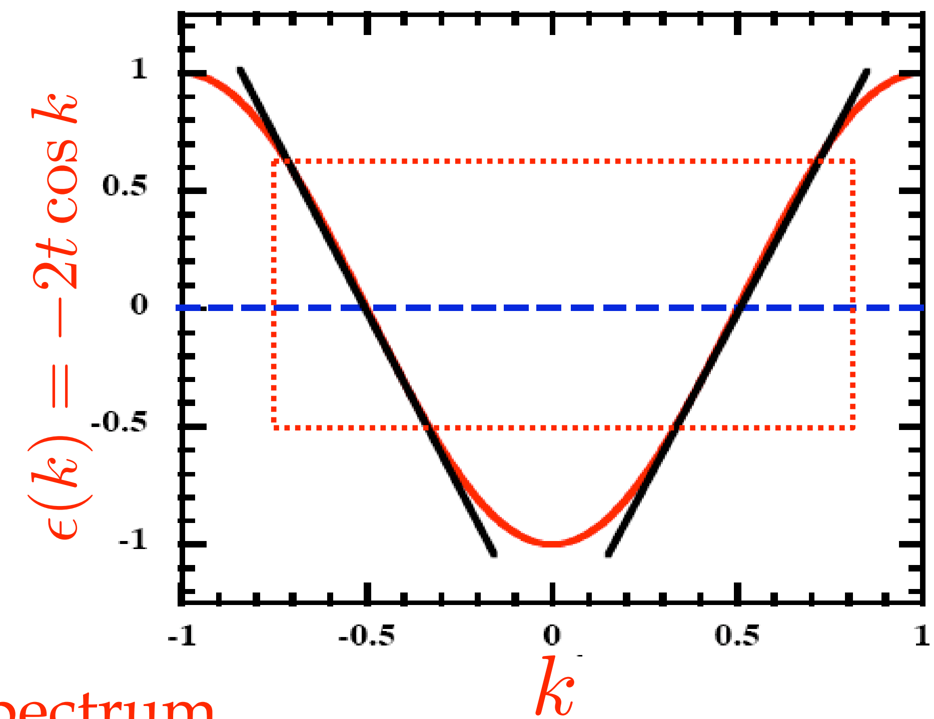
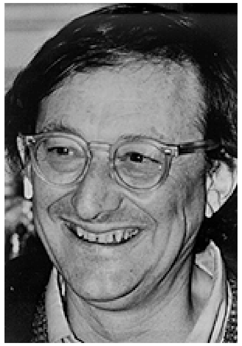


At low energies: linearized spectrum

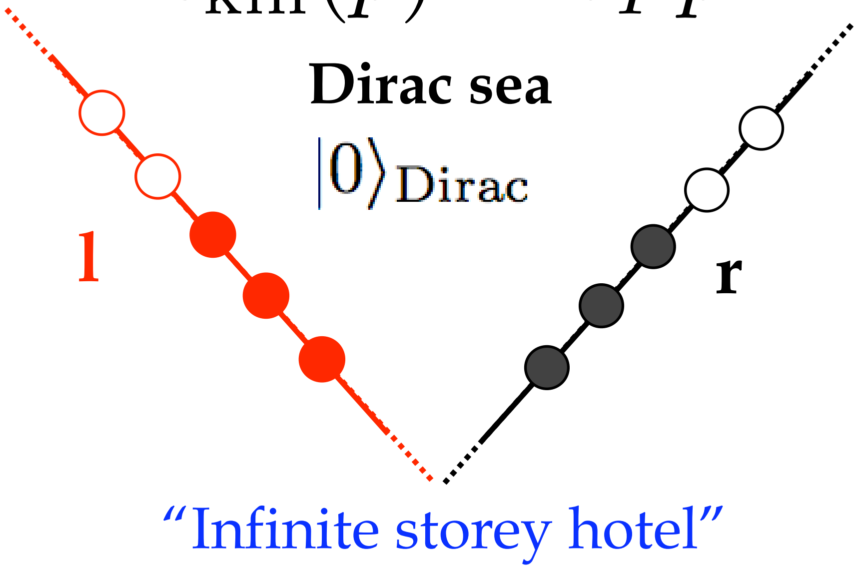
$$H \simeq \sum_p \hbar v_F p \left[: \psi_r^\dagger(p)\psi_r(p) + \psi_l^\dagger(p)\psi_l(p) : \right]$$

The Luttinger model (LM)

Joaquin M. Luttinger



$$\epsilon_{\text{kin}}(p) = v_F p$$



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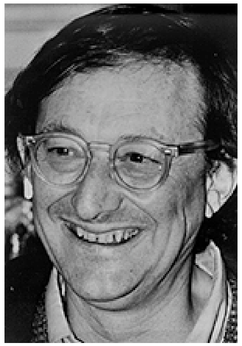
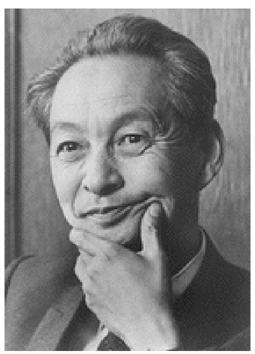
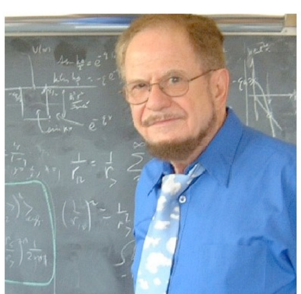
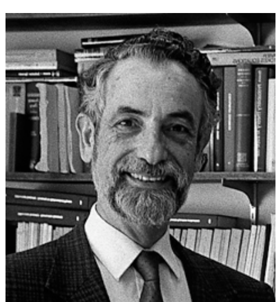


朝永振一郎



Daniel C. Mattis & Elliot H. Lieb





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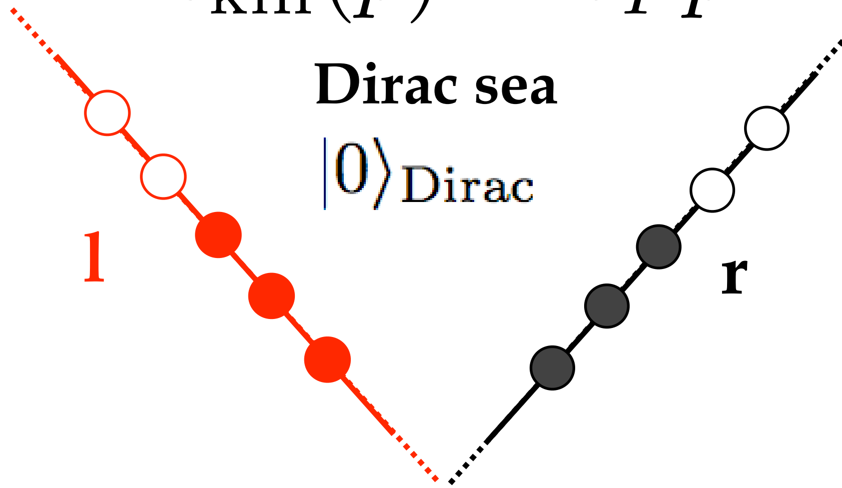

"Infinite storey hotel"

$$[\rho_\alpha(q), \rho_\beta(-q')] = \frac{qL}{2\pi}\delta_{q,q'}\delta_{\alpha,\beta} \quad (\alpha, \beta = r, l)$$

$$\rho_\alpha(q) = \sum_p : \psi^\dagger_\alpha(p+q)\psi_\alpha(p) :$$

[J. Math. Phys. (N.Y.) 6 (1965)]
[F. D. M. Haldane, J. Phys. C 14 (1981)]

Solution of the LM

[EH Lieb & DC Mattis J. Math. Phys. (N.Y.) 6 (1965)]
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$$H_{\text{LM}} = H_{\text{kin}} + H_{\text{int}},$$

$$H_{\text{kin}} = \sum_{p,\alpha=r,l} \hbar v_F p : \psi_\alpha^\dagger(p)\psi_\alpha(p) : ,$$

$$H_{\text{int}} = \frac{\hbar\pi}{2L} \sum_{q,\alpha=r,l} g_4(q) : \rho_\alpha(q)\rho_\alpha(-q) : + \frac{\hbar\pi}{2L} \sum_q g_2(q)\rho_r(q)\rho_l(q),$$

$$a(q) = -i\sqrt{\frac{2\pi}{|q|L}} \left[\theta(q)\rho_r(-q) - \theta(-q)\rho_l(q)\right],$$

$$b(q) = \cosh\varphi(q)\, a(q) + \sinh\varphi(q)\, a^\dagger(-q),\ \tanh\varphi(q) = \frac{g_2(q)}{v_F + g_4(q)}$$

$$H_{\text{LM}} = \sum_{q\neq 0} \hbar v(q)|q|\, b^\dagger(q)b(q) + \frac{\hbar\pi}{2L}(v_N N^2 + v_J J^2)$$

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$$
\begin{aligned}
H_{\rm LM} &= H_{\rm kin} + H_{\rm int}, \\
H_{\rm kin} &= \sum_{p,\alpha=r,l} \hbar v_F p : \psi^\dagger_\alpha(p)\psi_\alpha(p) : , \\
H_{\rm int} &= \frac{\hbar\pi}{2L} \sum_{q,\alpha=r,l} g_4(q) : \rho_\alpha(q)\rho_\alpha(-q) : + \frac{\hbar\pi}{2L}\sum_q g_2(q)\rho_r(q)\rho_l(q), \\
a(q) &= -i\sqrt{\frac{2\pi}{|q|L}}\left[\theta(q)\rho_r(-q) - \theta(-q)\rho_l(q)\right], \\
b(q) &= \cosh\varphi(q)\, a(q) + \sinh\varphi(q)\, a^\dagger(-q), \; \tanh\varphi(q) = \frac{g_2(q)}{v_F + g_4(q)} \\
H_{\rm LM} &= \sum_{q\neq 0} \hbar v(q)|q|\, b^\dagger(q)b(q) + \frac{\hbar\pi}{2L}(v_N N^2 + v_J J^2)
\end{aligned}
$$

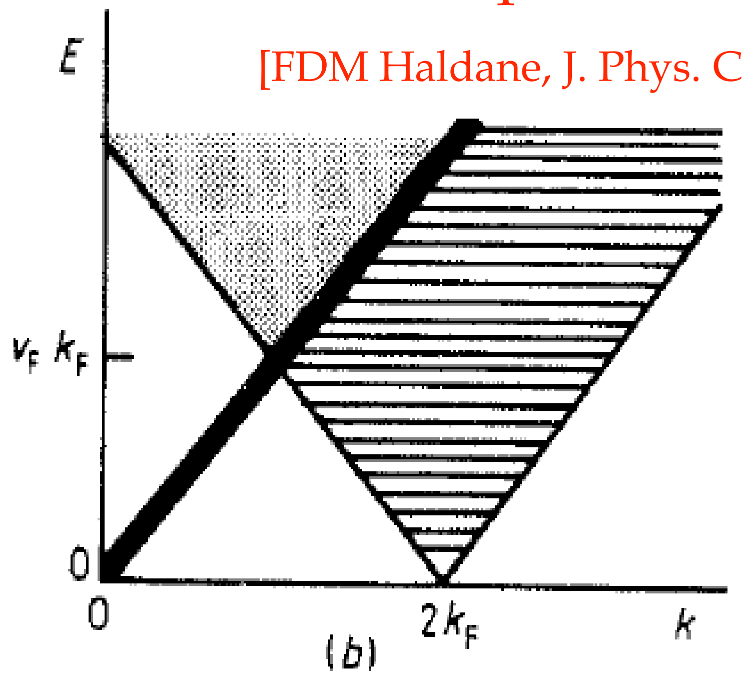
$$
a(q)\psi_r(x)|0\rangle_{\rm Dirac} = \left[i\left(\frac{2\pi}{qL}\right)^{1/2} e^{-iqx}\right]\psi_r(x)|0\rangle_{\rm Dirac},
$$

Bosonization
[DC Mattis,, Schotte & Schotte, A Luther, ...]

$$
\psi_R(x) = U_r \frac{e^{2\pi i(N+\frac{1}{2})x/L}}{\sqrt{L}}\, e^{i\Phi^\dagger_r(x)} e^{i\Phi_r(x)}, \quad \Phi_r(x) = \sum_{q>0}\left(\frac{2\pi}{qL}\right)^{1/2} e^{iqx}\, a(q)
$$

LM and Tomonaga-Luttinger liquids

Particle-hole spectrum in the LM and in other 1D models



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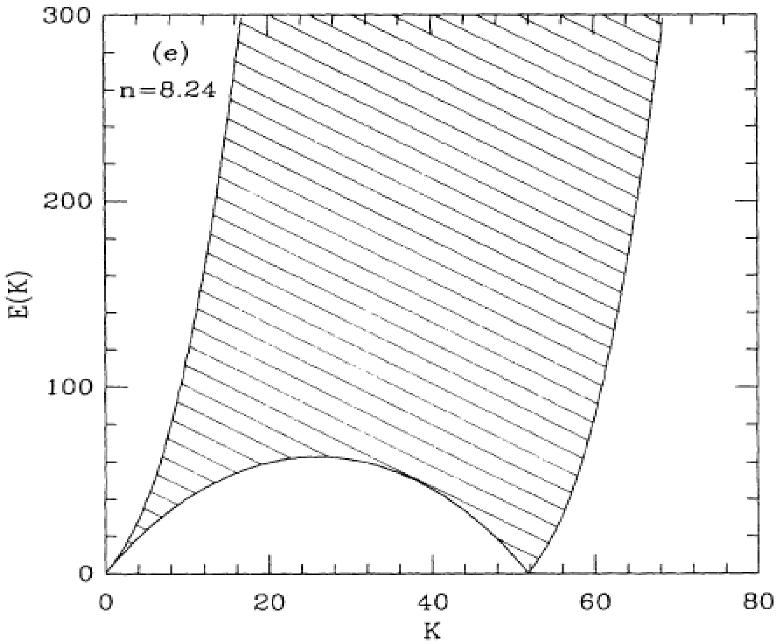

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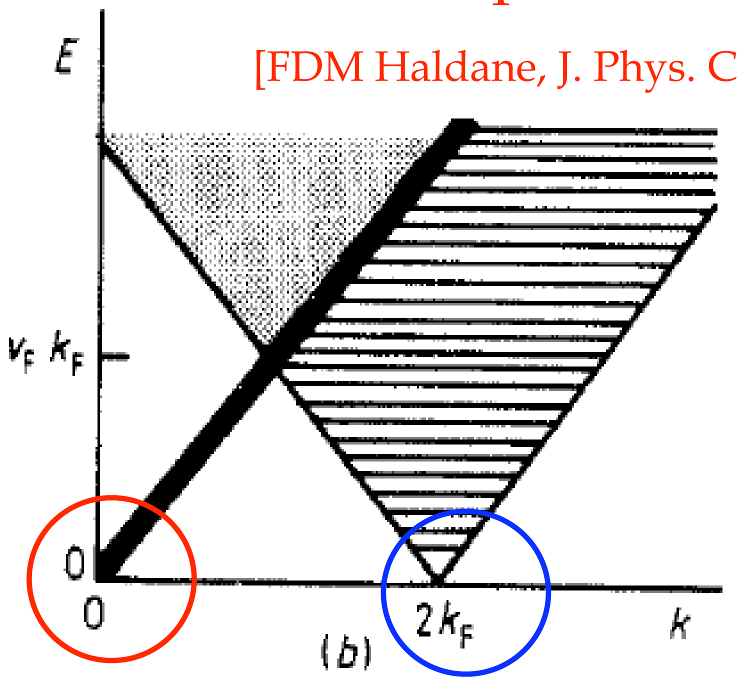

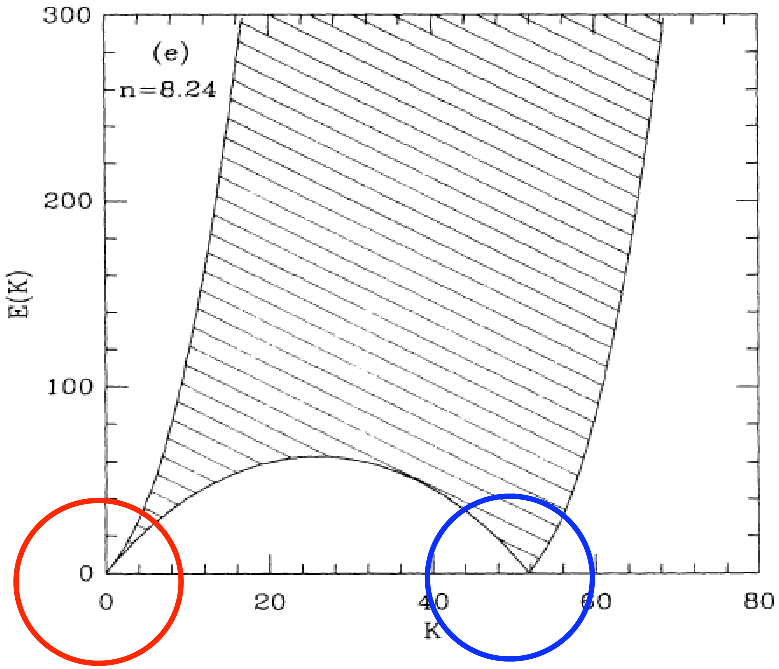

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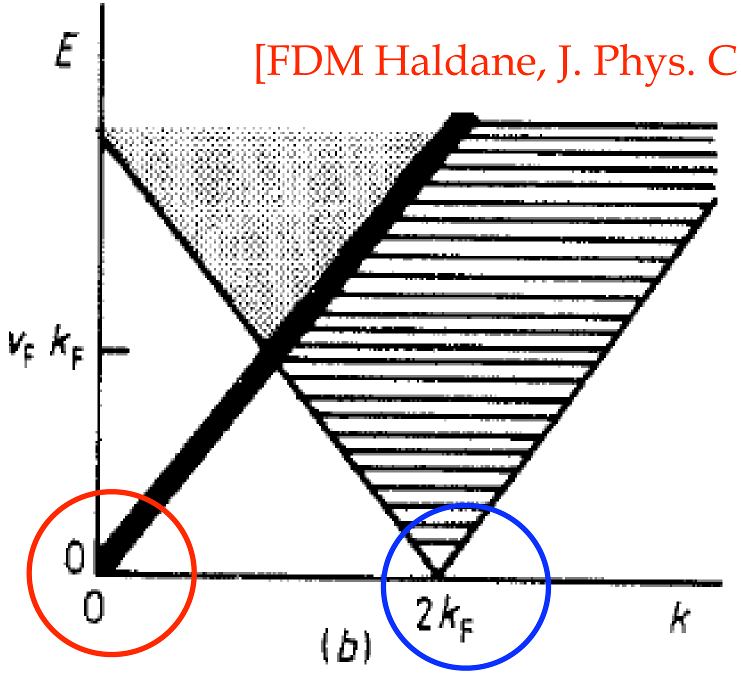

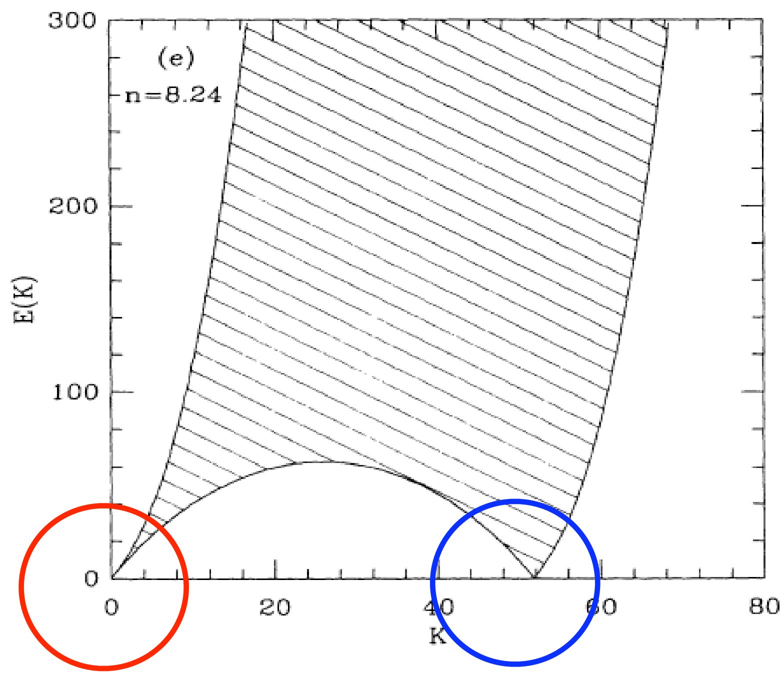

At low temperature and frequency...

