The Josephson effect through the BCS-BEC crossover

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The BCS-BEC crossover

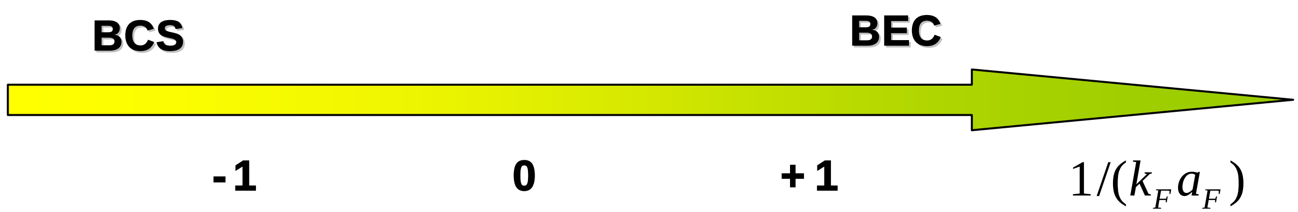
Gas of fermions interacting via an attractive potential *g(r)*.

- Weak *g*: Cooper pairs form at low temperature according to BCS picture. Largely-overlapping pairs form and condense at the same temperature (T_c).

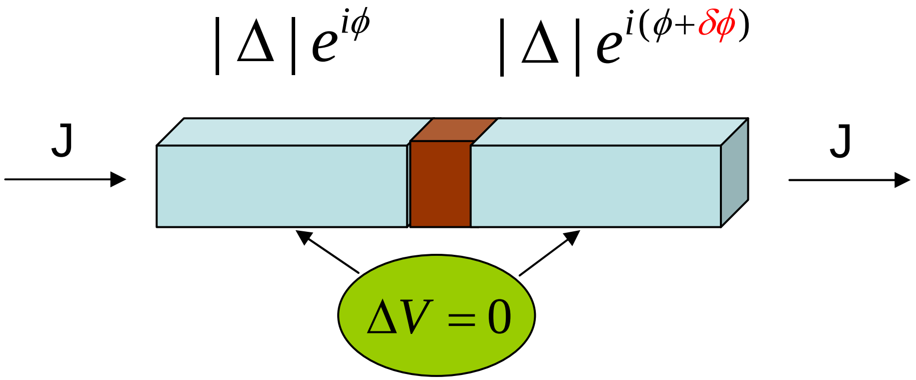
- Strong *g*: the pair-size shrinks and pair-formation is no longer a cooperative phenomenon.

Non-overlapping pairs (composite bosons) undergo Bose-Einstein condensation at low temperature. Pair-formation temperature and condensation critical temperature are unrelated.

BCS-BEC crossover realized experimentally with ultracold Fermi atoms by using appropriate Fano-Feshbach resonances. In this case the attractive potential is short-ranged and is parametrized completely in terms of the scattering length a_F. Effective coupling parameter: $1/(k_F a_F)$



The stationary Josephson effect



Join two superconductors by a weak link (e.g. a thin normal-metal or insulating barrier). A current can flow with no potential drop across the barrier if it does not exceed a critical value J_c .

The current is associated with a phase difference $\delta\phi$ of the order parameter Δ on the two sides of the barrier.

Josephson's relation:

$$J = J_c \sin(\delta\phi)$$

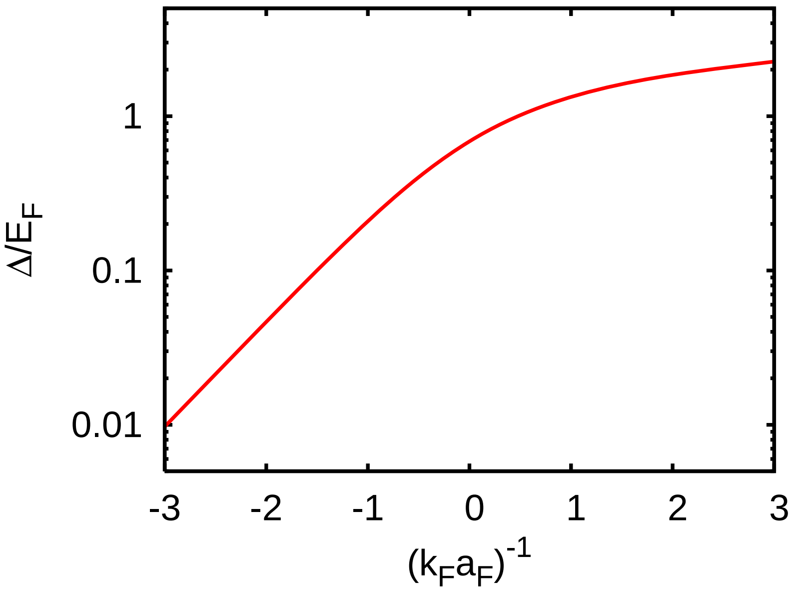
Same phenomenon occurs for two BECs separated by a potential barrier.

How does the Josephson's effect changes throughout the evolution between the two above quite different regimes?

In a BCS superconductor (weak attraction) the Josephson critical current is proportional to the order parameter:

$$J_c \propto \Delta$$

Does this remains true through the BCS-BEC crossover?



This would imply a monotonic increase of the Josephson critical current for increasing coupling strength.

Bogoliubov-de Gennes equations for superfluid fermions

For BCS superconductors, the microscopic treatment of the Josephson's effect relies on solving the BdG equations with an appropriate geometry:

$$\begin{bmatrix} \mathrm{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta(\mathbf{r})^* & -\mathrm{H}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{bmatrix} = \varepsilon_n \begin{bmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{bmatrix}$$

where

$$\mathrm{H}(\mathbf{r}) = -\frac{\nabla^2}{2m} + \mathrm{V}(\mathbf{r}) - \mu$$

and

$$\Delta(\mathbf{r}) = -g_0 \sum_n u_n(\mathbf{r}) v_n(\mathbf{r})^* \, [1 - 2 f(\varepsilon_n)]$$

SELF–CONSISTENT EQUATIONS

The BdG equations map in the BEC limit onto the GP equation for composite bosons (Pieri & Strinati PRL 2003) thus recovering the microscopic approach to the Josephson effect for the composite bosons.

The BdG equations are thus expected to provide (at least at T=0) a sensible description of the Josephson effect throughout the whole BCS-BEC crossover.

