Competition between magnetic and pairing exchange in confined systems

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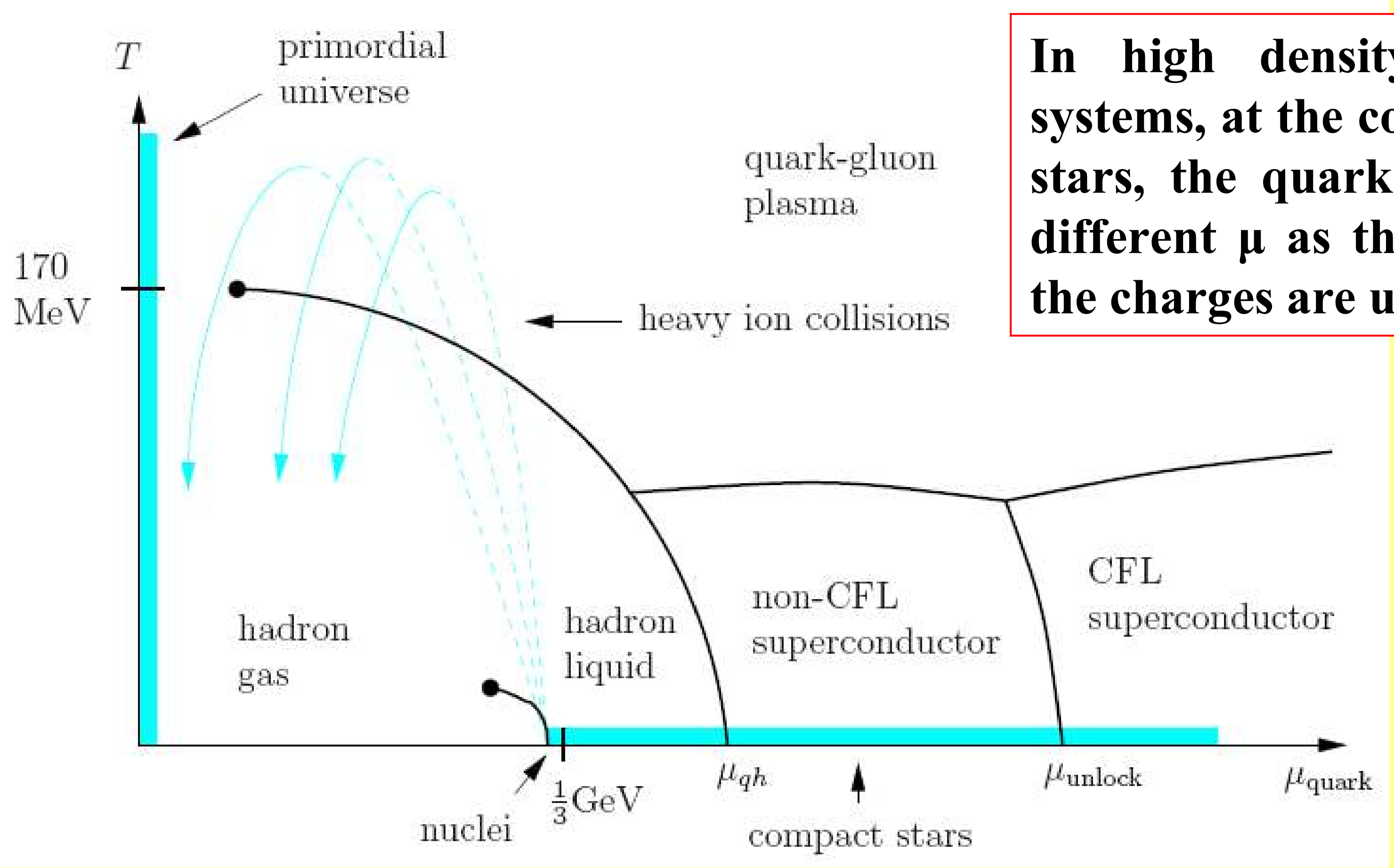
- ✓ Motivation: pairing with mismatched chemical potential @ coexistence of pairing and ferromagnetic correlations
- ✓ Mechanisms for the chemical potential imbalance of the two spin states: ferromagnetic exchange, mass asymmetry, external field
- ✓ Description: generalization of the Richardson model with inclusion of different depairing mechanisms
- ✓Mapping out the ground state diagram: exact solution
- ✓ Conclusions

Relevant systems: pairing with mismatched chemical potential

- Cold dense quark at the core of neutron stars
- Magnetized superconductors
- Ferromagnetic superconductors
- Ultracold atomic gas of fermions with unequal numbers of two components