(Same spectral degeneracies)

$$H \simeq H_{\rm LM} = \sum_k \hbar v |k|\, b^\dagger(k) b(k),$$

$$Z = {\rm Tr}\, e^{-H/T} \simeq {\rm Tr}\, e^{-H_{\rm LM}/T}$$

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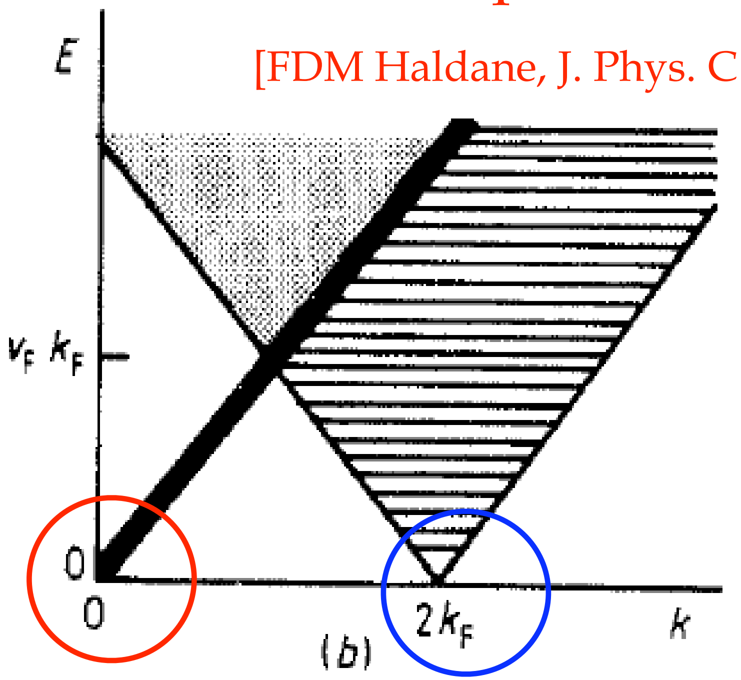

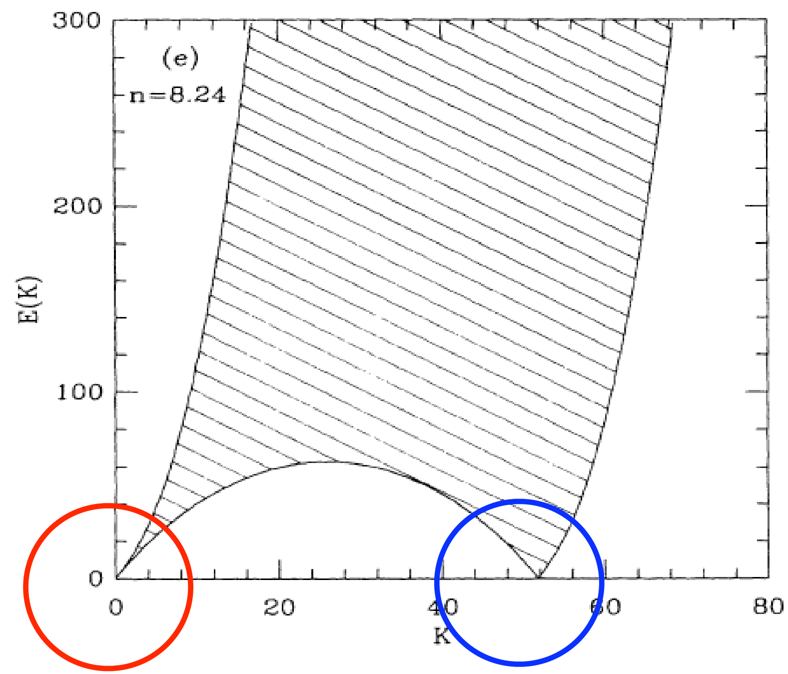

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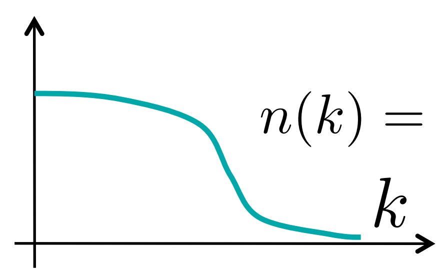
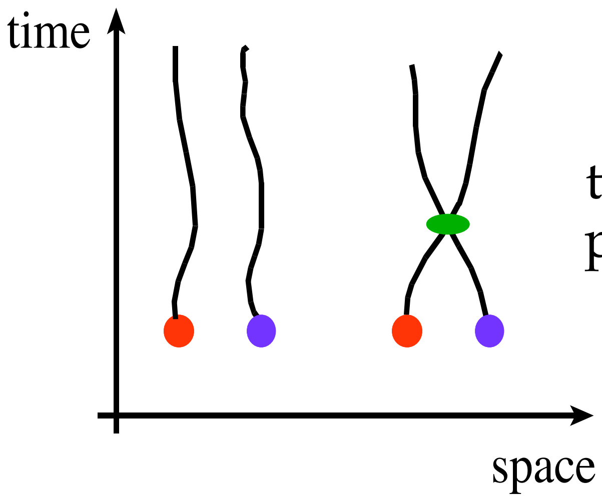
$$\langle O(x) O(0) \rangle \sim x^{-\alpha}$$



$$n(k) = {\rm const.} + |k-k_F|^\alpha {\rm sgn}(k-k_F)$$

Tomonaga-Luttinger liquids



What is it made of? Fermions? Bosons?

"In 1D [...] the symmetry of the wave function cannot be tested by a continuous change of coordinates that exchanges particles without close approach (collision). Thus interaction and statistics effects cannot be separated."

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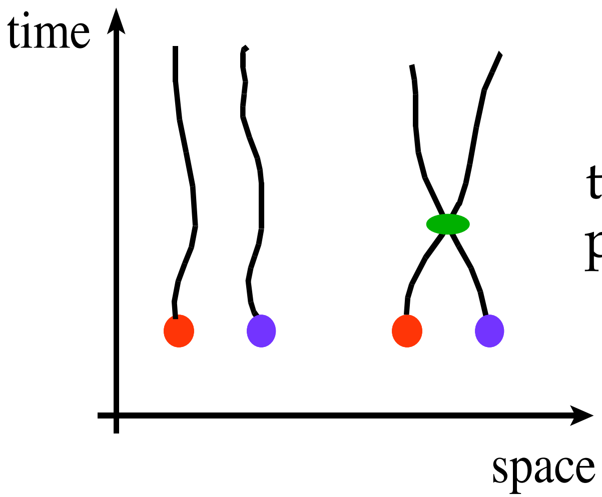

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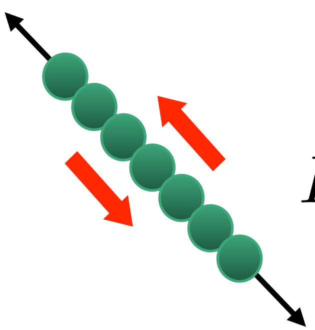
Collective modes exhaust the low-energy spectrum



$$H = \frac{\hbar}{2\pi} \int dx \left[v_J \left(\partial_x \phi\right)^2 + v_N \left(\partial_x \theta\right)^2 \right]$$

v_J: phase stiffness

v_N: density stiffness

$$\rho = \rho_r + \rho_l = \frac{1}{\pi}\partial_x \phi,$$

$$j = \rho_r - \rho_l = \frac{1}{\pi}\partial_x \theta$$

$$v_s = \sqrt{v_J v_N} \qquad K = \sqrt{\frac{v_J}{v_N}}$$

Hubbard models of cold atoms in low-D

Bose-Hubbard and Lieb-Liniger

Bose Hubbard model (not integrable) $\gamma = \frac{E_{\text{int}}}{E_{\text{kin}}} = \frac{U}{J}$

$$H_{\text{BH}} = -\frac{J}{2}\sum_m \left(b^\dagger_{m+1} b_m + \text{H.c.}\right) + \frac{U}{2}\sum_m (b^\dagger_m)^2 (b_m)^2$$

[T Stöferle *et al*, PRL 92 (2004), B. Paredes et al. Nature 429 (2004)]

Low filling limit: Lieb-Liniger model (BA integrable, PR 63 (19)

$$H_{\text{LL}} = \int_0^L dx\, \frac{\hbar^2}{2M}\left|\partial_x \Psi(x)\right|^2 + \frac{g}{2}(\Psi^\dagger(x))^2(\Psi(x))^2$$

[T. Kinoshita et al. Science 305 (2004)]

Dimensionless parameter $\gamma = \frac{E_{\text{int}}}{E_{\text{kin}}} = \frac{Mg}{\hbar^2 \rho_0}$

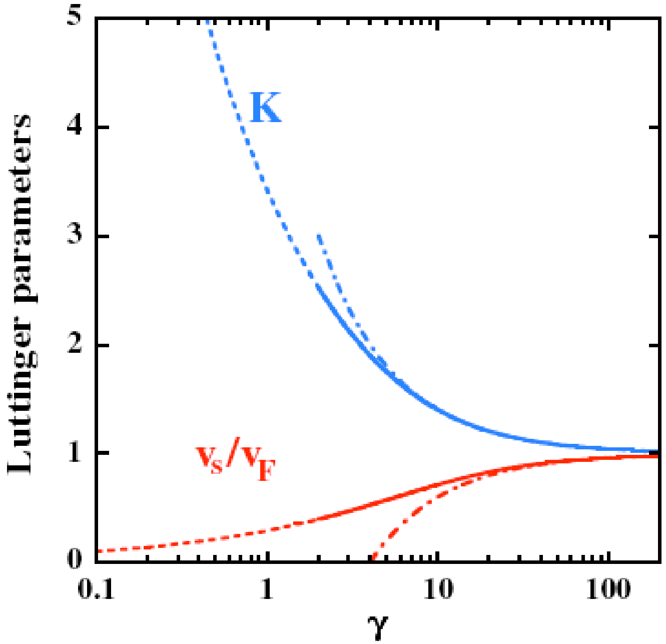
Low density ($\rho_0 \to 0$) means strong interaction $\gamma \to +\infty$

One-component 1D Bose gas: phase diagram

Lieb-Liniger model | **Bose-Hubbard model**

$\gamma = +\infty$ K = 1 (Tonks gas) U/J = +∞

Bethe-Ansatz results



$$v_F = \frac{\hbar k_F}{2M}, \quad k_F = \pi \rho_0$$

[MAC J Phys B 37 (2004)]

K

$$\lim_{U\to\infty} H_{\rm BH} = -\frac{J}{2}\sum_{\langle n,m\rangle} f_m^\dagger f_n, \quad f_m = e^{i\pi\sum_{l<m} b_l^\dagger b_l} b_m$$

Tonks regime $\gamma = \frac{U}{J} \gg 1 \quad f_0 = \frac{N_0}{M_0}$

$$K \simeq 1 + 4\gamma^{-1}\sin\pi f_0/\pi$$

$$v_s/v_F \simeq 1 - 4\gamma^{-1}(f_0\cos\pi f_0)$$

[MAC PRA 67 (2003), PRA 70R (2004)]

Weakly interacting regime

$$K \simeq \sqrt{\frac{J f_0}{U}},$$

$$v_s \simeq \frac{\pi a}{\hbar}\sqrt{J U f_0}$$

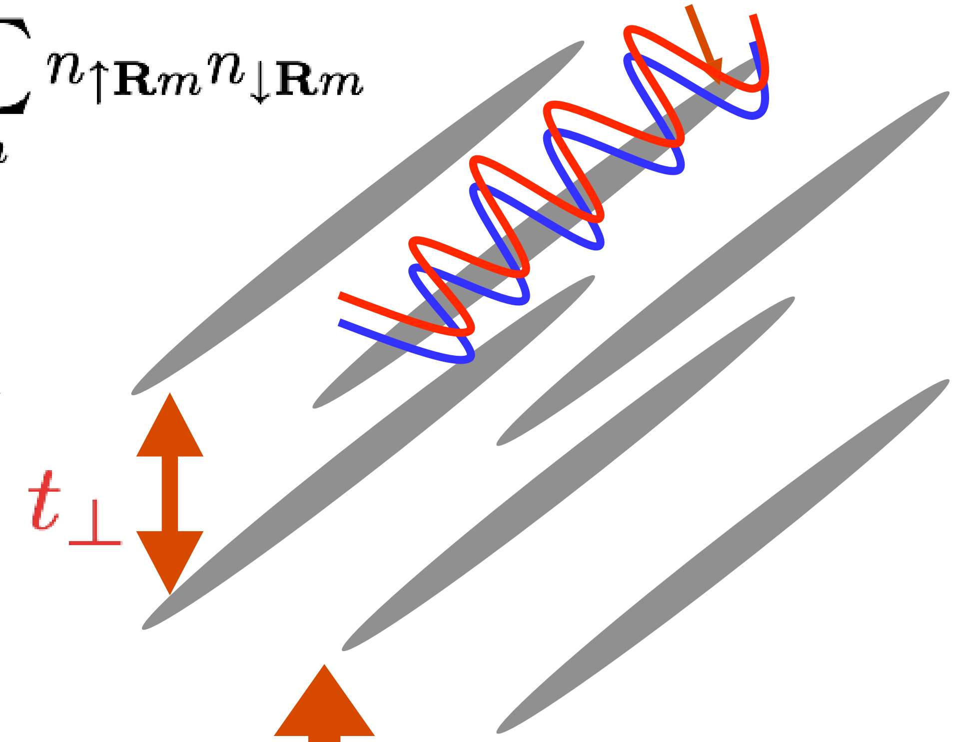
$\gamma \to 0$ K >> 1 (Quasi-condensate) U/J → 0

Fermions: Asymetric 1D Hubbard model

Internal-state dependent optical lattice
[O Mandel et al. PRL 91 (2003)]
or two different atom species (^{6}Li + ^{40}K)
[Innsbruck group]

$$H_{\mathbf{R}} = -\sum_{\langle m,n\rangle \sigma} t_\sigma \, c^\dagger_{\sigma \mathbf{R} m} c_{\sigma \mathbf{R} n} + U \sum_m n_{\uparrow \mathbf{R} m} n_{\downarrow \mathbf{R} m}$$

Constant $N_\uparrow$, $N_\downarrow$
(Canonical Ensemble)



Deep 2D optical lattice $\min\{t_\uparrow, t_\downarrow\} \gg t_\perp$

[T Stöferle, H Moritz, C Schori M Köhl, and T Esslinger, PRL 92 (2004)]

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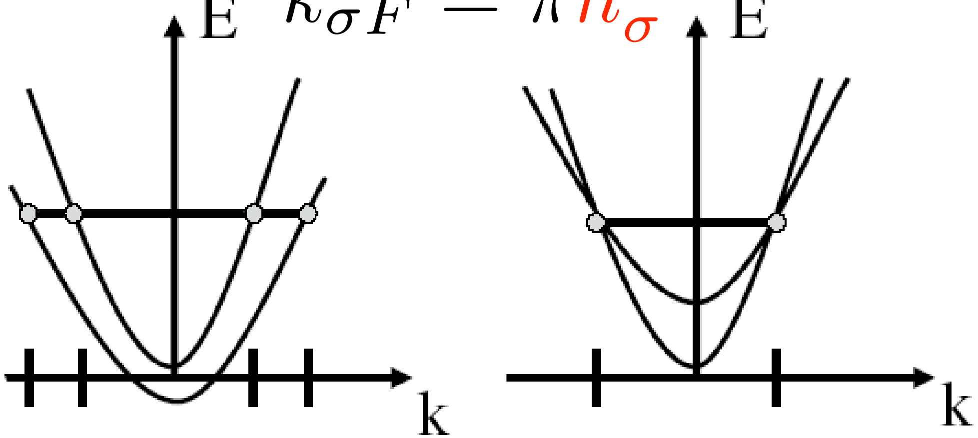
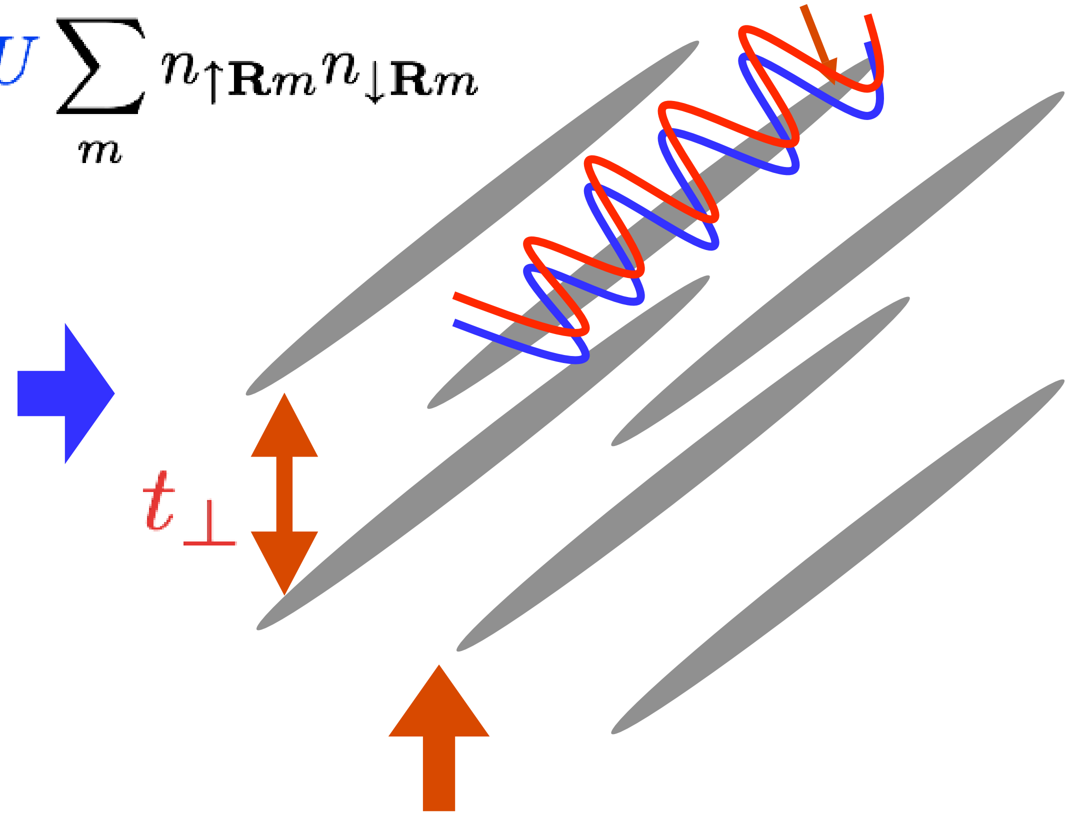
$$k_{\sigma F} = \pi n^0_\sigma$$