Geometry and boundary conditions

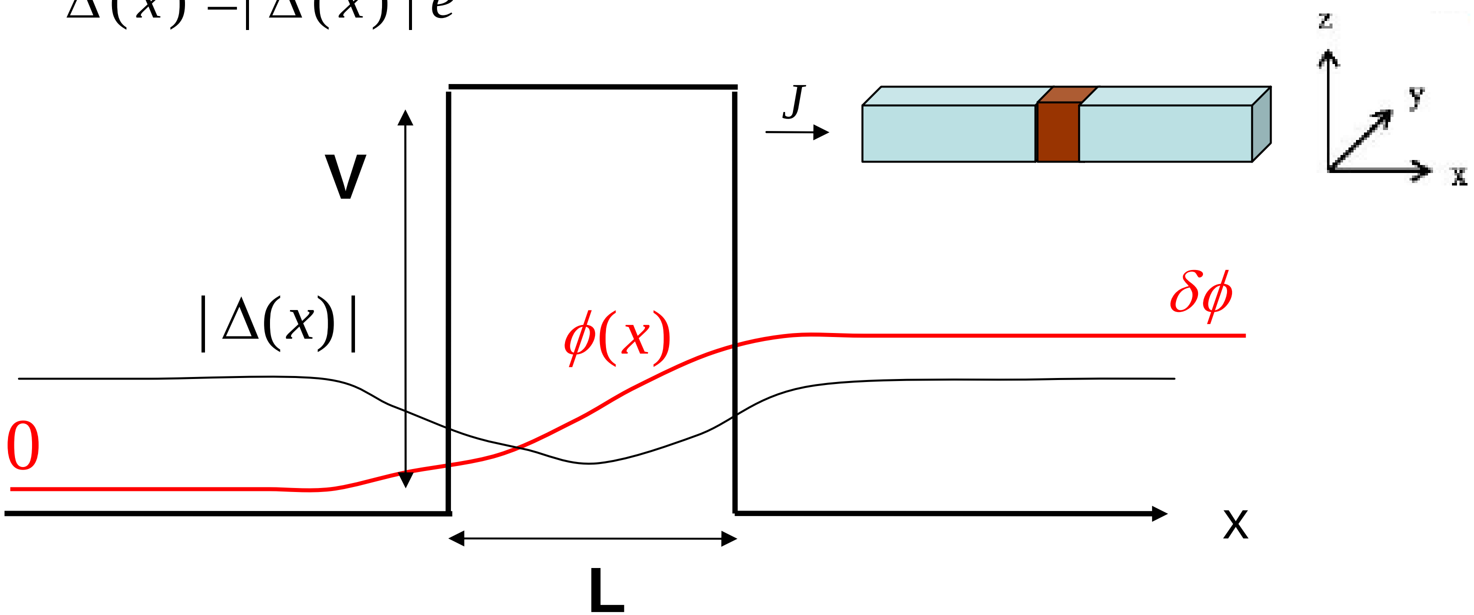
We assume the barrier to depend on one spatial coordinate only. Away from the barrier in the bulk the solution for a homogeneous superfluid flowing with velocity $\mathrm{v} = q/m$ (current $J = nq/m$) should be recovered.

We have thus the boundary conditions:

$$\Delta(x \to -\infty) = \Delta e^{2iqx}$$
$$\Delta(x \to +\infty) = \Delta e^{2iqx + i\delta\phi}$$

The order parameter $\Delta(x)$ accumulates a phase shift $\delta\phi$ across the barrier.
We set:

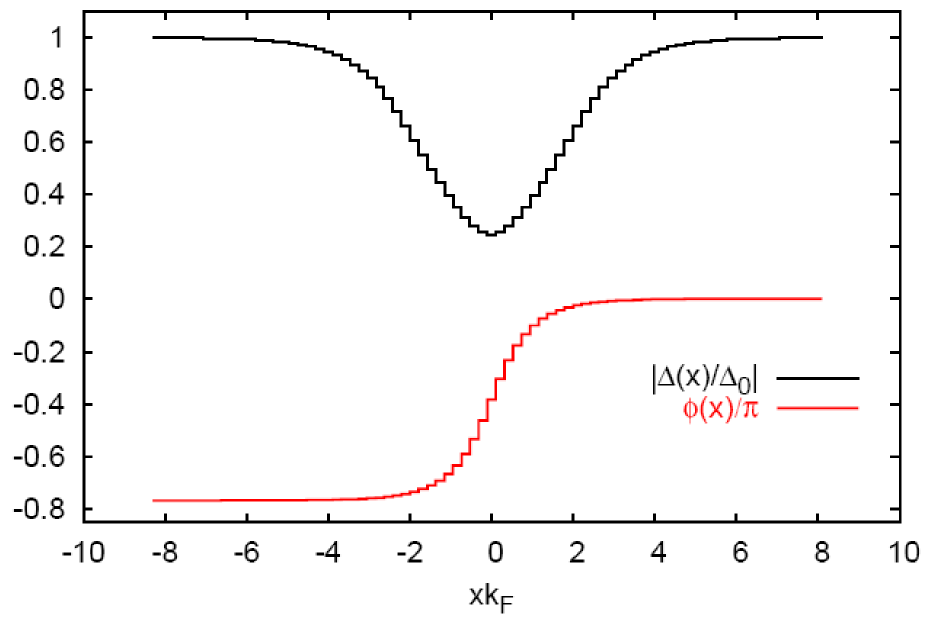
$$\Delta(x) = |\Delta(x)|\, e^{2iqx + i\phi(x)}$$