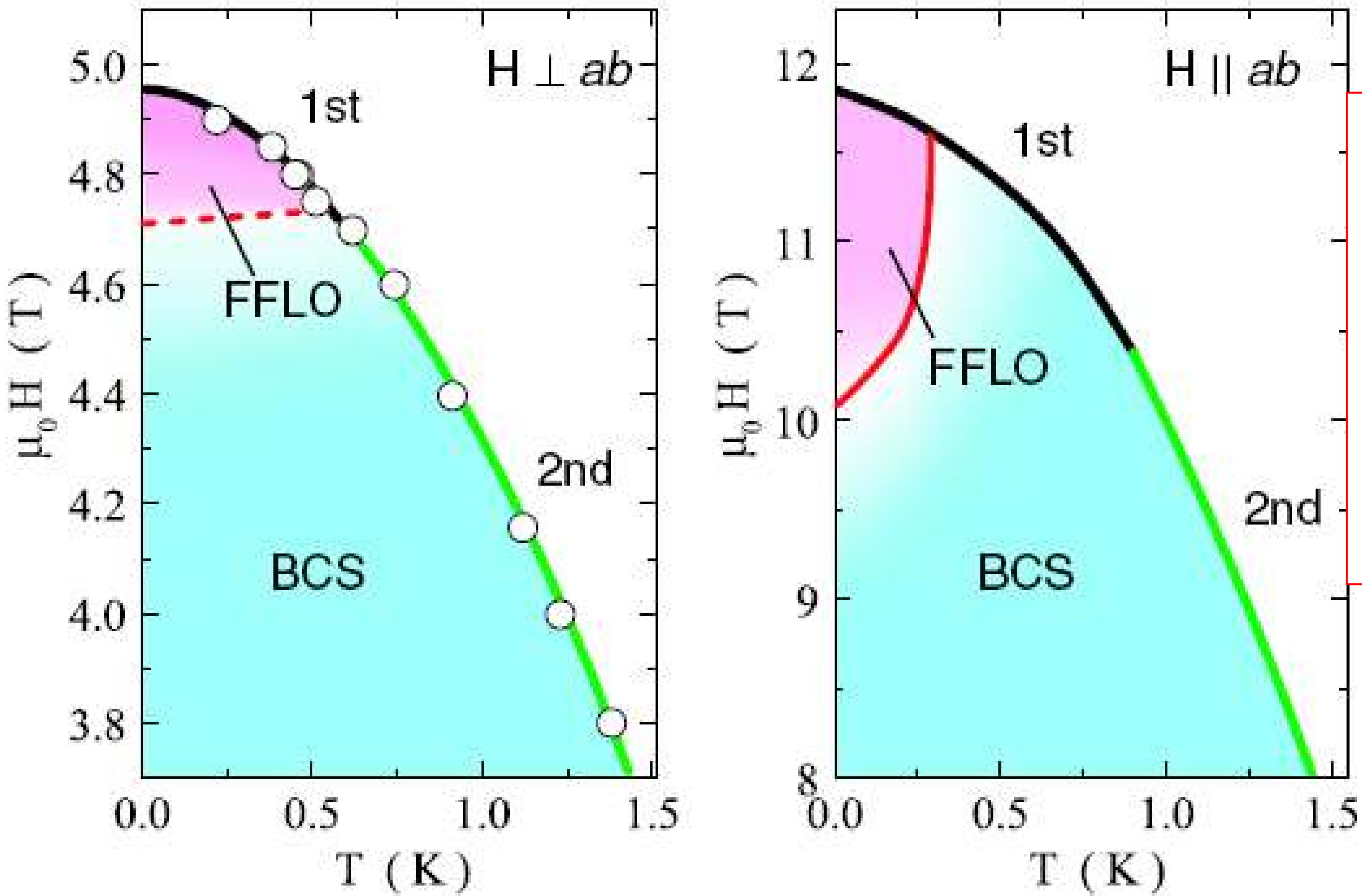
Relevant systems: pairing with mismatched chemical potential- 0



In high density interacting systems, at the core of neutron stars, the quark flavors have different μ as the masses and the charges are unequal

For references see: R. Caslbuoni and G. Nardulli, RMP 76, 263 (2004).