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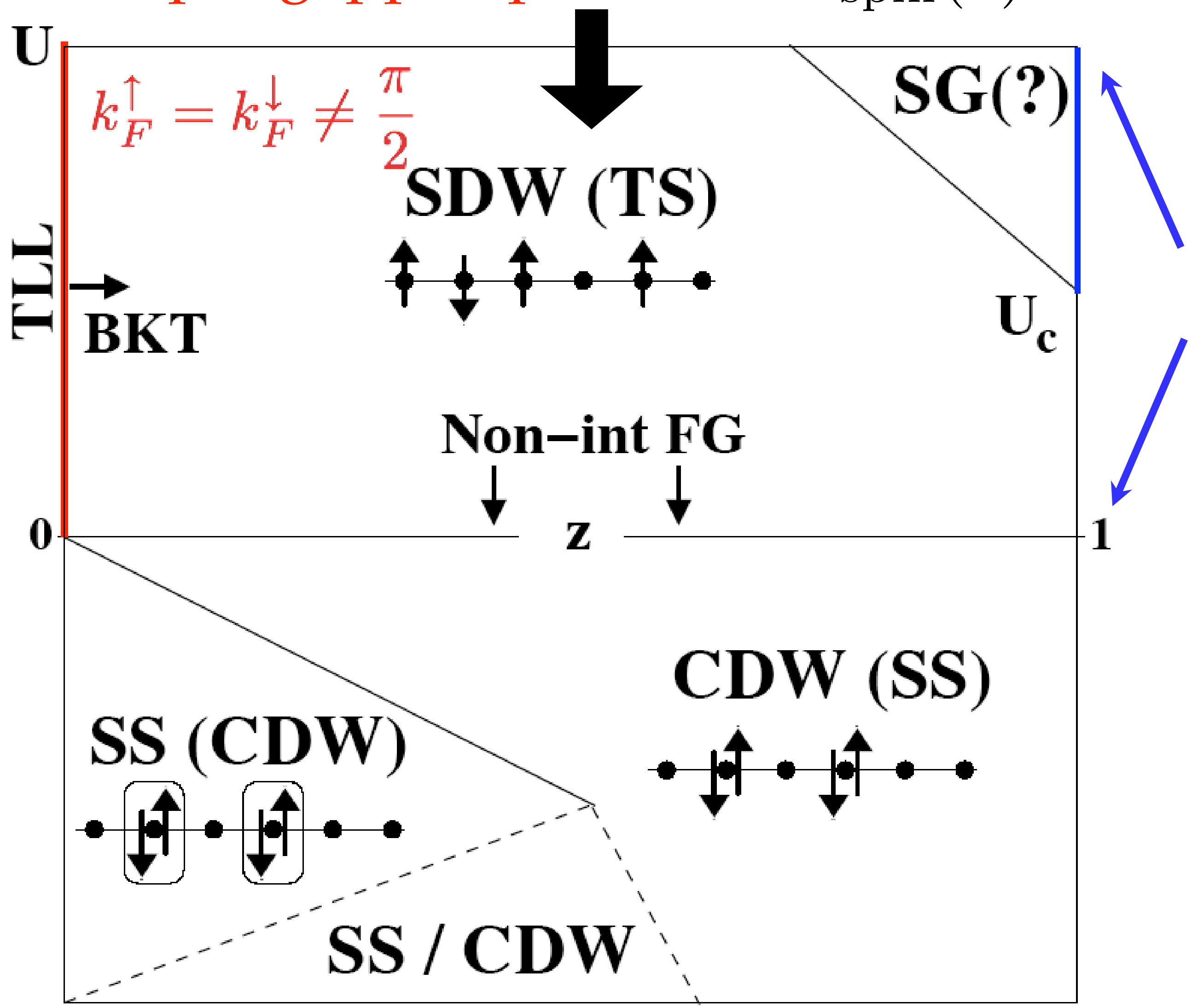
Schematic Phase Diagram

[MAC, AF Ho & T Giamarchi, PRL 95 (2005)]

Spin gapped phases $SU_{\text{spin}}(2) \to U(1) \times Z_2$



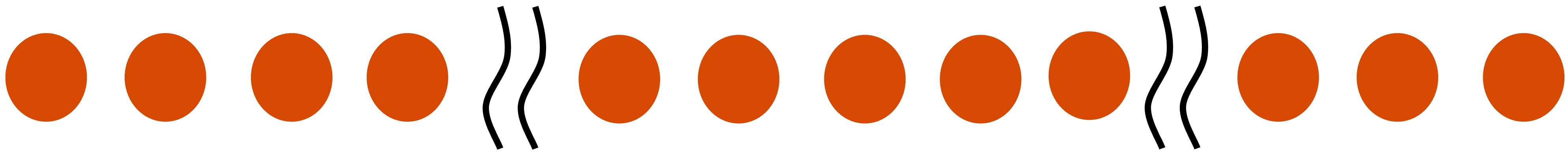
Falikov-Kimball Model

$$z = \frac{|t_{\uparrow} - t_{\downarrow}|}{t_{\uparrow} + t_{\downarrow}}$$

Explanation of Phase Diagram

"Almost crystaline" order in 1D

It looks like a crystal on a short distance scale, but it is disordered on a large distance scale. Periodic regions of all sizes.



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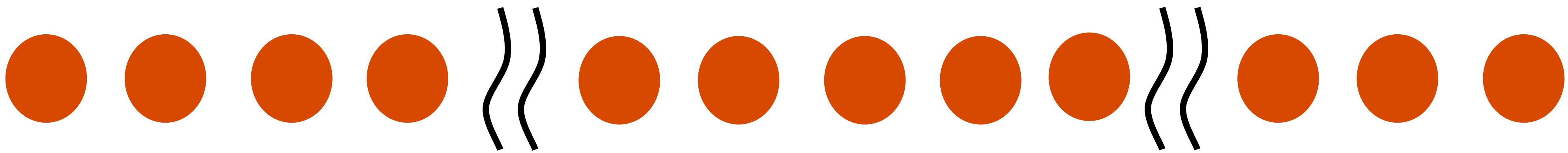

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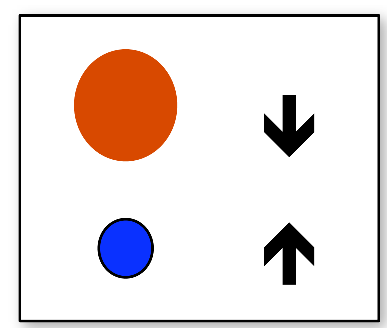

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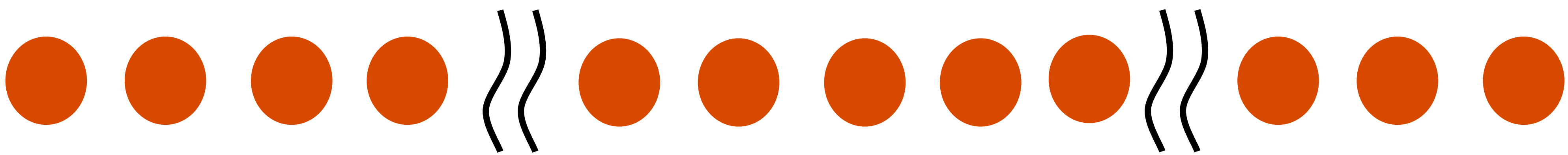

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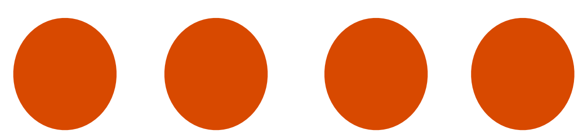

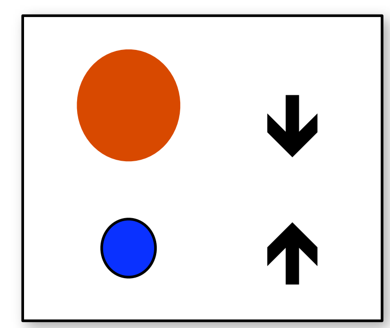

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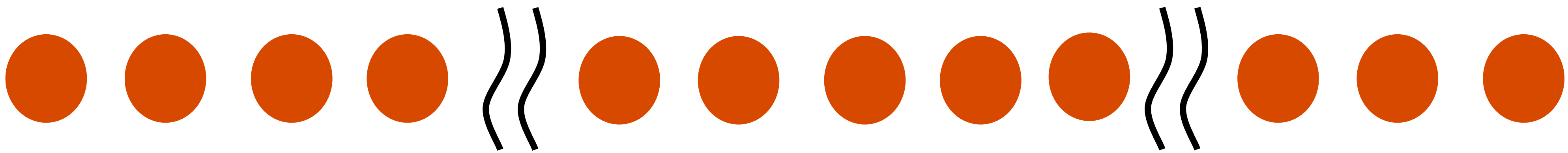

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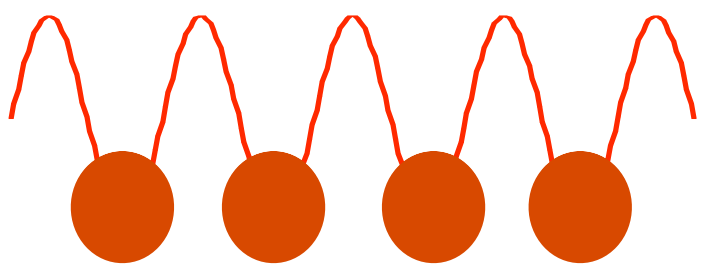

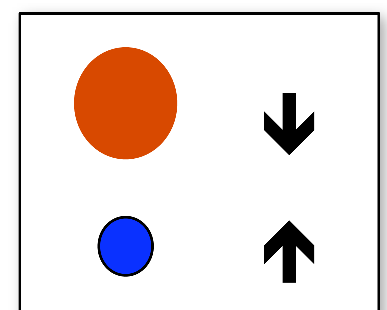

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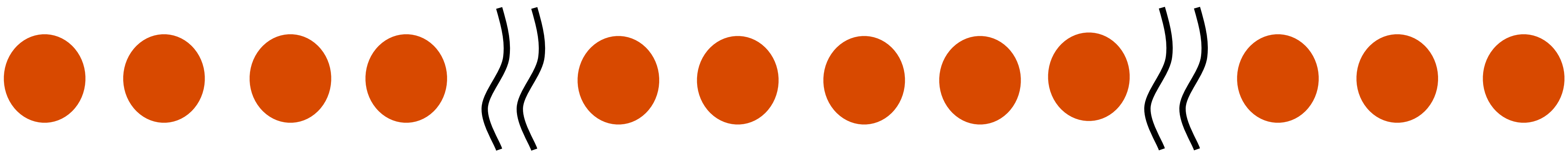

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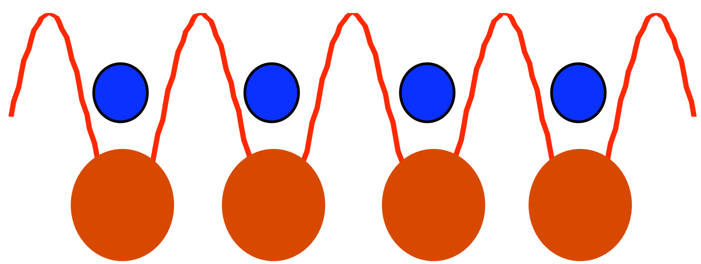

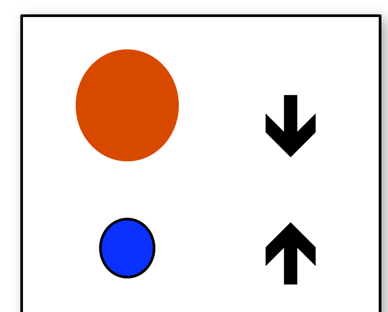

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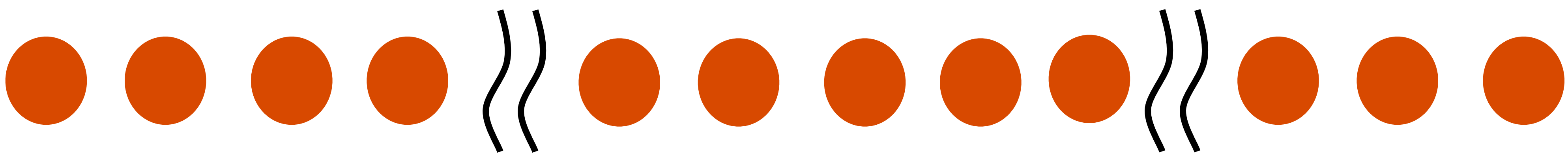

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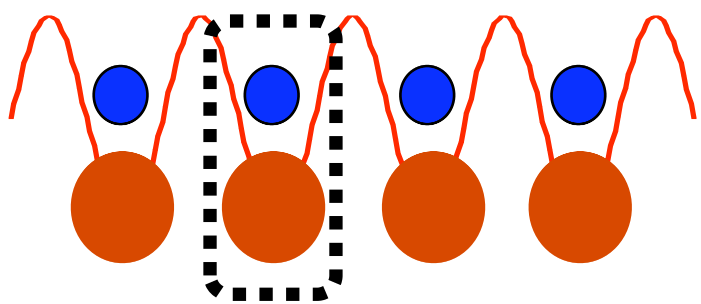

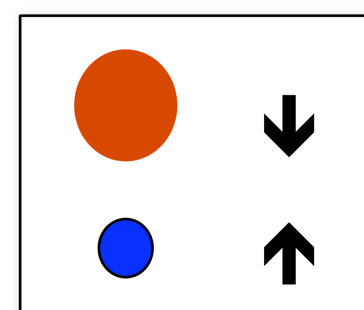

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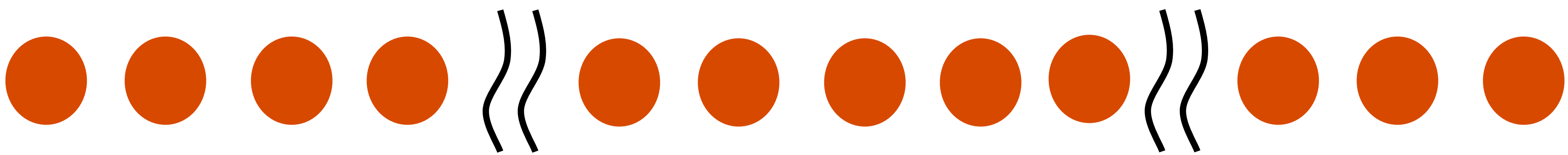

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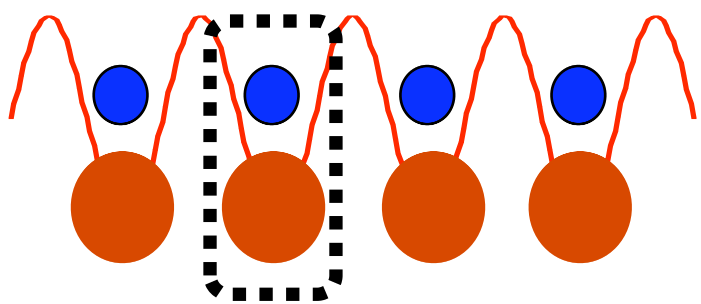
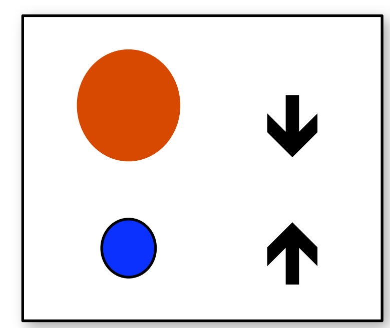


SS / CDW



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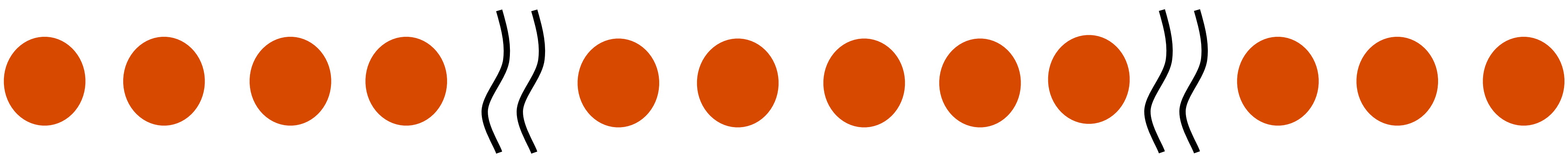

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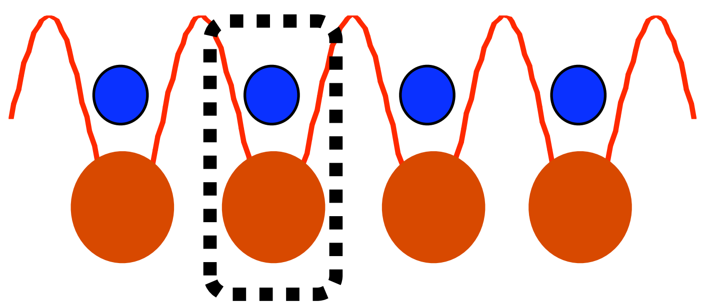

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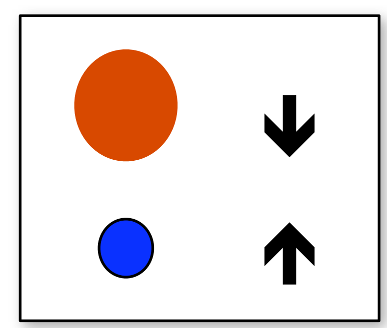

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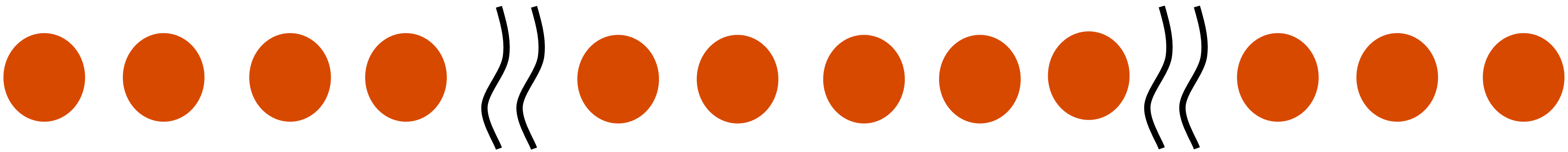

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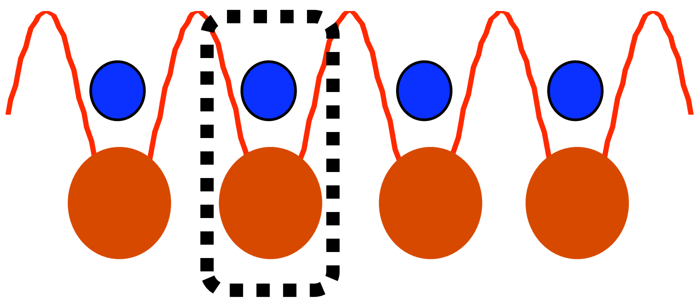