Numerical procedure

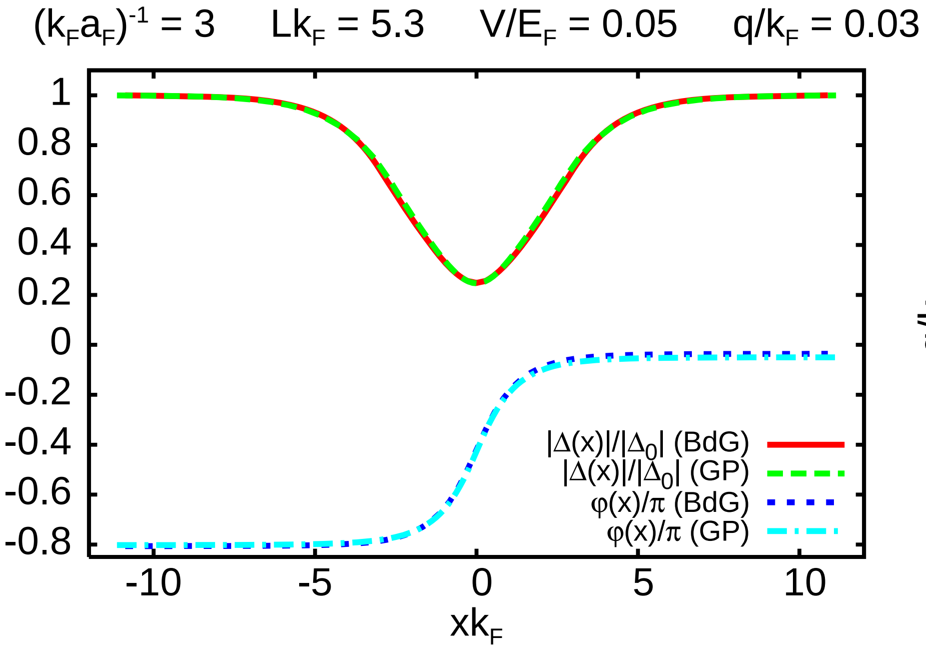
- Approximate $\Delta(x)$ with a sequence of steps (typically 80).
- In each region the solutions of BdG eqs. are plane waves.
- Impose continuity conditions at the boundaries of each region and boundary conditions at infinity.
- Integrate over continuous energies (scattering states) + discrete sum over Andreev-Saint James bound states and enforce self-consistency on a less dense grid (typically 20).
- At convergence calculate the current from the expression:

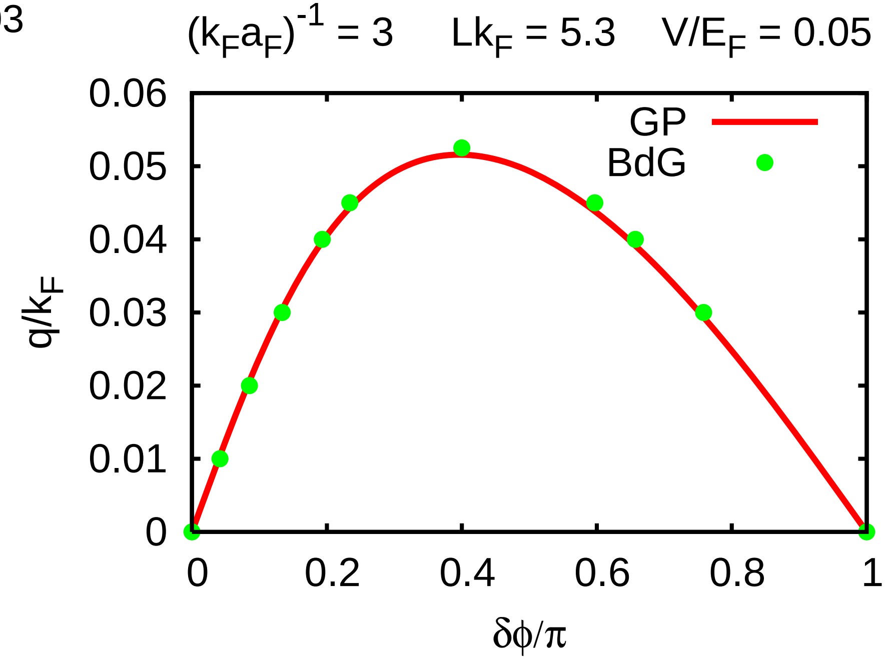
$$J(\mathbf{r}) = \frac{2}{m}\,\mathrm{Im}\sum_n \mathrm{v}_n(\mathbf{r})\vec{\nabla}\mathrm{v}_n^*(\mathbf{r})$$



Check of the numerical procedure in the BEC limit

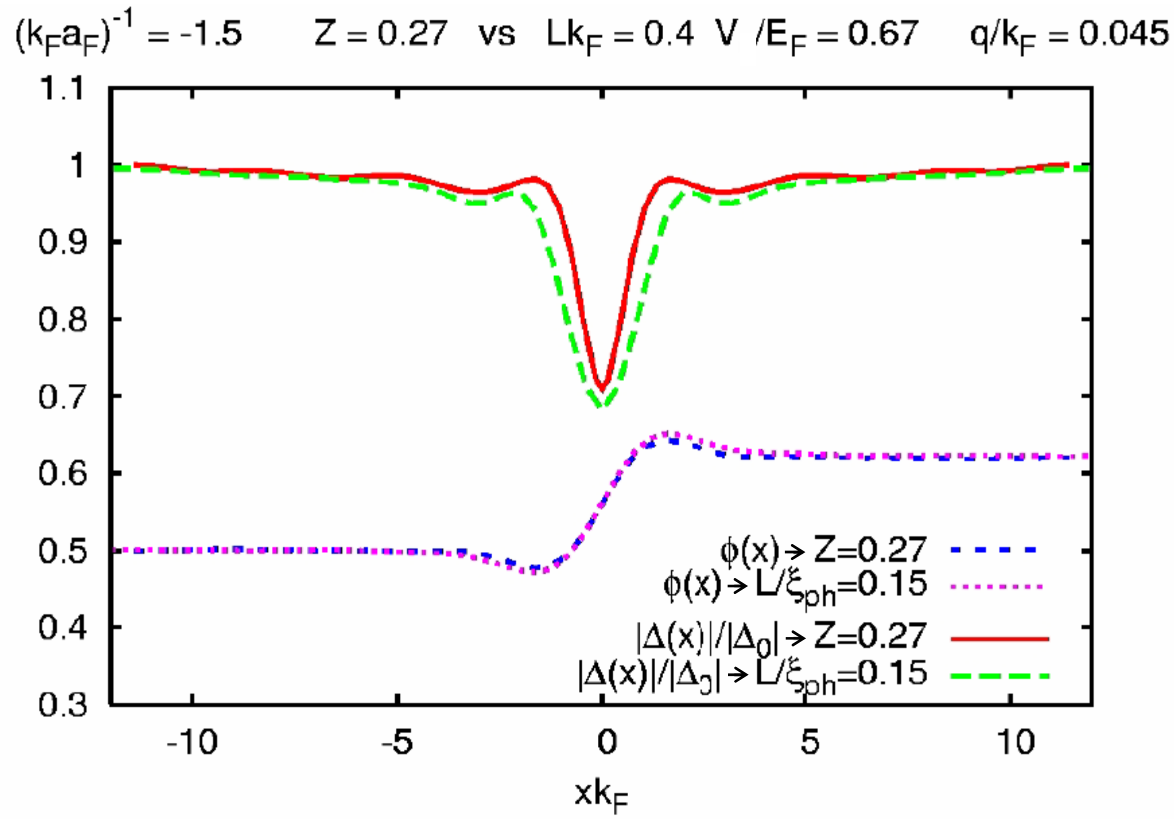
Compare the numerical solution of the BdG eqs. with the solution of the GP equation for bosons of mass $m_B = 2m$, scattering length $a_B = 2a_F$, in the presence of a barrier $V_B(x) = 2V(x)$.