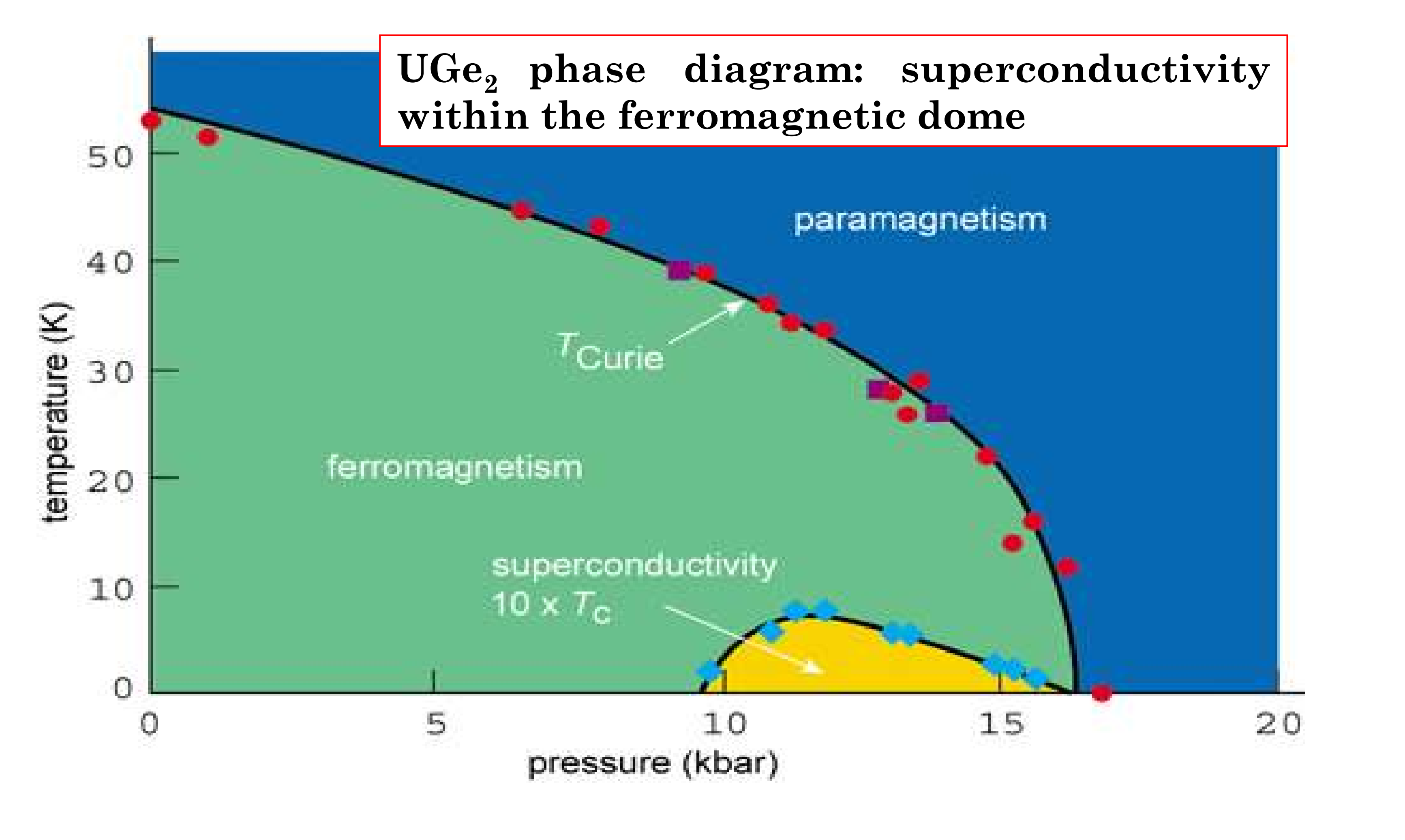
Relevant systems: pairing with mismatched chemical potential- I



Evidence of Fulde-Ferrell-Larkin-Ovchinnikov phase in $CeCoIn_5$

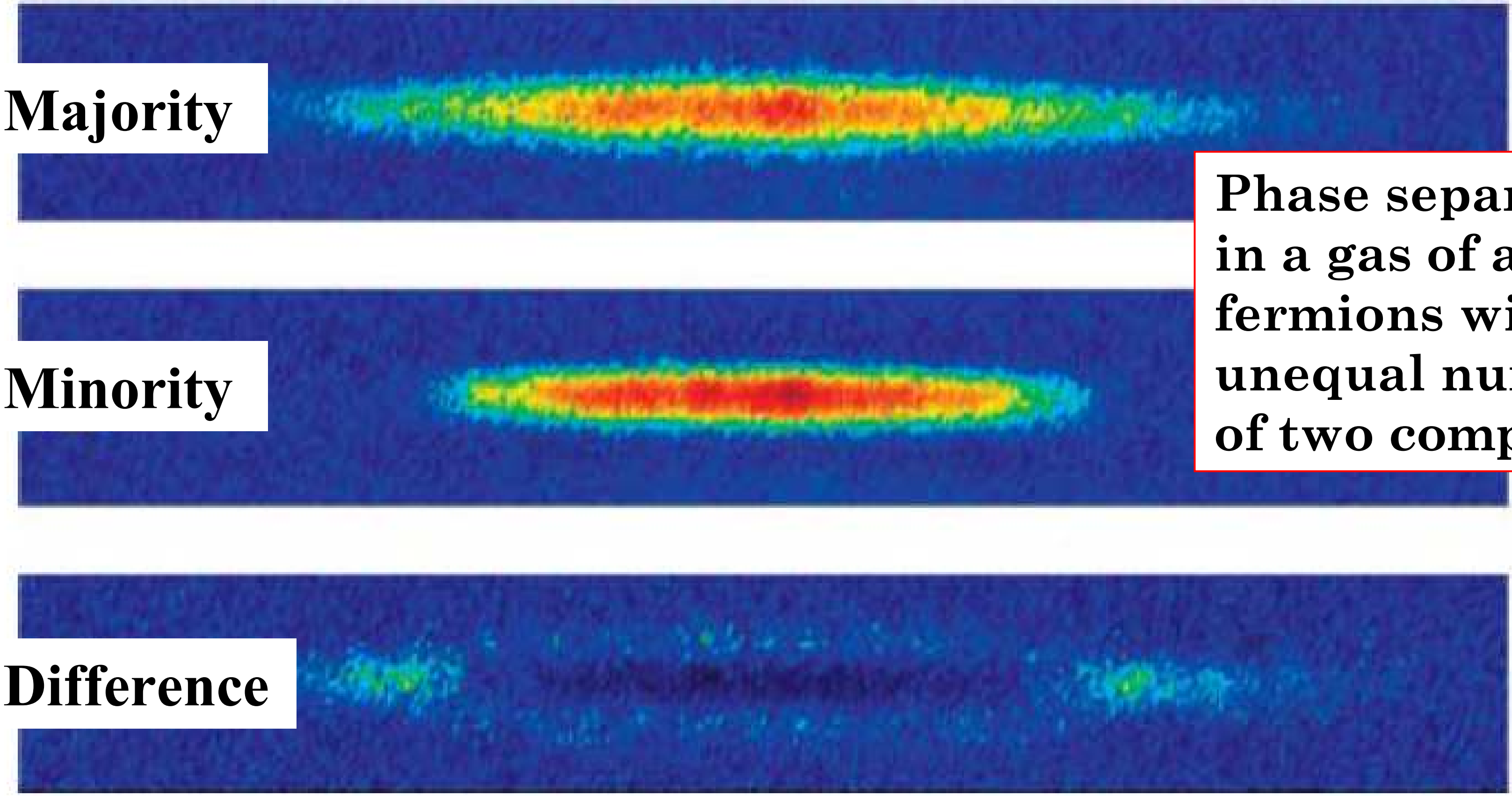
K. Kumagai, M. Saitoh, T. Oyaizu, Y. Furukawa, S. Takashima, M. Nohara, H. Takagi, and Y. Matsuda PRL 97, 227002 (2006)