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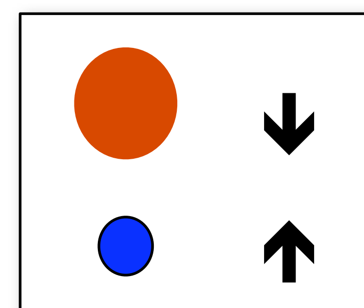

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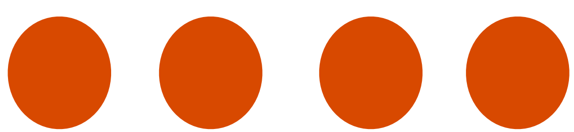

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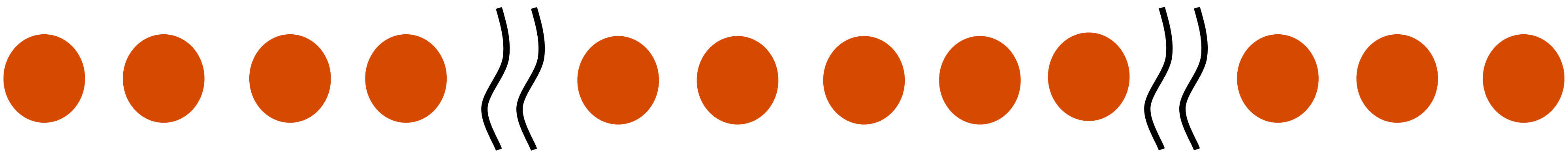

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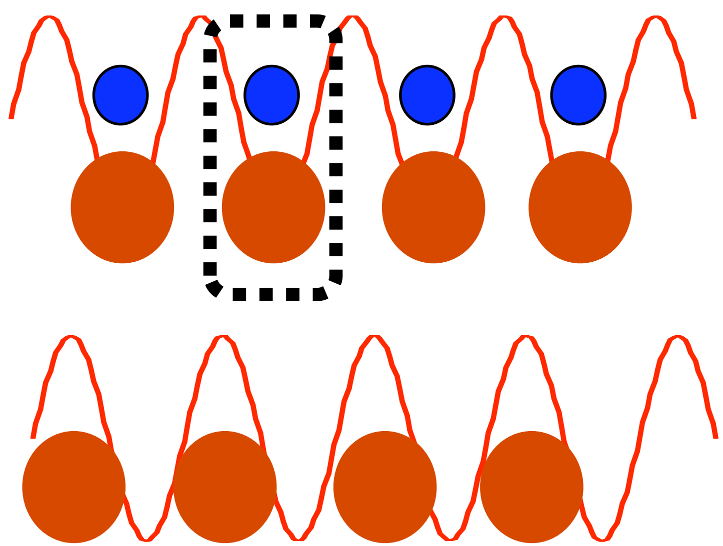

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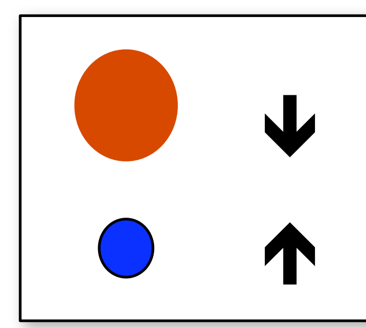

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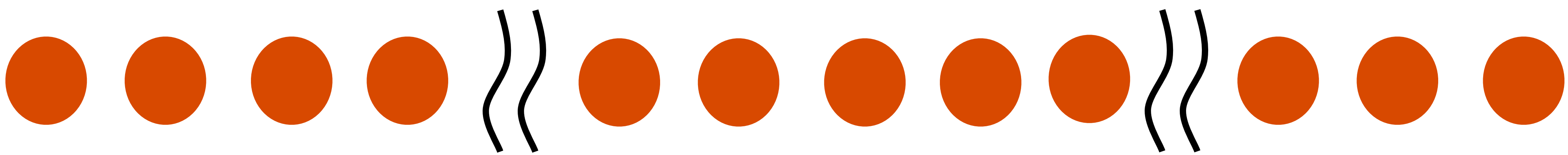

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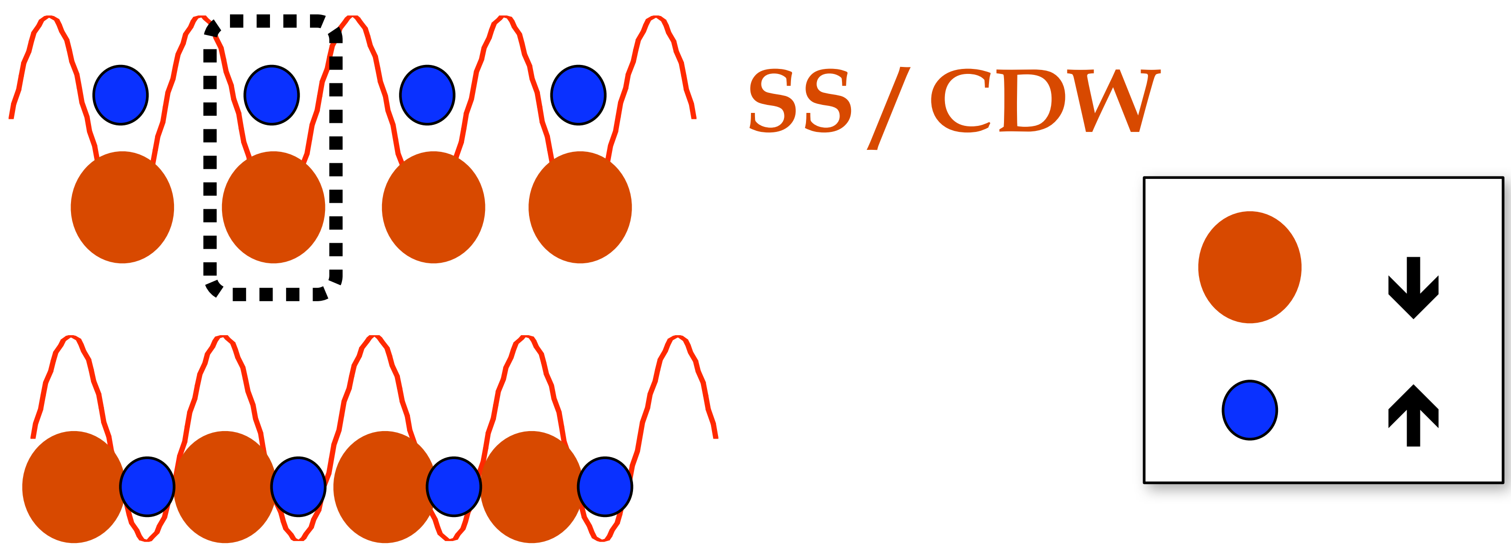

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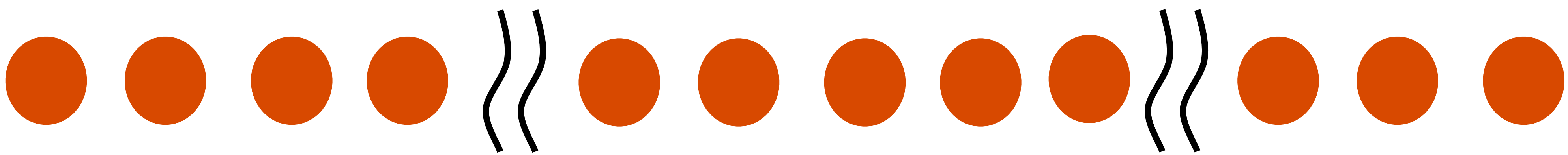
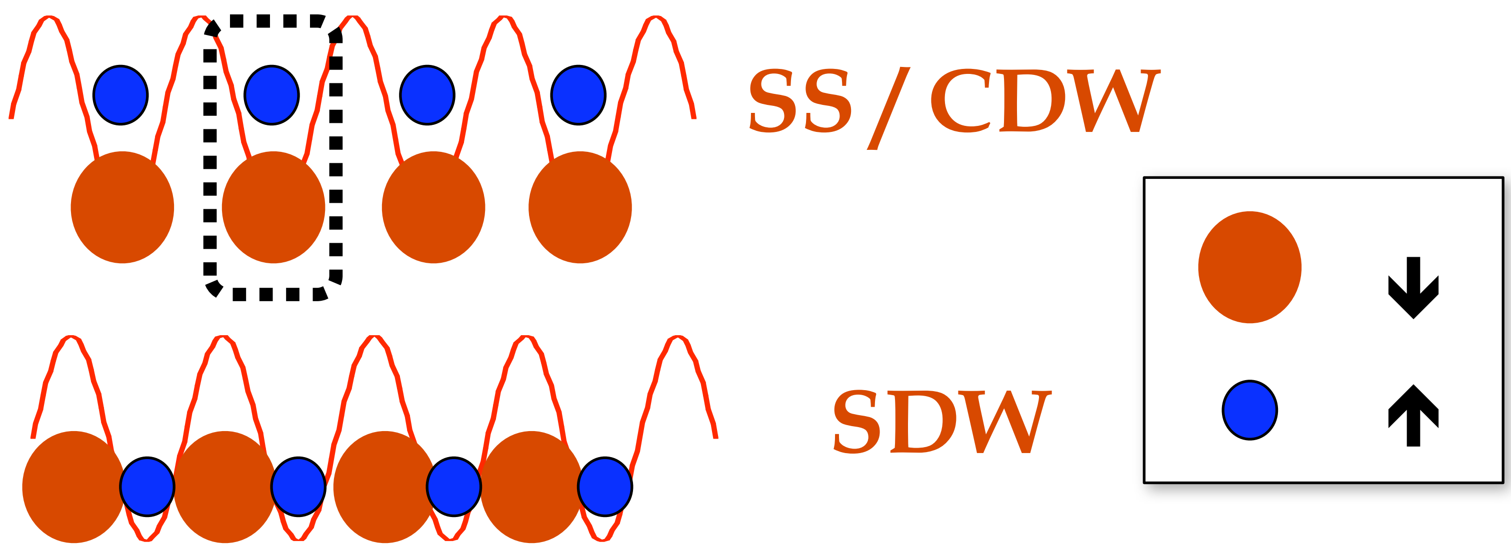


Adiabatic approximation $(t_\uparrow \gg t_\downarrow)$

$U < 0$

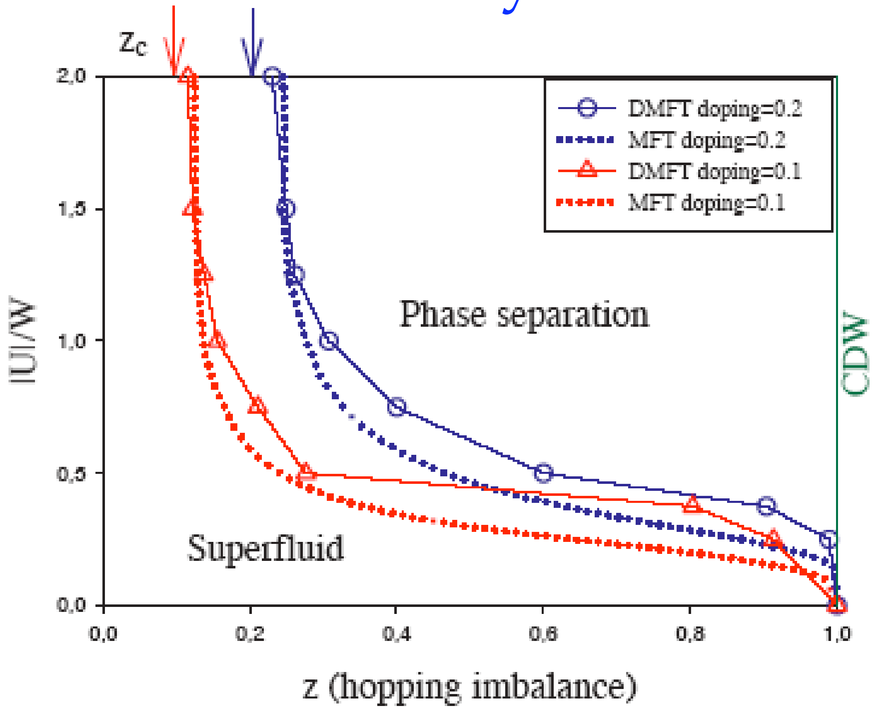
$U > 0$



What about d > 1?

[TL Dao, A Georges, and M Capone, arxiv/0407.2260]

DMFT U < 0 Asymmetric Hubbard



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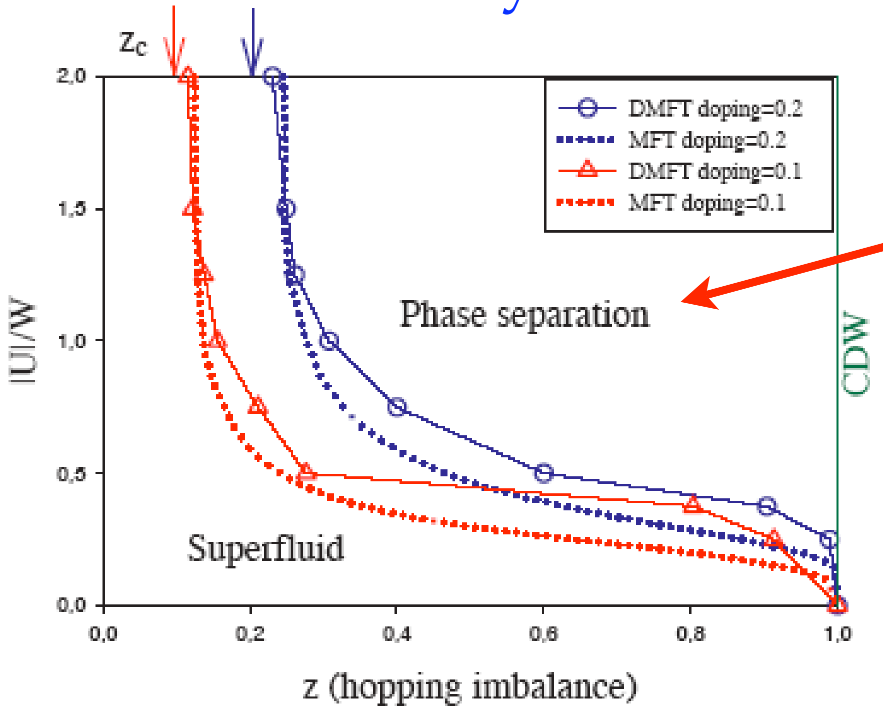

Coexistence of SF and CDW

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[TL Dao, A Georges, and M Capone, arxiv/0407.2260]

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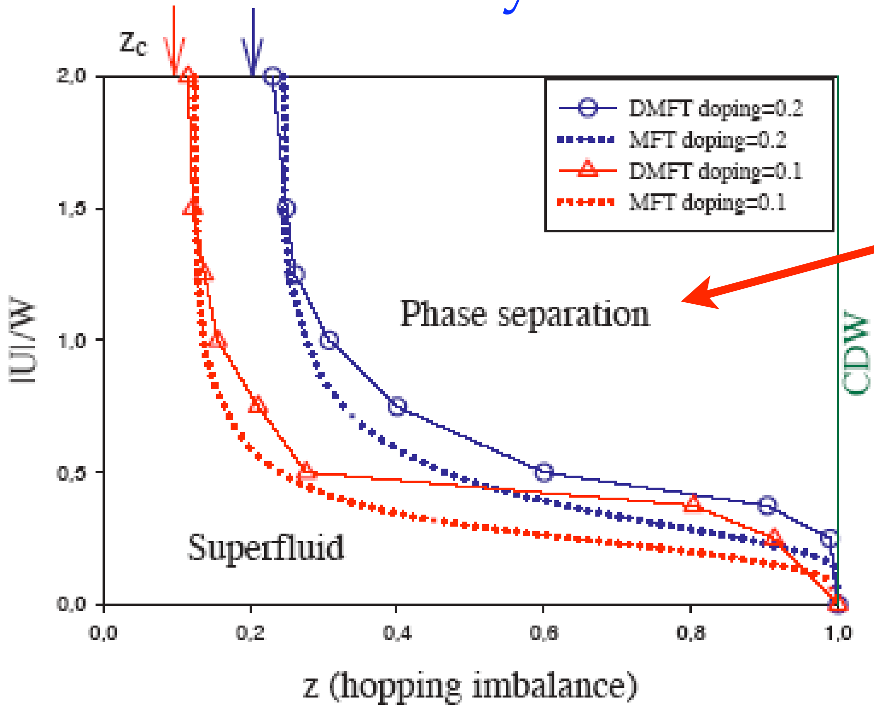
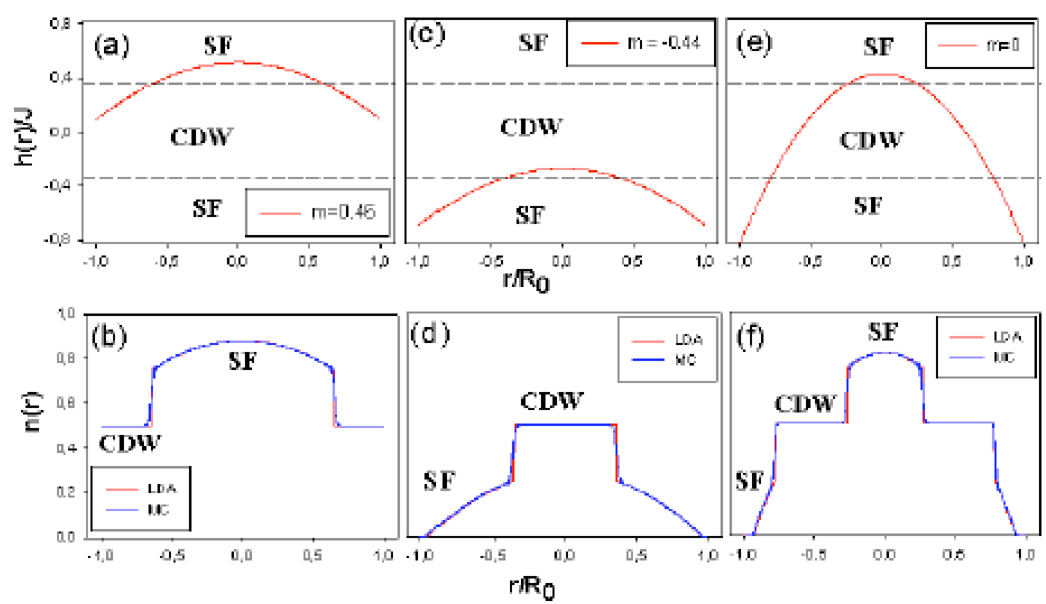


Coexistence of SF and CDW

Harmonic trap (LDA)

