INVERSE TRANSPORT FROM ANGULARLY AVERAGED MEASUREMENTS AND TIME HARMONIC ISOTROPIC SOURCES

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Inverse transport consists in reconstructing optical parameters -such as the total absorption $\sigma(x)$, where x is position, and the scattering coefficient $k(x, v, v')$, indicating the strength of scattering from direction v' into direction v at position x- from measurements of the density of particles $u(x, v)$ -typically for outgoing particles at a domain's boundary- generated by sources $f(x, v)$ -typically prescribing incoming particles into the domain.

Uniqueness of the identification of the optical parameters in a domain has been established in many settings where the source is allowed to be arbitrary in the phase space (x, v) for x at the domain's boundary and the measurements are spatially and angularly resolved at a domain's boundary. In practice, however, sources are not arbitrary and measurements are not angularly resolved, either because it would take too long to perform all measurements, or because particle counts would be too low, hence data too noisy. Yet no theory of uniqueness of identification exists in such settings.

I will present recent results obtained in collaboration with Ian Langmore and François Monard showing uniqueness of the reconstruction of $k = k(x)$ with $\sigma(x)$ known from diffusion-type measurements, i.e., from measurements of spatial currents in experiments involving isotropic sources $f = f(x)$. I will consider the cases of boundary measurements as well as volume measurements outside of the domain where $k(x)$ is to be reconstructed.

The reconstruction of optical parameters from angularly averaged measurements is severely ill-posed, in contrast to reconstructions from angularly resolved measurements where Hölder-type stability estimates are sometimes available. One possible way to remedy such ill-posedness is to use time harmonic sources $f = e^{i\omega t} f(x, v)$. It was shown in recent work in collaboration with Andreas Hielscher and Kui Ren that time harmonic sources allowed us to obtain sharper reconstructions of the optical parameters than CW sources (when $\omega = 0$). I will review available results and present theoretical explanations for such a behavior.