Relevant systems: pairing with mismatched chemical potential -II



S.S. Saxena et al., Nature 406, 587(2000)

Relevant systems: pairing with mismatched chemical potential-III



Phase separation in a gas of atomic fermions with unequal numbers of two components

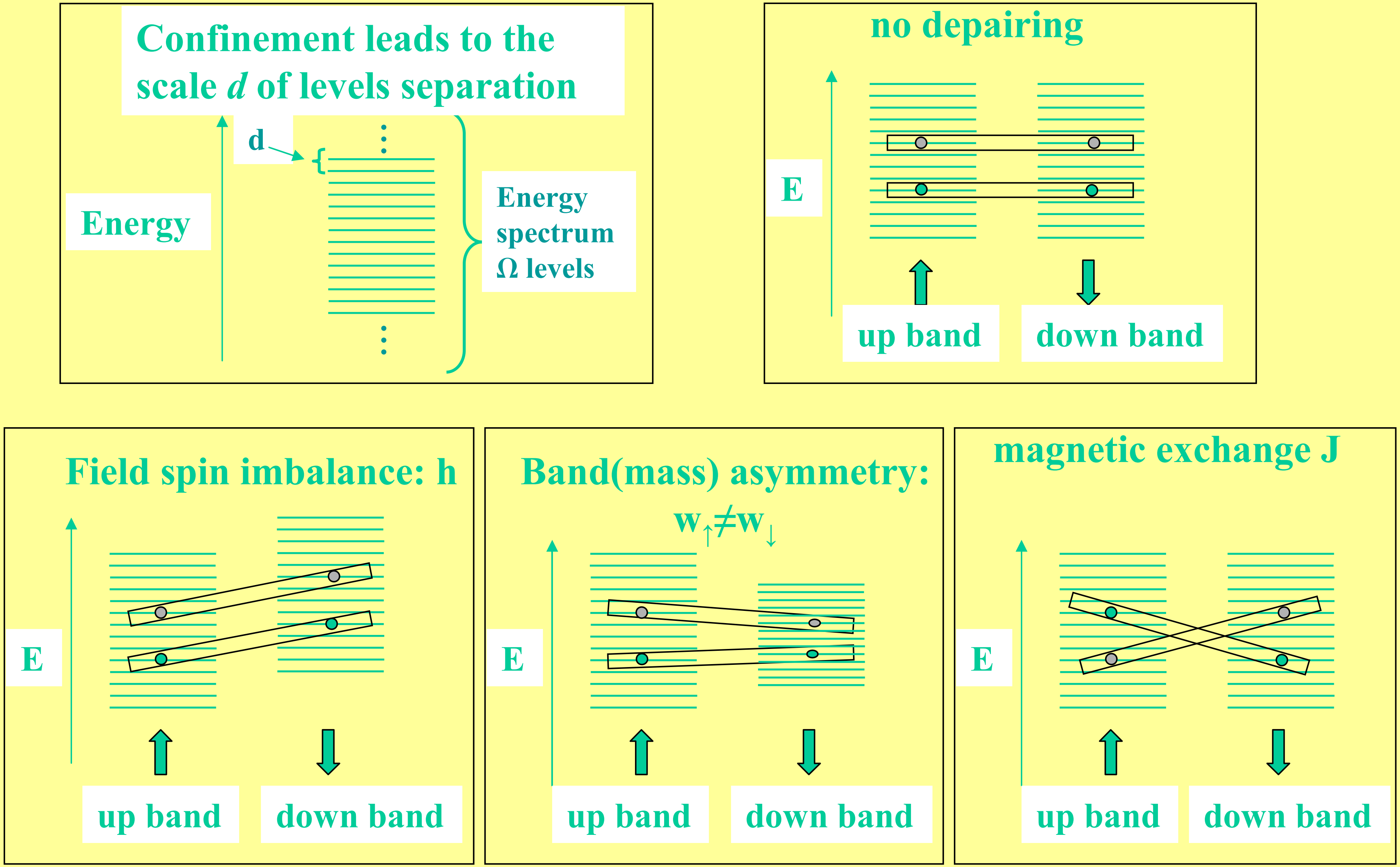
G.B. Partridge, W. Li, Ramsey I. Kamar, Y. Liao, R.G. Hulet Science 311, 503 (2006)