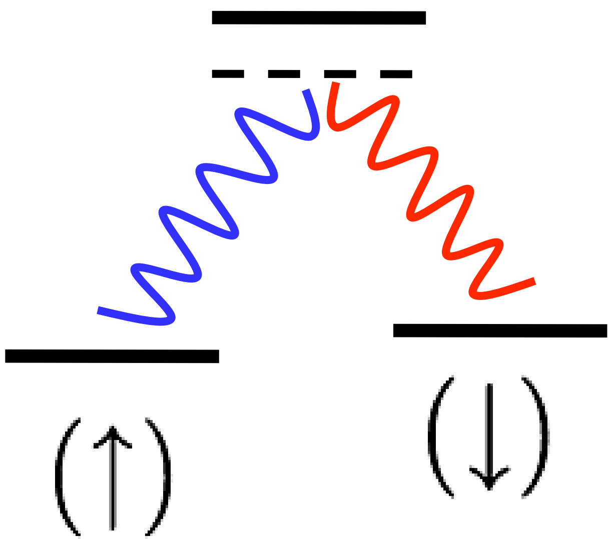


Reading out: Detecting the spin gap

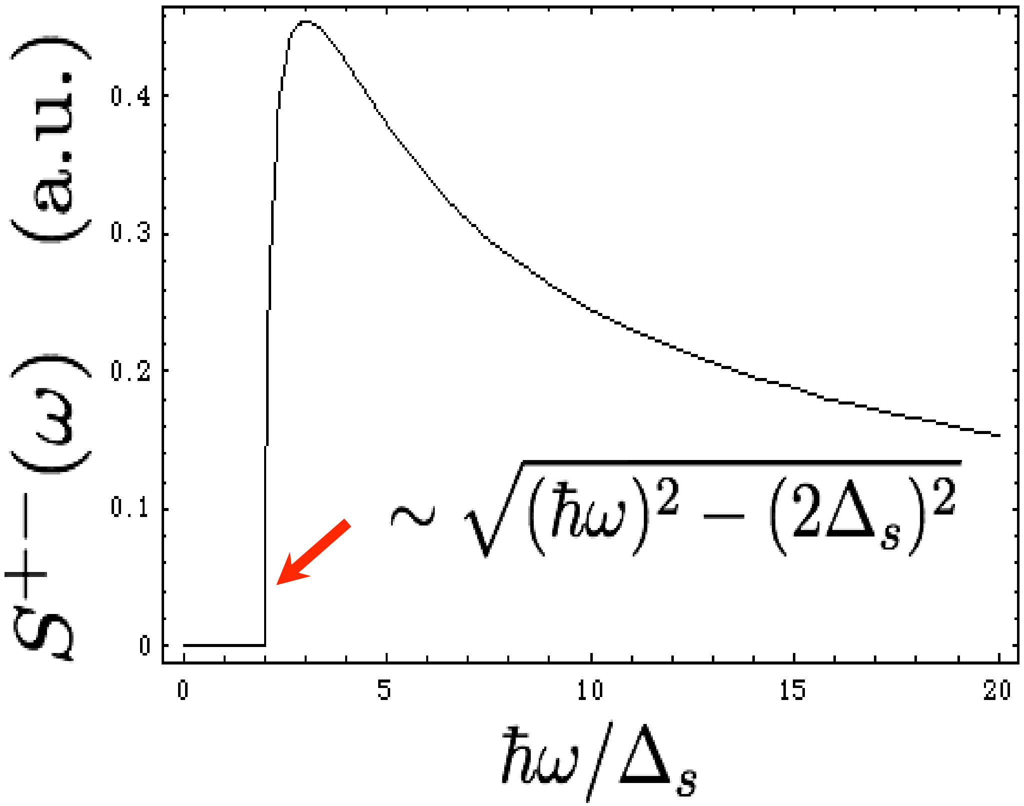
[MAC, AF Ho & T Giamarchi, PRL 95 (2005)]

A Raman laser induces transitions between hyperfine states

[HP Buchler *et al* PRL 93 (2004)]

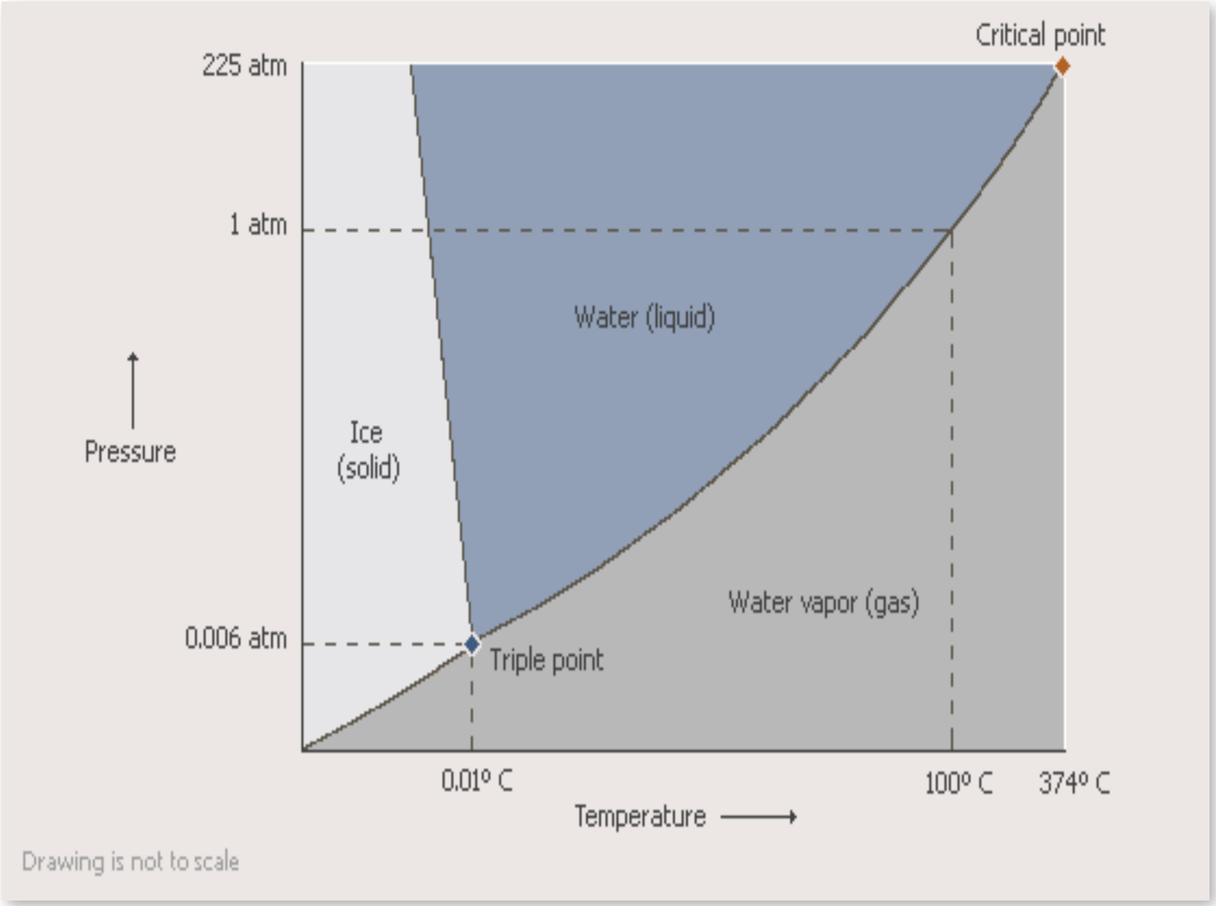


$$S^{+-}(t) = \langle S_T^{+}(t) S_T^{-}(0) \rangle$$



Non-equilibrium phenomena in 1D quantum gases

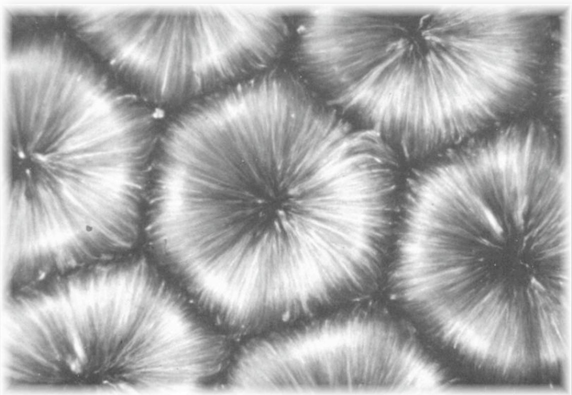
Non-equilibrium steady states

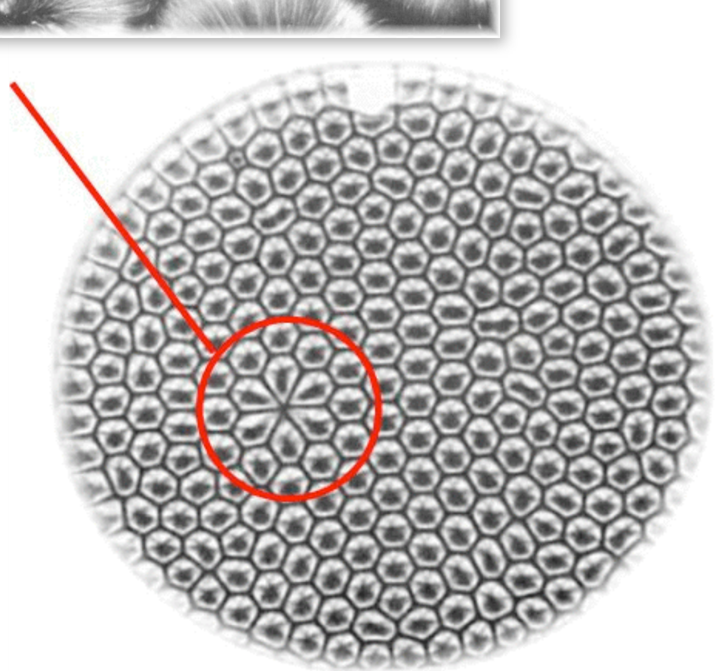


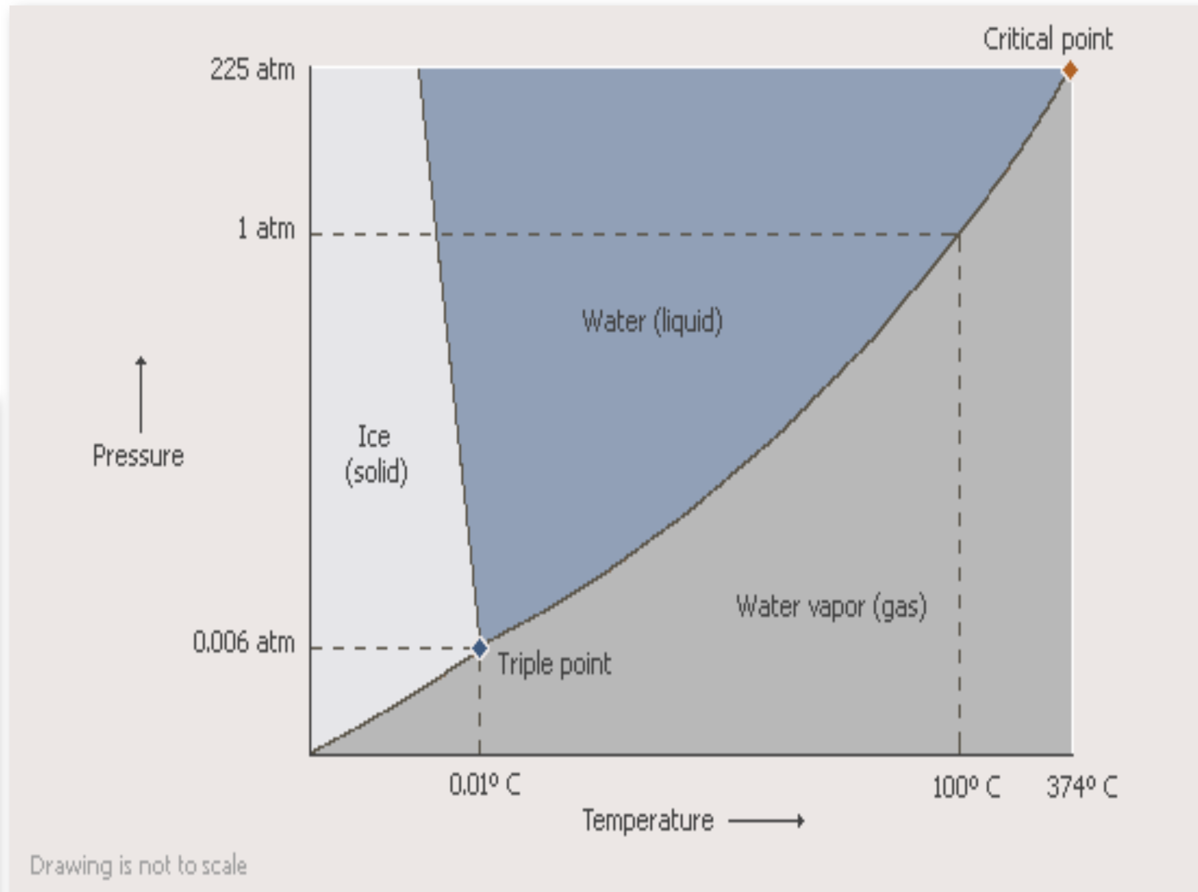
Equilibrium phase diagram of H_2O

Non-equilibrium steady states

Rayleigh-Bénard convection cells







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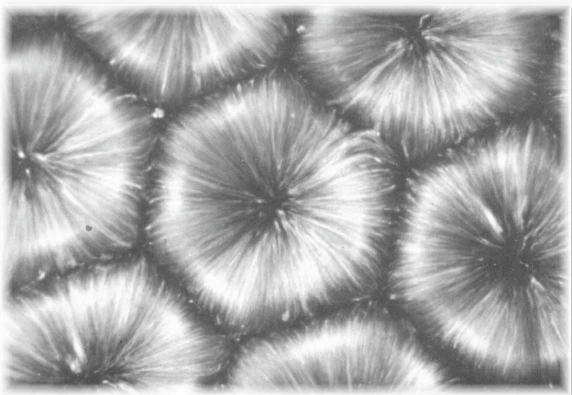
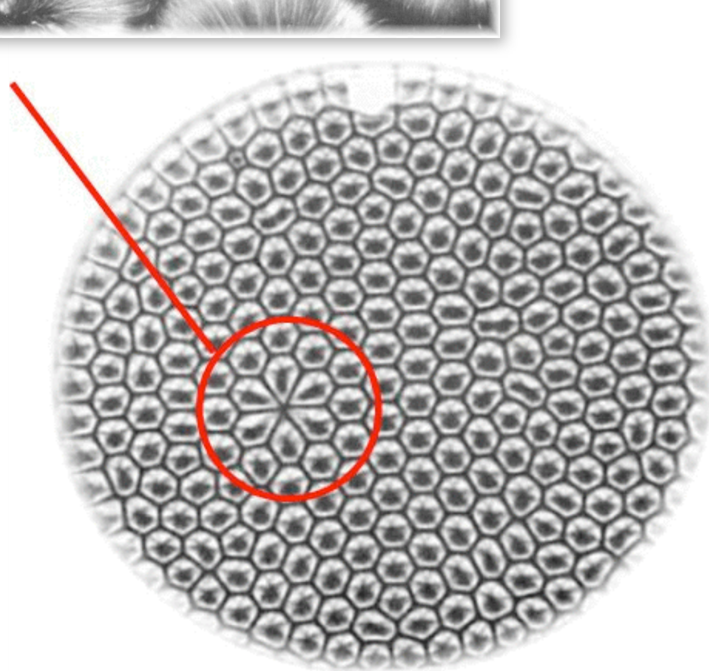
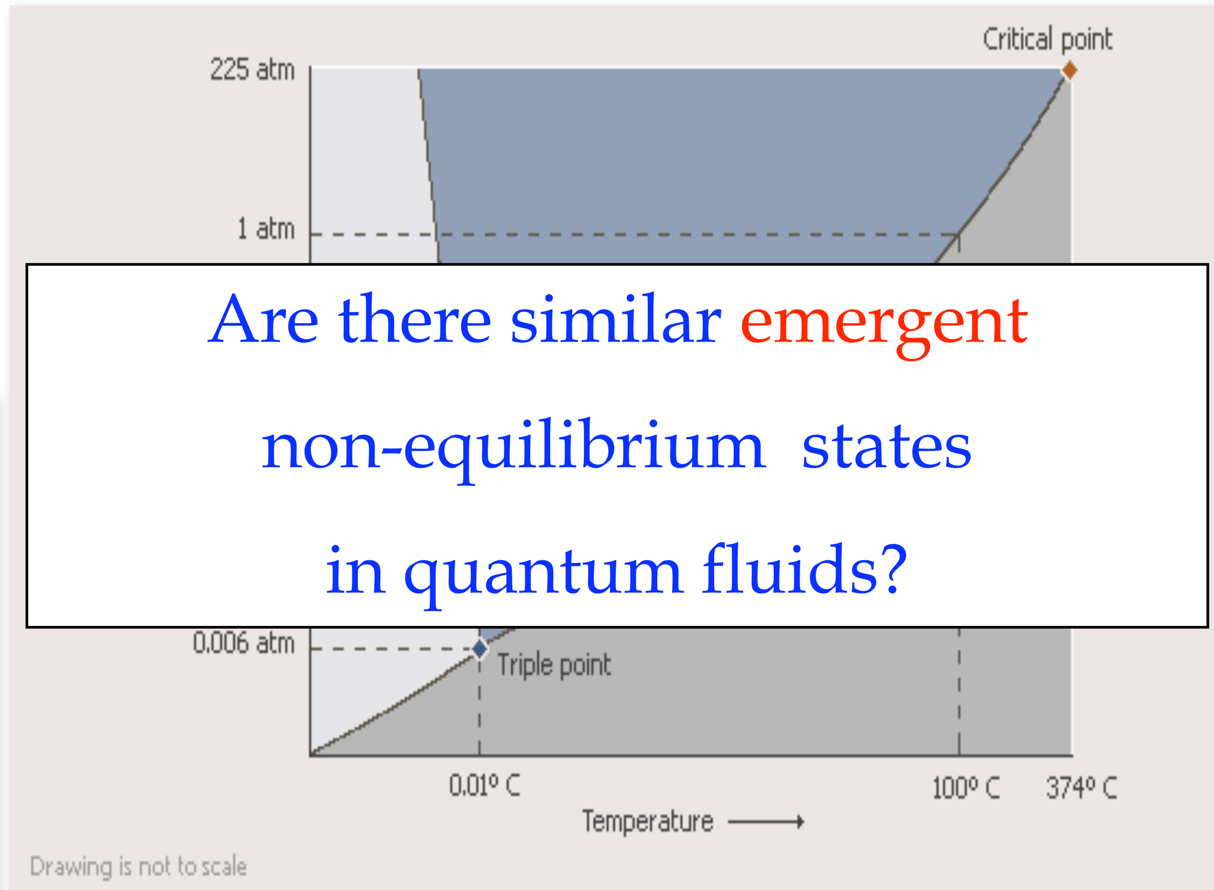






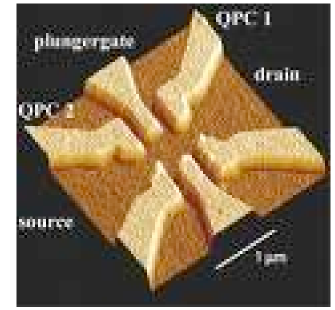
Are there similar emergent

non-equilibrium states

in quantum fluids?