Spin imbalance: mechanisms and possible ground state configurations

- Applied field
- Ferromagnetic exchange
- Mass asymmetry or kinetic energy imbalance
- Spin population imbalance

- Fulde-Ferrell-Larkin-Ovchinnikov state: pairs possess a nonzero center of mass momentum that breaks translational invariance
- Homogeneous Sarma or breached pair state: phase with gapless excitations
- Mixed phase: regions of a paired BCS superfluid are surrounded by an unpaired normal particles
- States with phase separation

Microscopic ingredients



Effective modelling: generalized Richardson Hamiltonian

$$\mathbf{H} = \sum_{j,\sigma=+,-} w_\sigma \epsilon_j \; c^\dagger_{j\sigma} c_{j\sigma} - g \sum_{j,j'} c^\dagger_{j+} c^\dagger_{j-} c_{j'-} c_{j'+} - J \sum_{j,j'} \hat{S}_j \cdot \hat{S}_{j'} - h \, \hat{S}^z$$

- w_σ stands for the factor controlling the bandwidths amplitude for different spin polarizations
- g describes the electron-electron interaction via pairing
- J describes the electron-electron interaction via magnetic exchange
- h is the Zeeman energy in the z direction, acting as a chemical potential spin imbalance

the problem is exactly solvable

Z.Ying, M.Cuoco, C.Noce, H.Zhou PRB 74, 012503 (2006)
Z.Ying, M.Cuoco, C.Noce, H.Zhou PRB 74, 214506 (2006)

Sketching exact solution: 1

The Hamiltonian H can be rewritten into two commuting parts $H = H_T + H_S$ where:

$$
\begin{aligned}
H_T &= \sum_j (w_+ + w_-)\epsilon_j \hat{T}_j^z - \frac{1}{2} g \sum_{j,k} (\hat{T}_j^+ \hat{T}_k^- + \hat{T}_k^+ \hat{T}_j^-), \\
H_S &= \sum_j (w_+ - w_-)\epsilon_j \hat{S}_j^z - J \sum_{j,j'} \hat{S}_j \cdot \hat{S}_{j'} - h\, \hat{S}^z,
\end{aligned}
$$

- Singly-occupied levels do not participate in the pair scattering, thus staying “blocked” according to the Pauli principle
- Similarly, the double (empty) states do not enter the spin dynamics
- the effective magnetic field $(w_+ - w_-)\epsilon_j$ in the spin channel corresponds to the kinetic energy $(w_+ + w_-)\epsilon_j$ of a pair, and the transverse part of the magnetic exchange J has its counterpart in the pairing amplitude g

Sketching exact solution: 2

$N = 2(n+m)$ electrons → $2m$ electrons fill a set B of singly occupied

→ n pairs are distributed among the set U of $N_U = \Omega - 2m$ unblocked levels

- a generic eigenstate of H can be expressed as $|n,m\rangle = \prod_{\beta=1}^{m+S^z} |\psi_\beta\rangle \prod_{\mu=1}^{n} |\psi_\mu\rangle$ where:

$$|\psi_\beta\rangle = \sum_{j\epsilon B} \frac{\hat{S}_j^+}{(w_+ - w_-)\epsilon_j - \overline{E}_\beta} |-\rangle ,$$

$$|\psi_\mu\rangle = \sum_{j\epsilon U} \frac{c_{j+}^+ c_{j-}^+}{(w_+ + w_-)\epsilon_j - E_\mu} |0\rangle .$$

- $|-\rangle = \prod_{i\epsilon B} c_{i-}^+ |0\rangle$, with $|0\rangle$ being the vacuum state, and S^z is the z projection of the total spin of the electrons in the blocked levels
- the n parameters E_μ and the $m+S^z$ $\overline{E}_\beta$ are the solutions of the two sets of the Richardson equations

$$\frac{1}{g} + \sum_{\nu=1(\nu\neq\mu)}^{n} \frac{2}{E_\nu - E_\mu} = \sum_{j\in U} \frac{1}{(w_+ + w_-)\epsilon_j - E_\mu} ,$$

$$\frac{1}{J} + \sum_{\alpha=1(\alpha\neq\beta)}^{m+S^z} \frac{2}{\overline{E}_\alpha - \overline{E}_\beta} = \sum_{j\in B} \frac{1}{(w_+ - w_-)\epsilon_j - \overline{E}_\beta} .$$

Possible ground state configurations

FM-SC → $|n, m\rangle = \prod_{\beta=1}^{m+S^z} |\psi_\beta\rangle \prod_{\mu=1}^{n} |\psi_\mu\rangle$

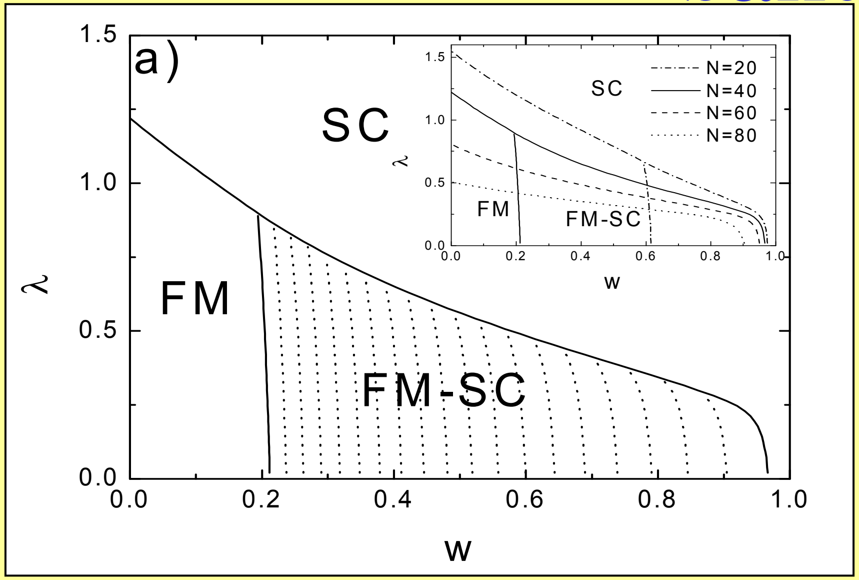
SC → $|n, 0\,\rangle = \prod_{\mu=1}^{n} |\psi_\mu\rangle$

FM → $|0\,, m\rangle = \prod_{\beta=1}^{m+S^z} |\psi_\beta\rangle$

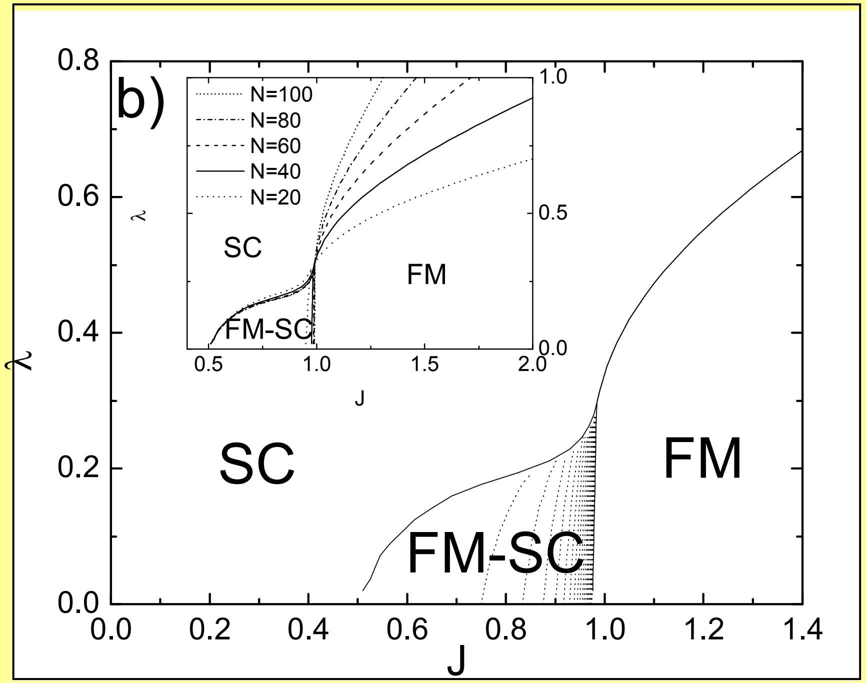
$$|\psi_\beta\rangle \;=\; \sum_{j\epsilon B} \frac{\hat{S}_j^+}{(w_+ \;\; - w_-)\epsilon_j - \overline{E}_\beta} |-\rangle\,,$$

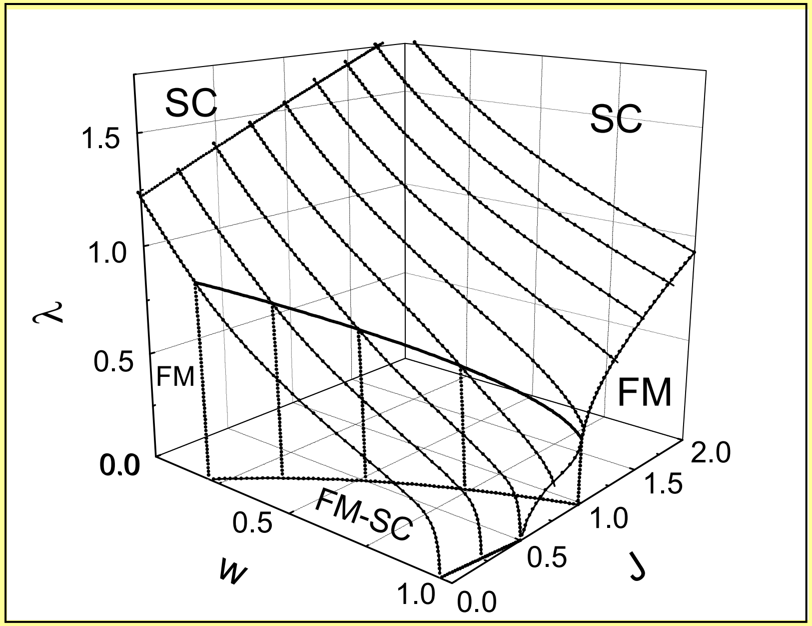
$$|\psi_\mu\rangle \;=\; \sum_{j\epsilon U} \frac{c_{j+}^+ c_{j-}^+}{(w_+ \;\; + w_-)\epsilon_j - E_\mu} |0\rangle\,.$$