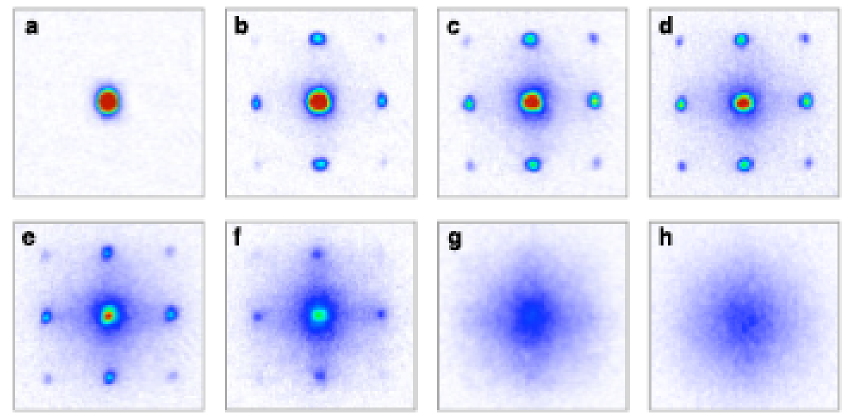
Equilibrium phase diagram of H_2O

Quantum Fluids out of equilibrium



Problems with solid state/liquid He quantum fluids:

- Not easily tunable
- Quantum decoherence is a killer



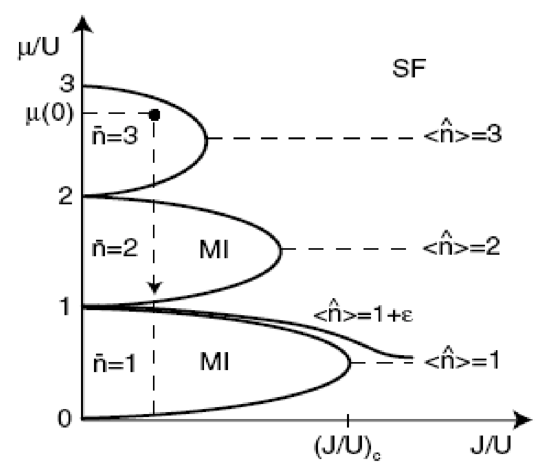
Cold atoms in an optical lattice

[D. Jaksch et al. PRL 81 (1998)]
[M Greiner et al. Nature, 415 (2002)]

Interacting bosons on a lattice

[MPA Fisher et al. PRB 40 (1989)]

$$H_{\rm BH} = -J \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} b^\dagger_{\mathbf{R}} b_{\mathbf{R}'} + U \sum_{\mathbf{R}} n_{\mathbf{R}}(n_{\mathbf{R}} - 1)$$



A Statistical description of non-equilibrium states?

Quench at t = 0 (sudden approximation):

$$|\Phi(t>0)\rangle = e^{-iH_f\, t/\hbar}|\Phi(0)\rangle = e^{-iH_f\, t/\hbar}|\Phi_0\rangle$$

Not an eigenstate of Hf !!

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Operators after the quench:

$$O(t>0) = \langle\Phi(t)|\hat{O}|\Phi(t)\rangle = \langle\Phi_0|e^{iH_f\, t/\hbar}\hat{O}e^{-iH_f t/\hbar}|\Phi_0\rangle$$

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Does the system reach a (quasi-) stationary state? If so,

$$\bar{O} = \lim_{T\to+\infty}\lim_{t_0\to+\infty}\frac{1}{T}\int_{t_0}^{T+t_0} dt\, O(t)$$

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Is there any statistical ensemble such that $\bar{O} = \mathrm{Tr}\hat{\rho}_{\mathrm{quench}}\,\hat{O}$?

Is $\hat{\rho}_{\mathrm{quench}} = \rho_{\mathrm{eq}} = e^{-(H_f-\mu N)/T_{\mathrm{eff}}}$? (ergodic hypotesis)

Won't be looking at the creation of topological defects

Absence of thermalization in 1DBG

nature

LETTERS

[T Kinoshita, T Wenger & D Weiss, Nature (2006)]

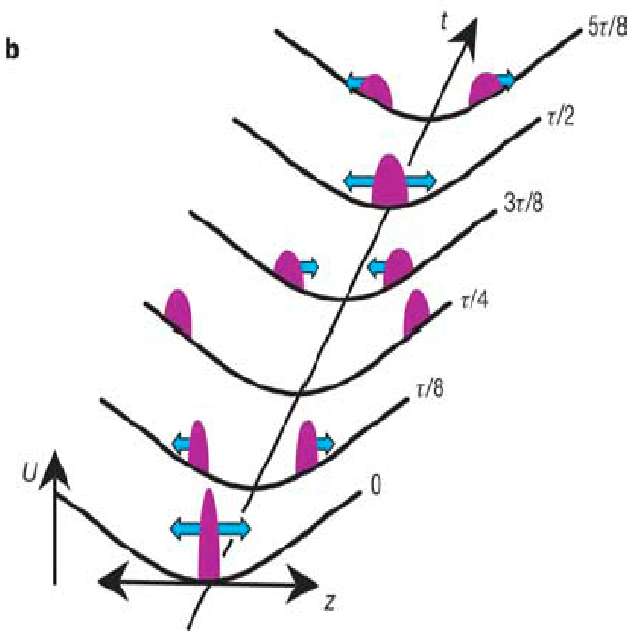
A quantum Newton's cradle

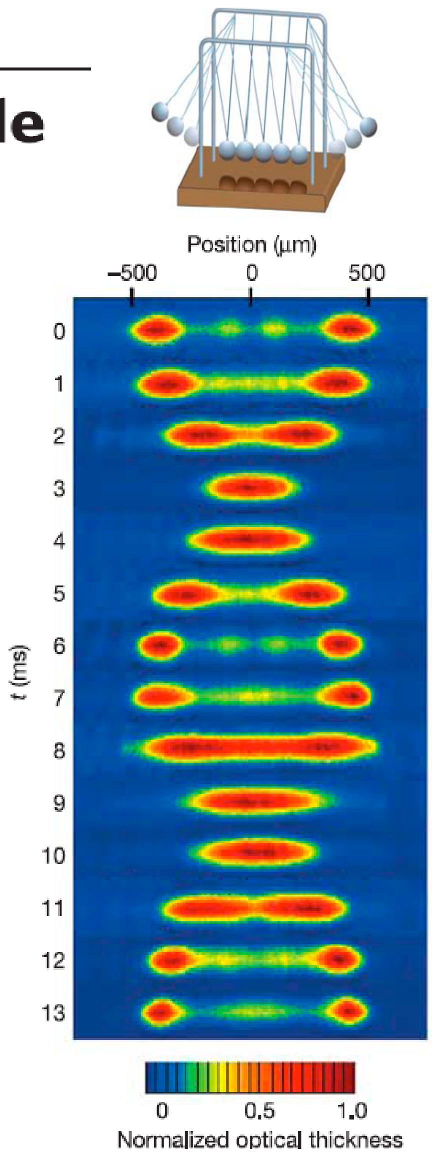
Toshiya Kinoshita[1], Trevor Wenger[1] & David S. Weiss[1]

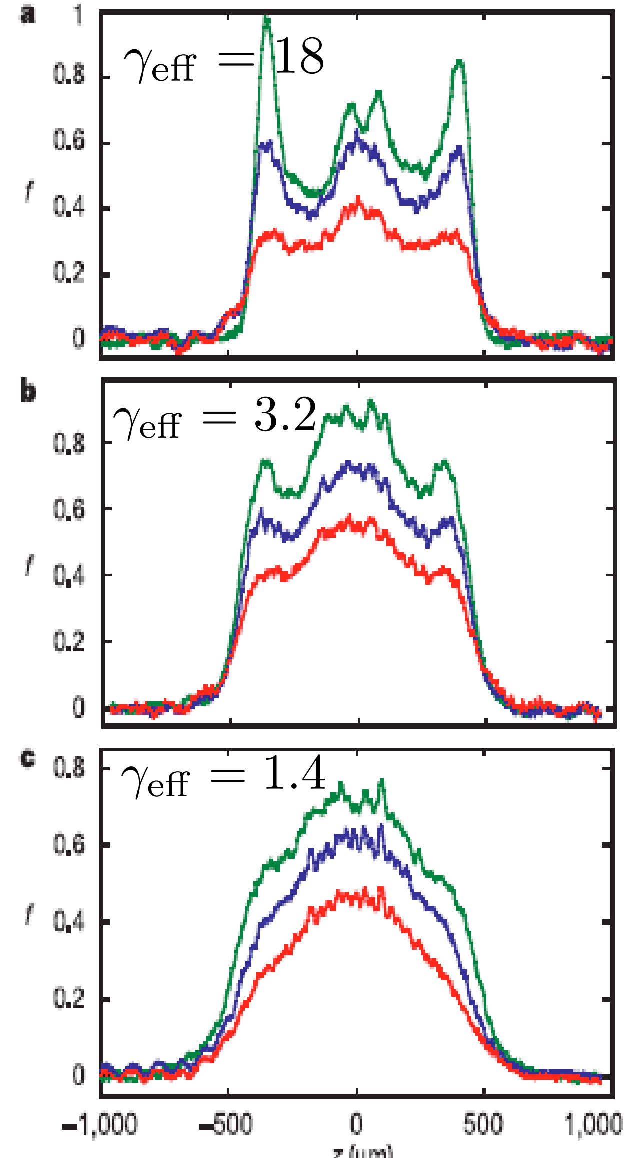
$$p_1 + p_2 = p_1' + p_2',$$

$$\frac{p_1^2}{2M} + \frac{p_2^2}{2M} = \frac{p_1'^2}{2M} + \frac{p_2'^2}{2M},$$

$$p_1 = p_1',\ p_2 = p_2', \qquad p_1 = p_2',\ p_2 = p_1'$$







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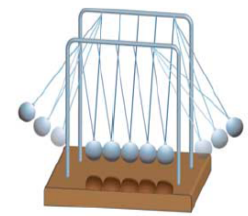

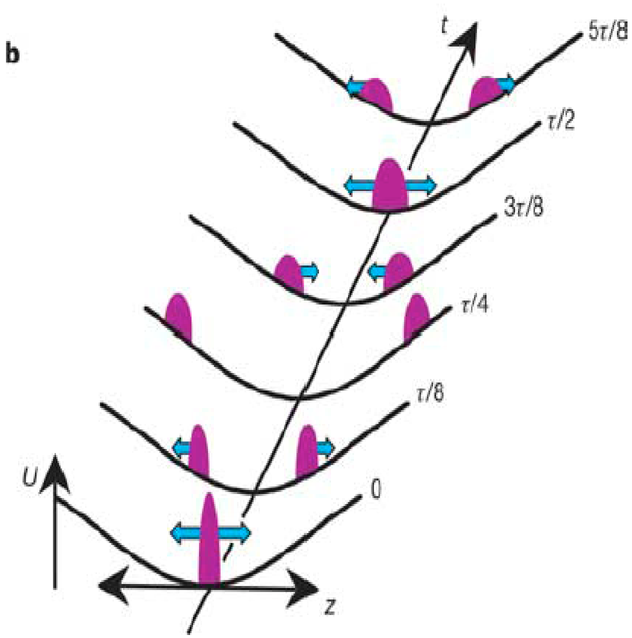

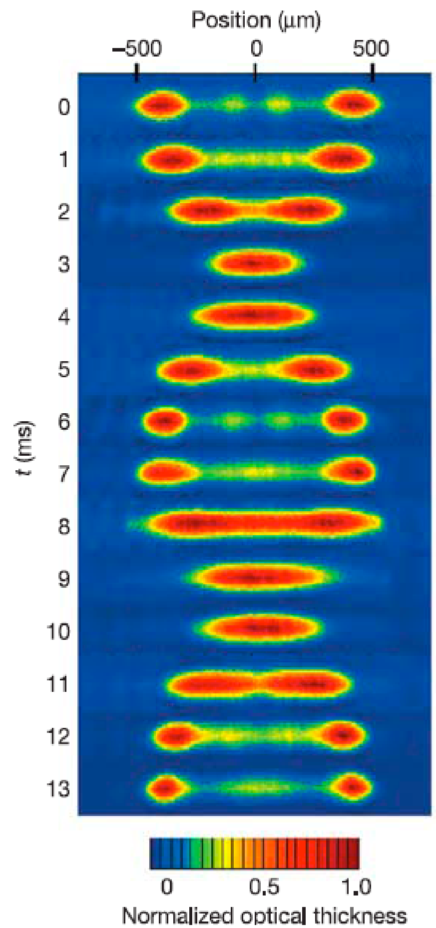

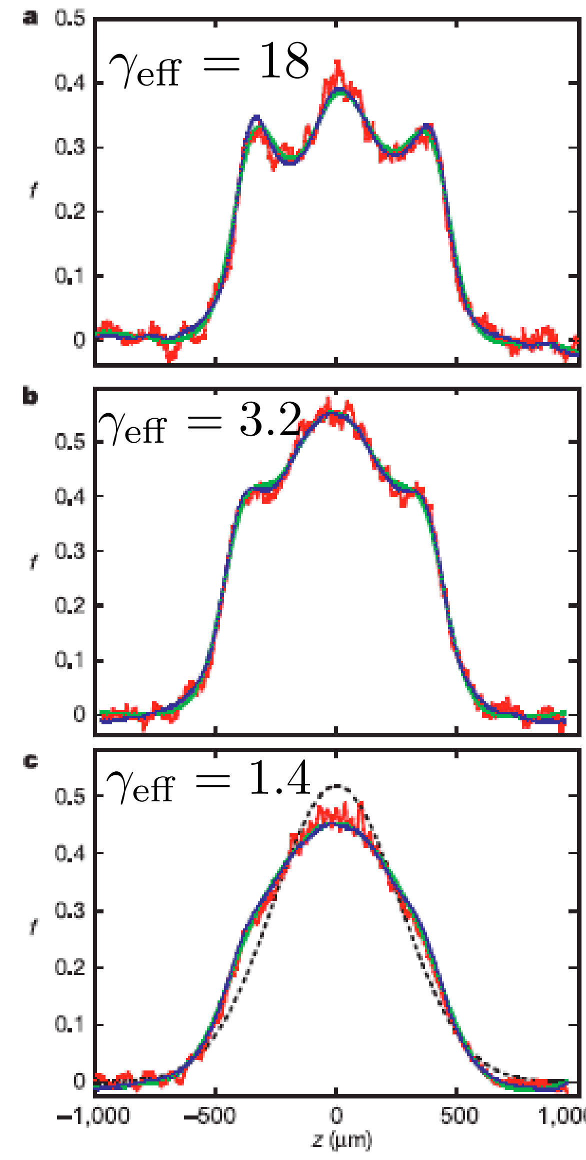

Free expansion of a Tonks gas

[M Rigol, B Dunjko, V Yurovsky, and M Olshanii PRL 98 (2007)]

Integrable XY model

$$H = -J \sum_{\langle n,m \rangle} \sigma_m^+ \sigma_n^- = -\sum_p (2J\cos p)\ f^\dagger(p) f(p)$$

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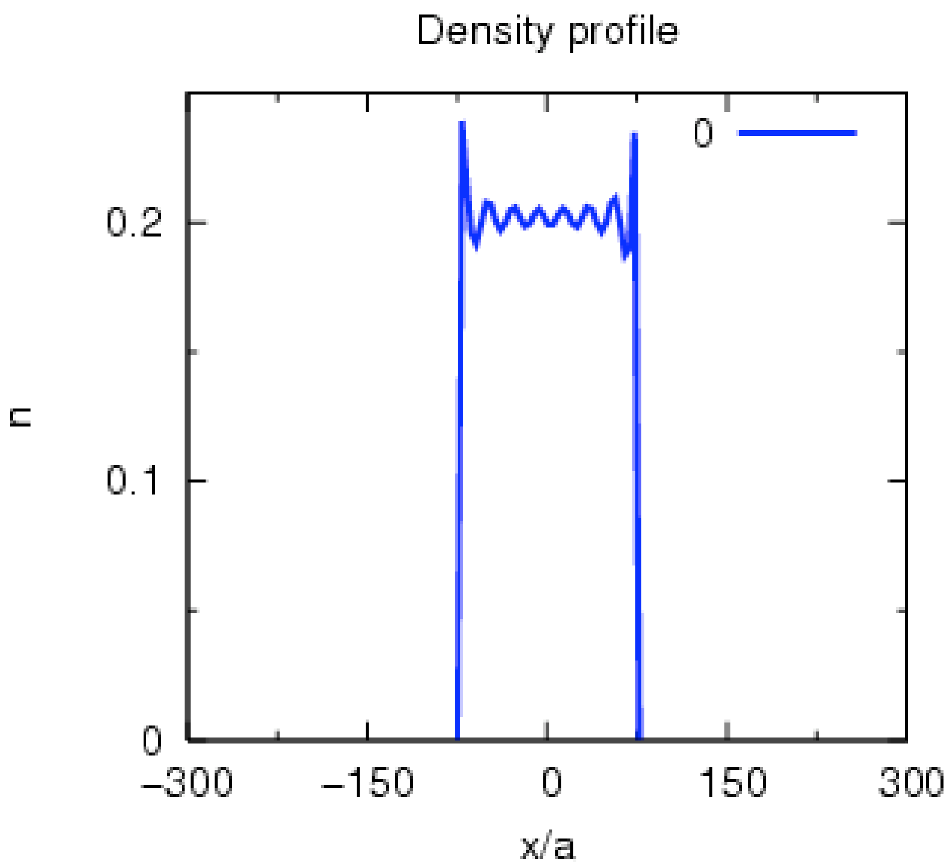