Stoner exchange vs asymmetric spin dependent bandwidth



- Coexistence of polarized spins FM and SC is enhanced for the w mechanism
- For the J case: the FM-SC is limited at the region around the Stoner threshold





Strong pairing and polarizing field: role of antiferromagnetic correlations

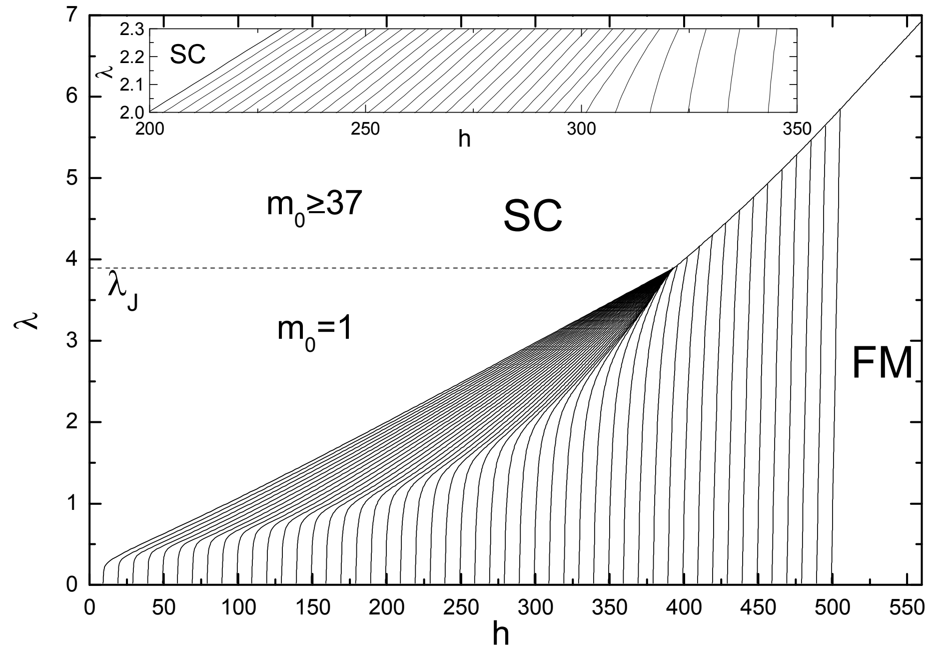
- **In general, in presence of strong coupling the FM and SC states cannot be accomodated together**

- **The chemical spin imbalance tends to enlarge the region of the ground state diagram with FM character**

- **Is it possible to induce a SC-FM configuration with moderate/strong pairing?**

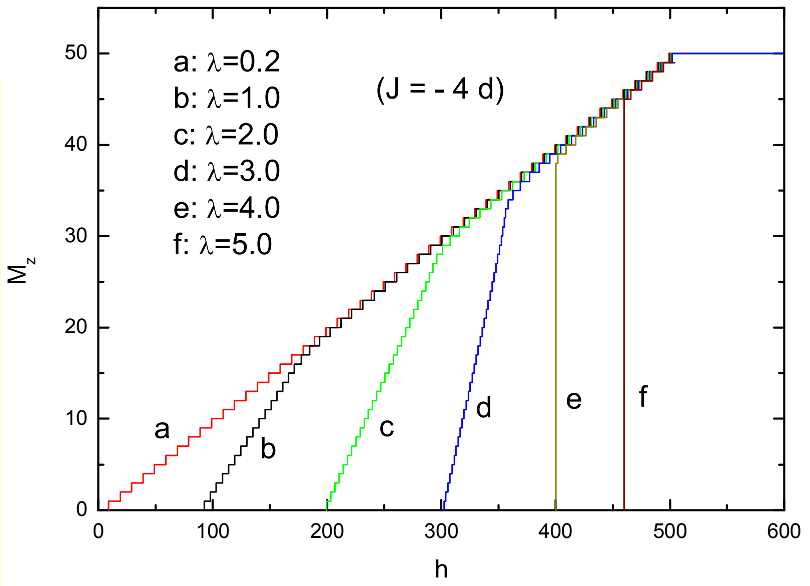
- **The antiferromagnetic exchange can represent a mechanism for softening of the depairing process**

Field response in presence of AF and pairing exchange

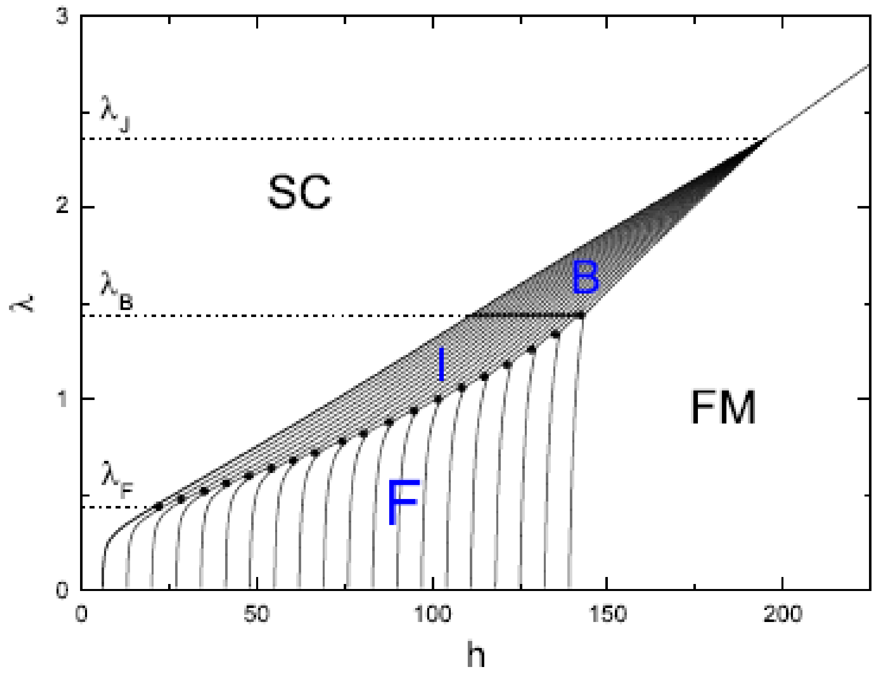


- GS diagram for J=-4d
- Two stage magnetization induced by the spin-imbalance
- Jump of the number of polarized spins at the interface of SC- SF

Critical value of λ separating smooth to sharp changeover in the competition between pairing and spin imbalance



Inhomogeneous profile of the pair distribution function in presence of AF



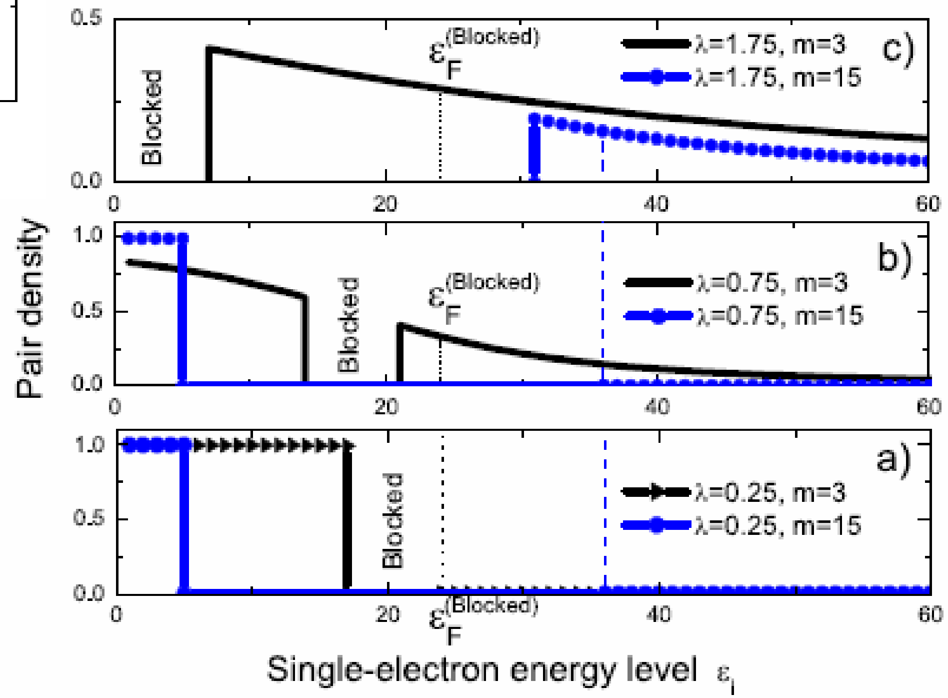
- F and I are made by two Fermi surfaces
- B is a one Fermi surface configuration
- Topological transition from F (I) to B

a) F configuration

b) I configuration

c) B configuration

→



Conclusions

- **We have obtained the ground state diagram for a system of correlated pairs in presence of spin imbalance**
- **We scan the full phase of parameters by means of the exact solution of the problem upon examination**
- **Pairing coexisting with polarized states is better realized for asymmetric spin bandwidth**
- **For strong pairing correlations in presence of a polarizing field is crucial the role of a mechanism for softening the depairing process: antiferromagnetic exchange can work for that**
- **Opening of a window for coexisting paired and polarized spins due to antiferromagnetic exchange and spin imbalance field includes states with an inhomogeneous profile in the energy distribution**