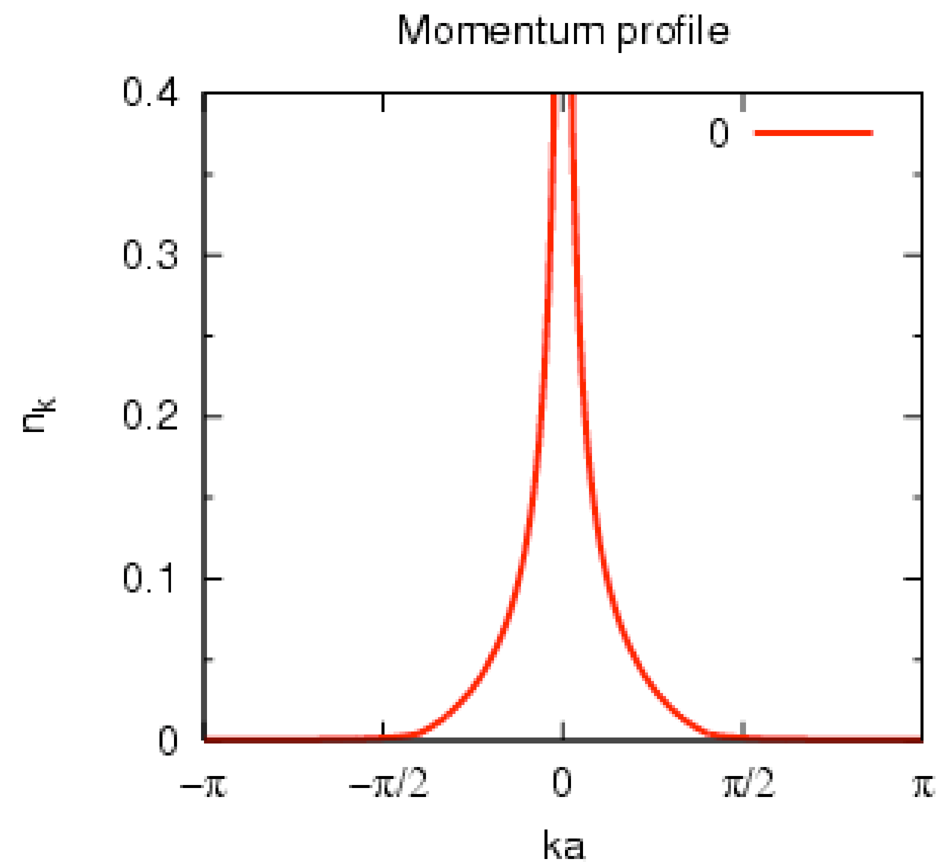

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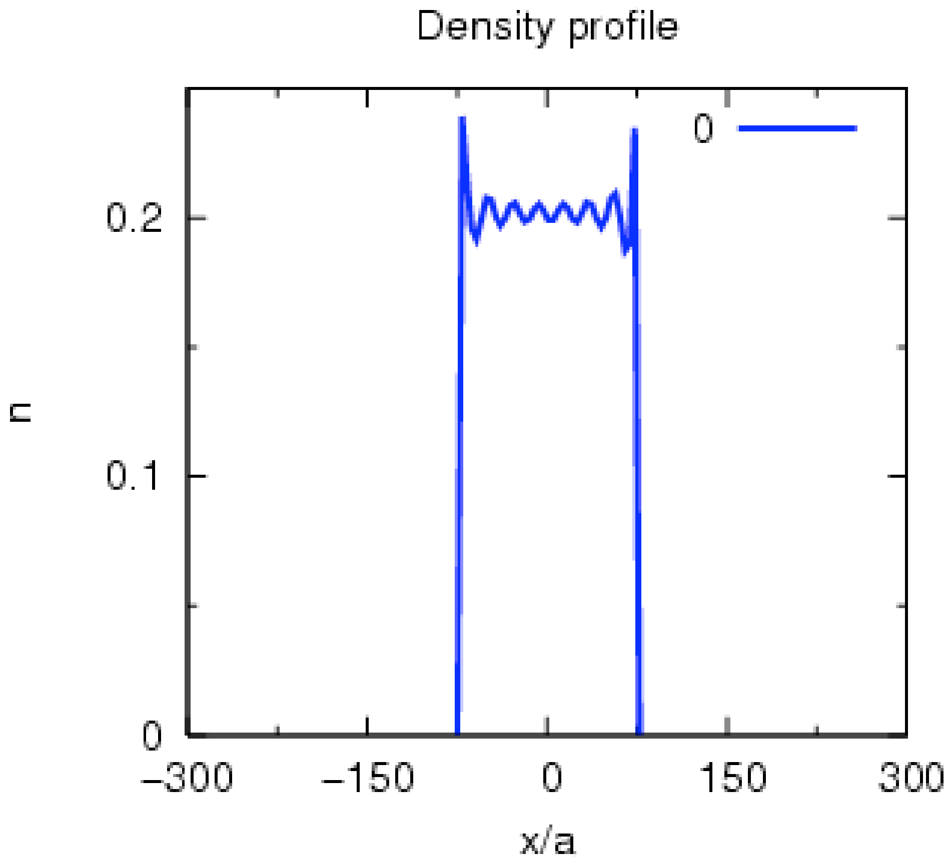

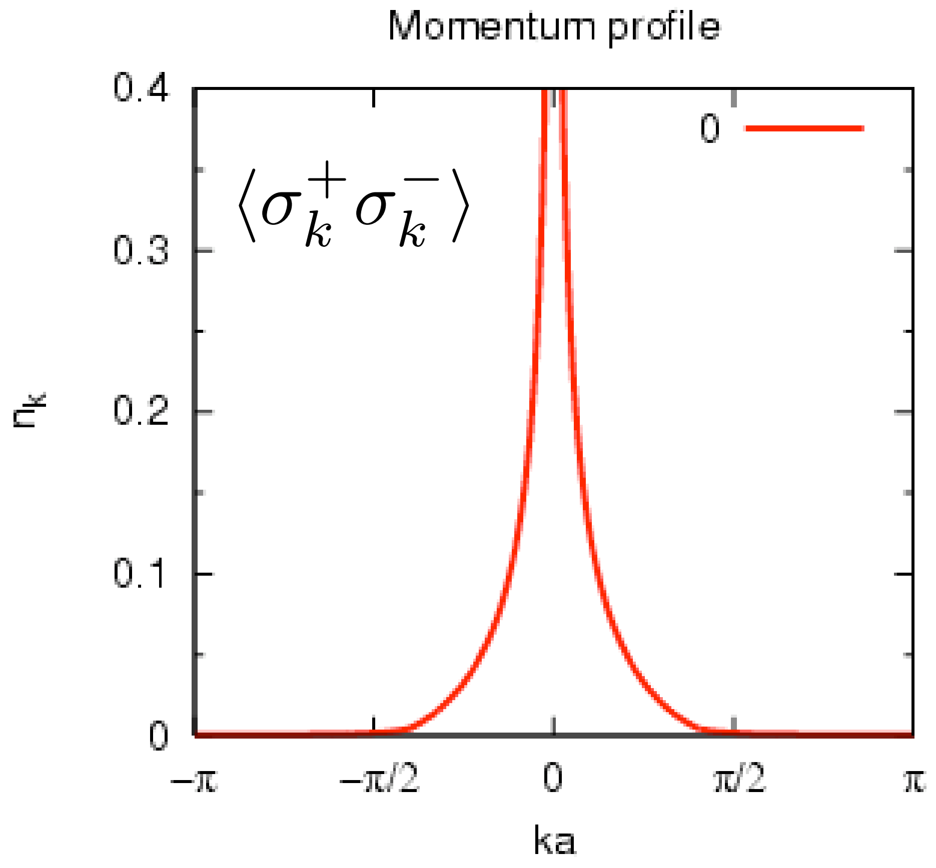

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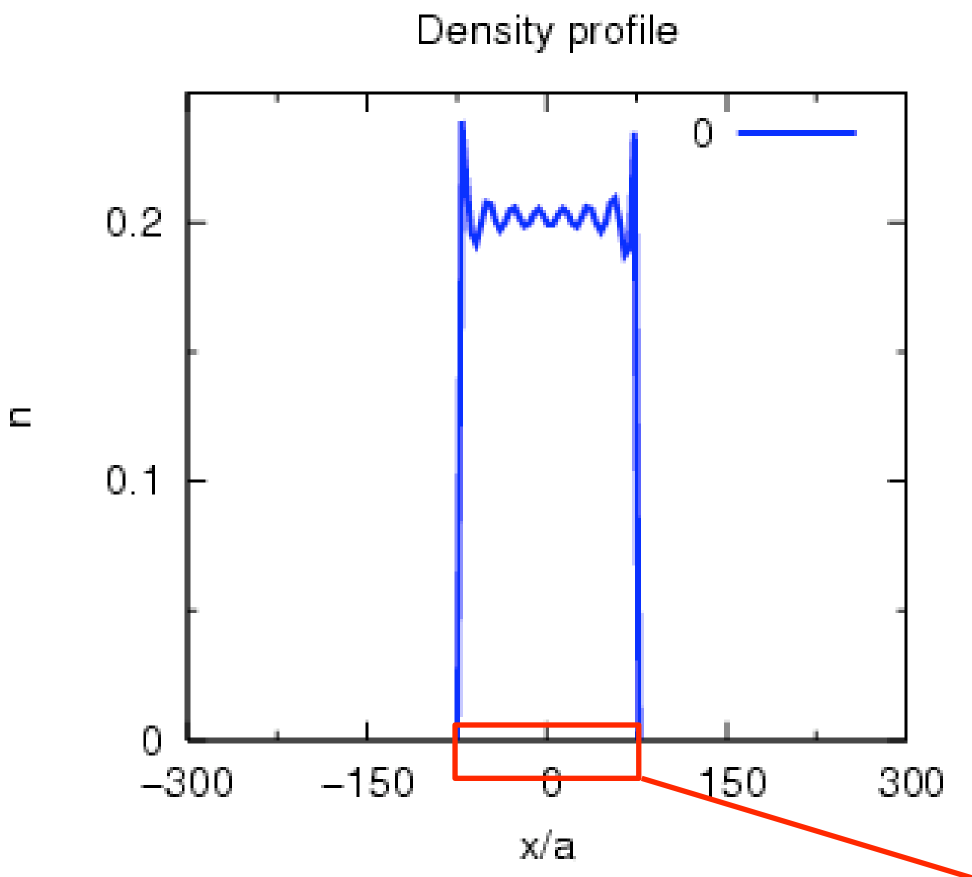

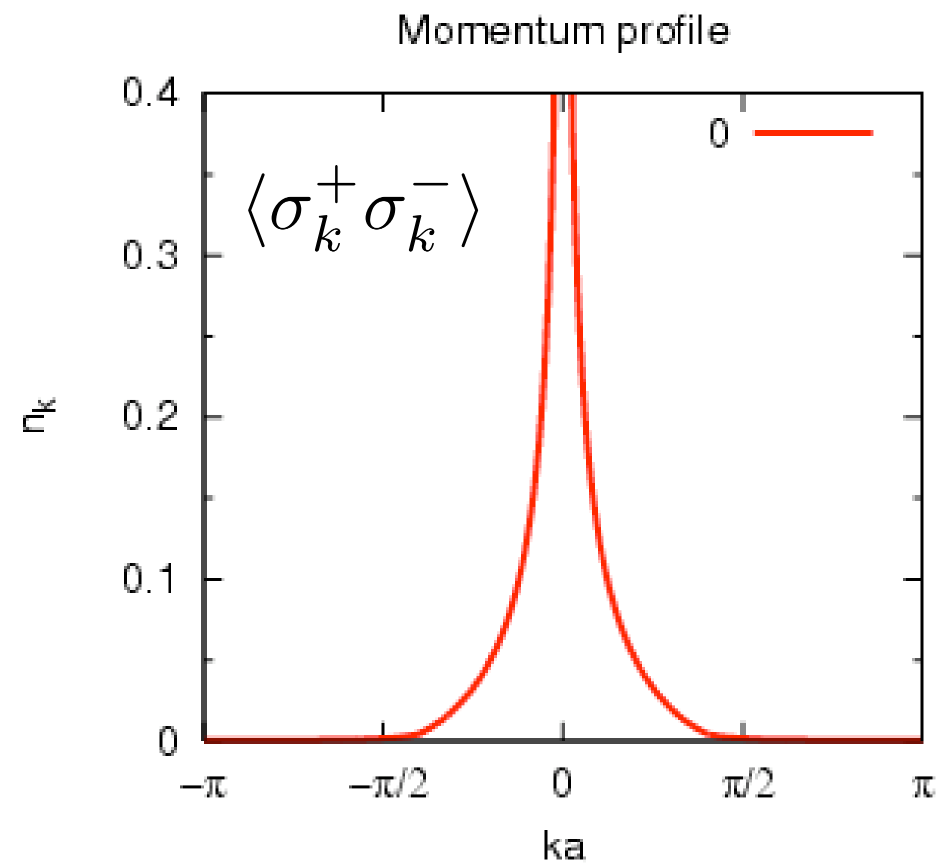

$$L(t=0) = 150\,a, N = 30$$
$$|\Phi(t=0)\rangle$$

Quantum quench in the LM

$$H_{\rm kin} = \sum_{q\neq 0} \hbar v_F |q| \; a^\dagger(q) a(q) \qquad H_{\rm LM} = \sum_{q\neq 0} \hbar v |q| \; b^\dagger(q) b(q)$$

Non-interacting fermions (t ≤ 0) Interacting fermions (t > 0)

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Non-interacting fermions (t ≤ 0)

Interacting fermions (t > 0)

Equilibrium solution $$b(q) = \cosh\varphi(q)\, a(q) + \sinh\varphi(q)\, a^\dagger(-q)$$

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Non-equilibrium (quench) solution

$$\begin{aligned} a(q,t) &= e^{iH_{LM}t/\hbar} a(q) e^{-iH_{LM}t/\hbar} = f(q,t) a(q) + g^*(q,t) a^\dagger(-q), \\ f(q,t) &= \cos v|q|t - i \sin v|q|t\ \cosh 2\varphi(q), \\ g(q,t) &= i \sin v|q|t \sinh 2\varphi(q) \end{aligned}$$

[MAC, PRL 97 (2006)]

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[MAC, PRL 97 (2006)]

One-particle density matrix

$$C_{\psi_r}(x, t > 0) = \langle 0| e^{iH_{LM}t/\hbar} \psi_r^\dagger(x)\psi_r(0) e^{-iH_{LM}t/\hbar} |0\rangle_{\mathrm{Dirac}}$$

Quench in the LM

~ Interaction range

[MAC, PRL 97 (2006)]

Thermodynamic limit

$$C_{\psi_r}(x, t > 0) = C^{\text{free}}_{\psi_r}(x) \, \left|\frac{R}{x}\right|^{\gamma^2} \left|\frac{x^2 - (2vt)^2}{(2vt)^2}\right|^{\gamma^2/2}$$

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[MAC, PRL 97 (2006)]

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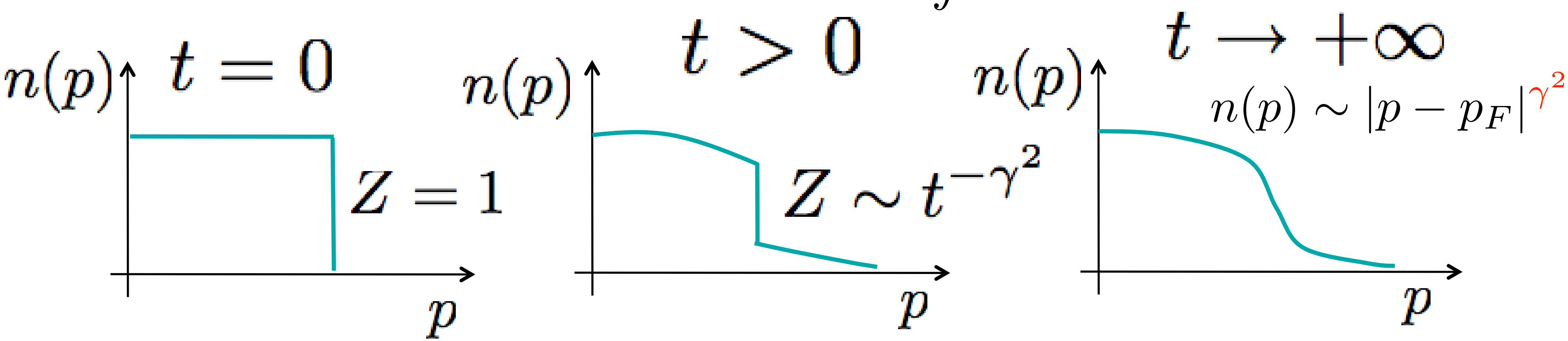
Momentum distribution at time t

$$n(p,t) = \int dx\, e^{-ipx} C_{\psi_r}(x,t>0)$$



Quench in the LM

[MAC, PRL 97 (2006)]

Thermodynamic limit

~ Interaction range (R)

$$C_{\psi_r}(x,t>0) = C^{\text{free}}_{\psi_r}(x)\left|\frac{R}{x}\right|^{\gamma^2}\left|\frac{x^2-(2vt)^2}{(2vt)^2}\right|^{\gamma^2/2}$$

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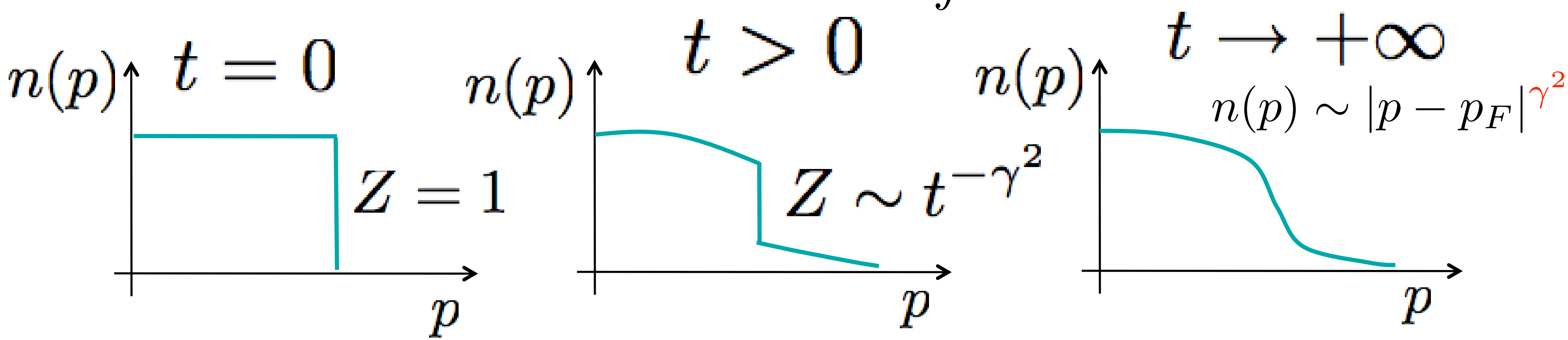

Non-equilibrium exponent $\gamma^2 = \sinh^2 2\varphi > \gamma^2_{\text{eq}} = 2\sinh^2\varphi$

Quench in the LM

~ Interaction range

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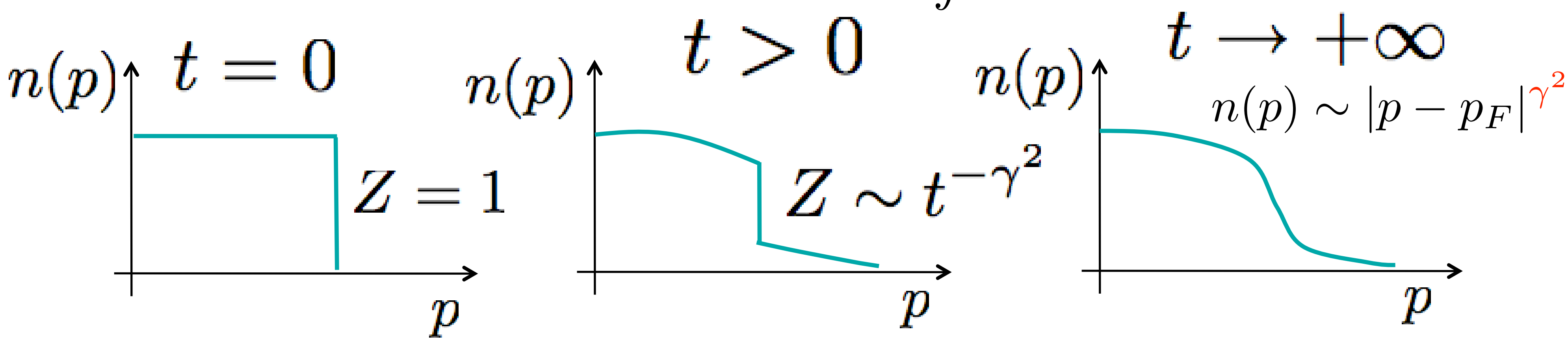

Non-equilibrium exponent

$$\gamma^2 = \frac{1}{4}\left(K - K^{-1}\right)^2 > \gamma_{eq} = \frac{1}{2}\left(K - K^{-1} - 2\right)$$

Quench in the LM

~ Interaction range

[MAC, PRL 97 (2006)]

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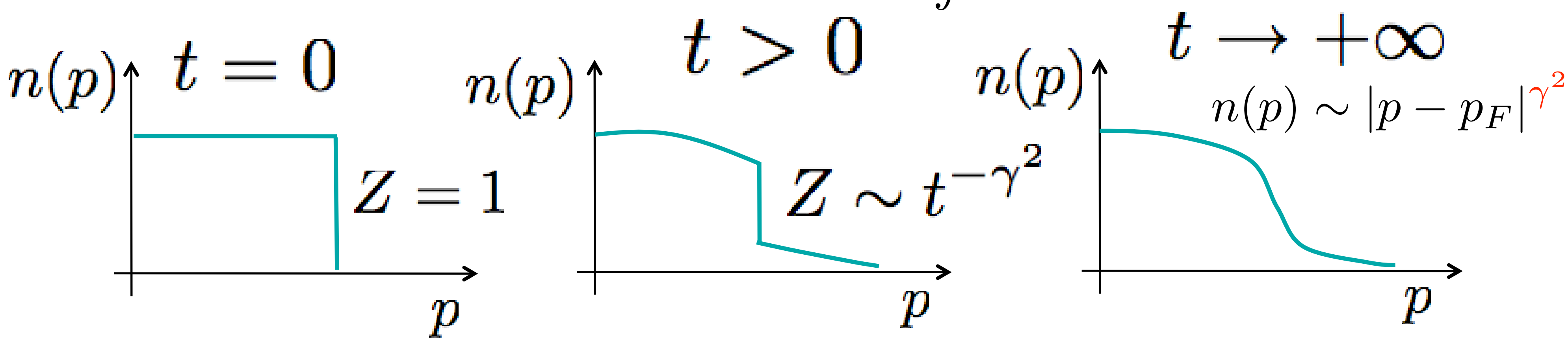
Momentum distribution at time t $\quad n(p,t) = \int dx\, e^{-ipx} C_{\psi_r}(x, t>0)$



Non-equilibrium exponent $\quad \gamma^2 = \frac{1}{4}\left(K - K^{-1}\right)^2 > \gamma_{eq} = \frac{1}{2}\left(K - K^{-1} - 2\right)$

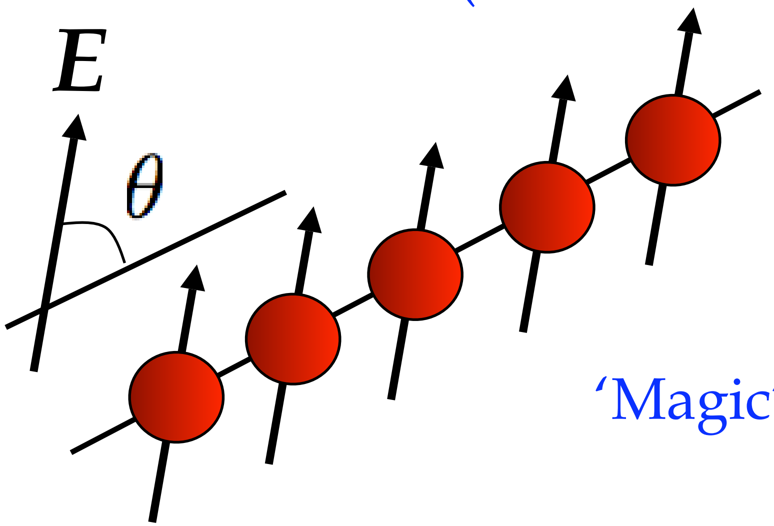
Maximum entropy $\quad \rho_{\text{gG}} = \frac{e^{\sum_q \lambda_q I_q}}{Z_{\text{gG}}}, \quad I_q = b^\dagger(q) b(q)$

$$\lim_{t\to+\infty} C_{\psi_r}(x,t) = C^{\text{gG}}_{\psi_r}(x) = \text{Tr}\, \rho_{\text{gG}}\, \psi^\dagger_r(x)\psi(0)$$

Relevant (to) experiments?

1D dipolar gas of (spin polarized) fermionic atoms/molecules

(not the LM but a Tomonaga-Luttinger liquid)



$$V_{\rm dip}(x) \simeq \frac{1}{4\pi\epsilon_0}\frac{D^2(1-3\cos\theta)}{[x^2+R^2]^{3/2}}$$

'Magic' angle : $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\rm dip} = 0$

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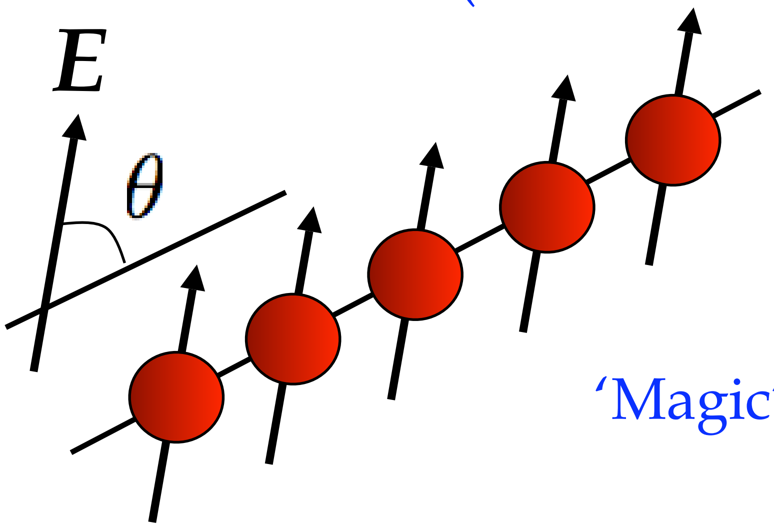

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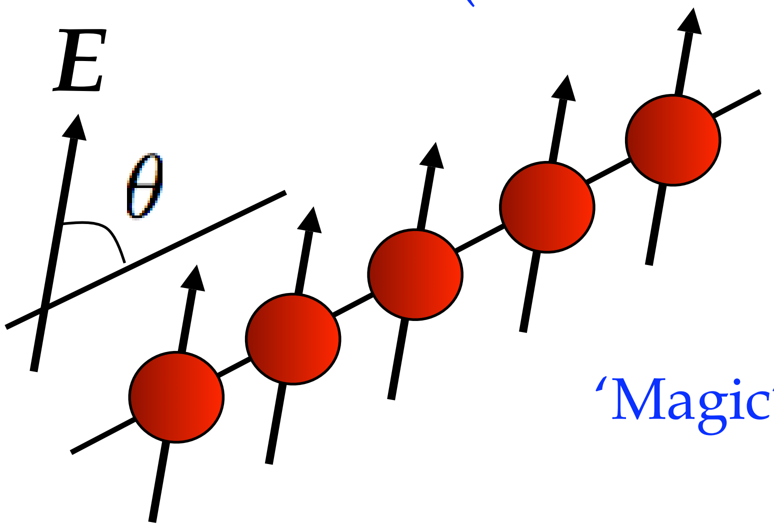
'Magic' angle : $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\rm dip} = 0$

Quench (very rapid change in the direction of E away from magic angle)

Relevant (to) experiments?

1D dipolar gas of (spin polarized) fermionic atoms/molecules

(not the LM but a Tomonaga-Luttinger liquid)



$$V_{\rm dip}(x) \simeq \frac{1}{4\pi\epsilon_0}\frac{D^2(1-3\cos\theta)}{[x^2+R^2]^{3/2}}$$

'Magic' angle : $\theta_m = \cos^{-1}\left(\frac{1}{3}\right) \Rightarrow V_{\rm dip} = 0$

Quench (very rapid change in the direction of E away from magic angle)

vs.

Relevant (to) experiments?

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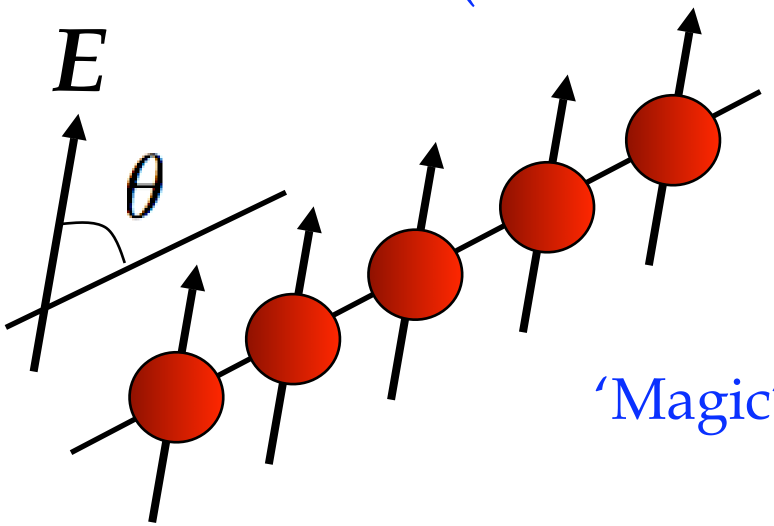

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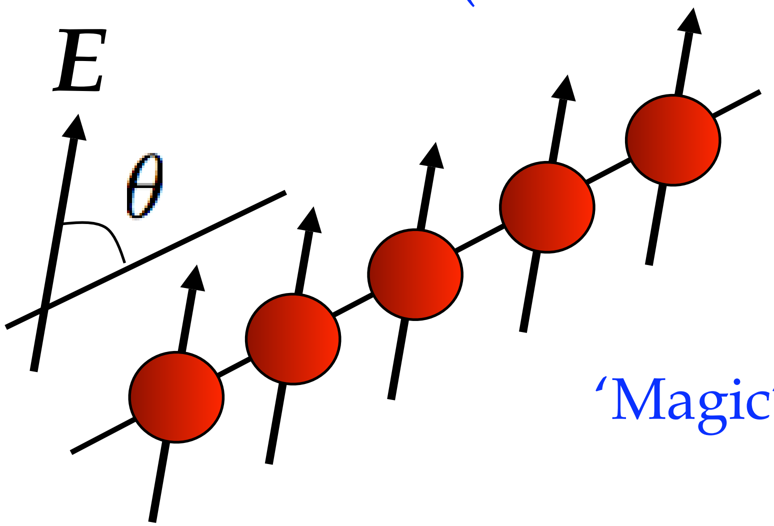

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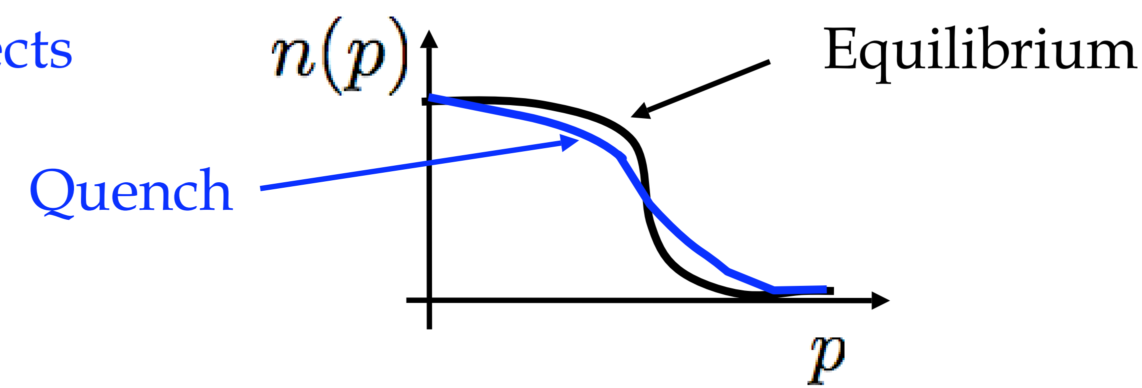
Finite temperature effects

$$t_{\rm Relax} \simeq \frac{\hbar}{T}$$



[MAC, PRL 97 (2006)]

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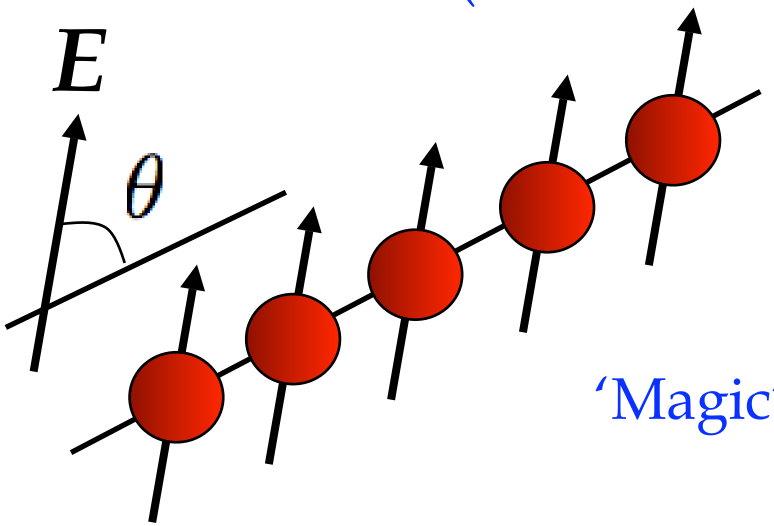

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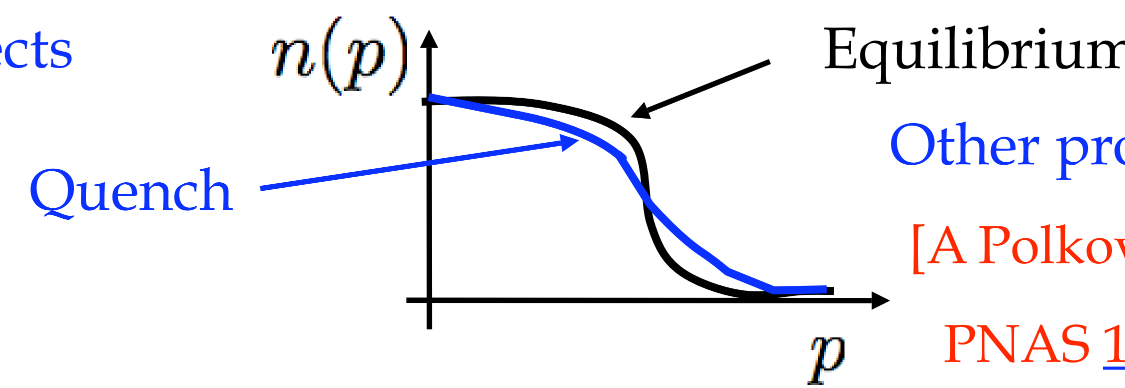


Other probes: Noise

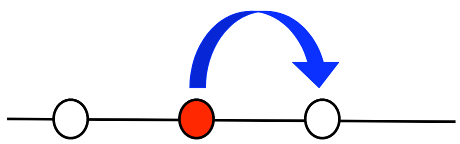
[A Polkovnikov et al PNAS 103 (2006)]

Summary

- Ultracold atomic gases are highly controllable quantum systems with long coherence times.
- True one-dimensional models can be realized using e.g. optical lattices.
- New situations (non-existent in the solid state) can be created: conserved magnetization, non-equilibrium states.
- We can address old and new problems: phase diagrams and phase properties and thermalization in absence of reservoirs.

Tonks regime: An effective Hamiltonian

$$n_m \leq 1 \qquad \gamma = \frac{U}{J} \gg 1$$



$$H_F = -\frac{J}{2}\sum_{\langle m,n\rangle} f_m^\dagger f_n + H_{\rm int}^1 + H_{\rm int}^2,$$

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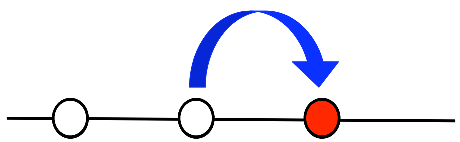

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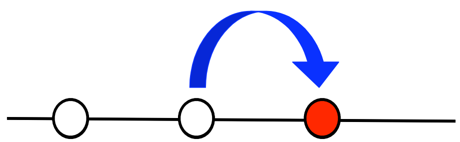

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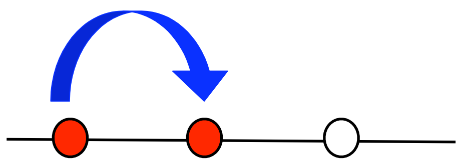

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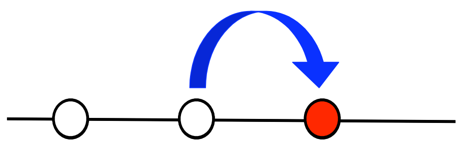

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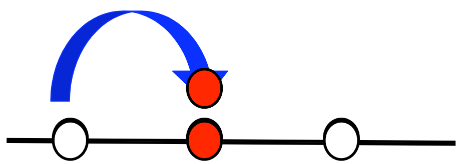

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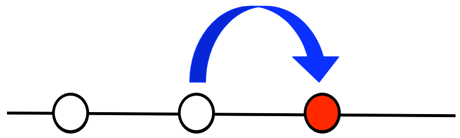

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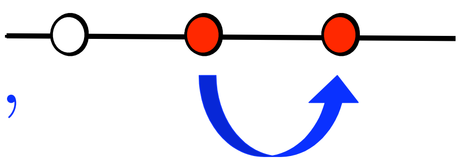

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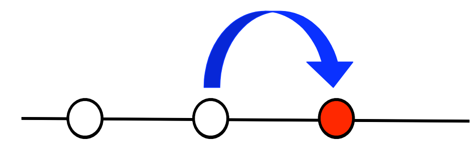

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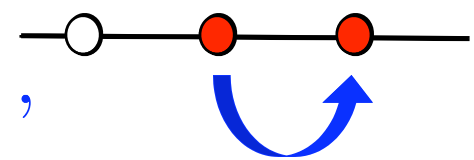

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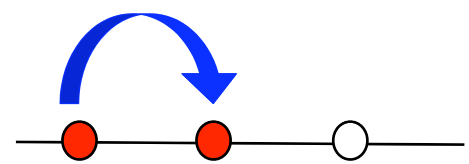

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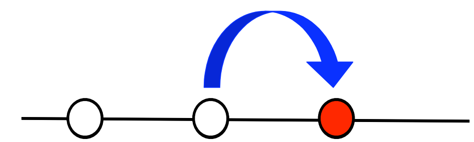

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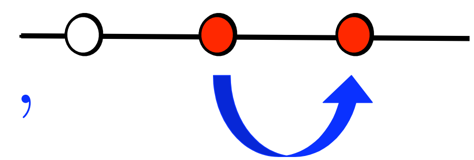

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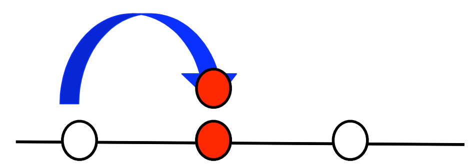

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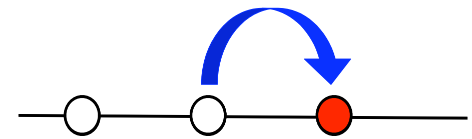

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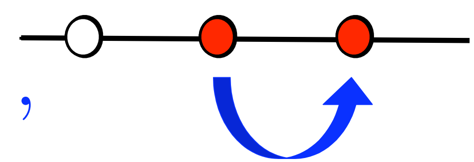

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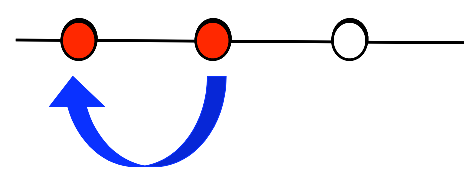

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