Comparison is very good!

Comparison with delta-like barrier in BCS limit



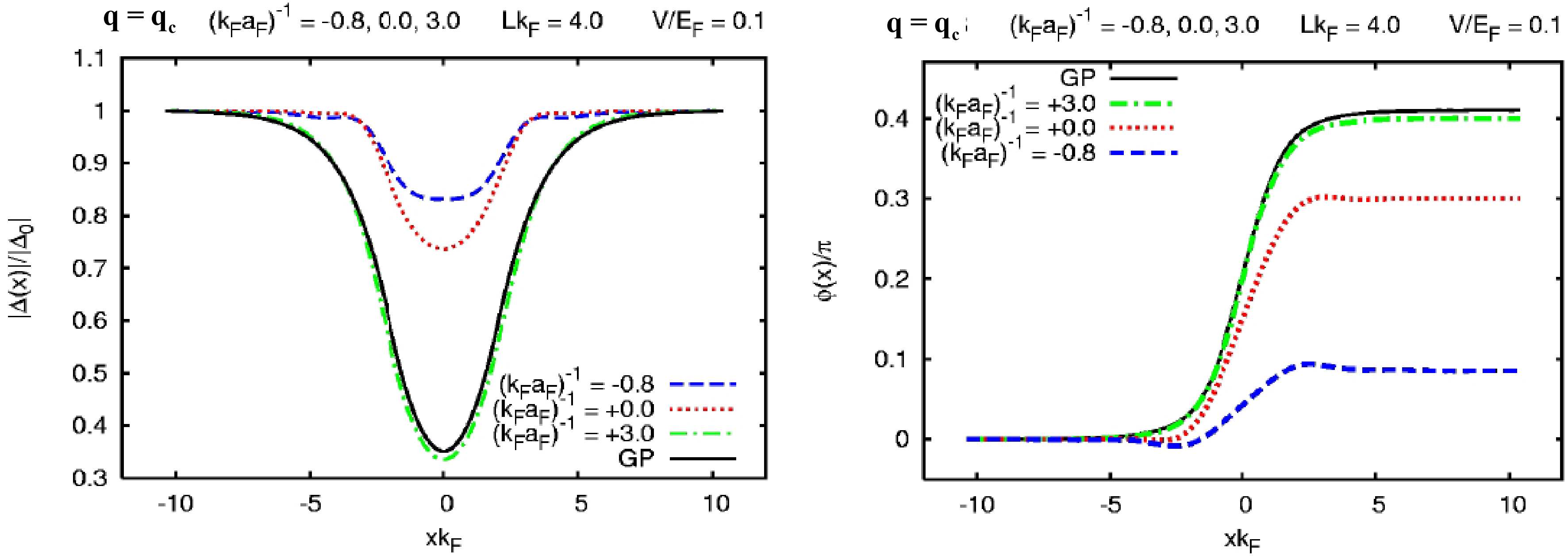
$$Z = (Lk_F) \times (V/E_F)$$

When approaching the BCS limit with fixed barrier parameters, results for a delta-like barrier are invariably recovered:

The coherence length $\xi >> L$ → the barrier is seen as point-like.

Friedel oscillations are clearly visible in the BCS limit.

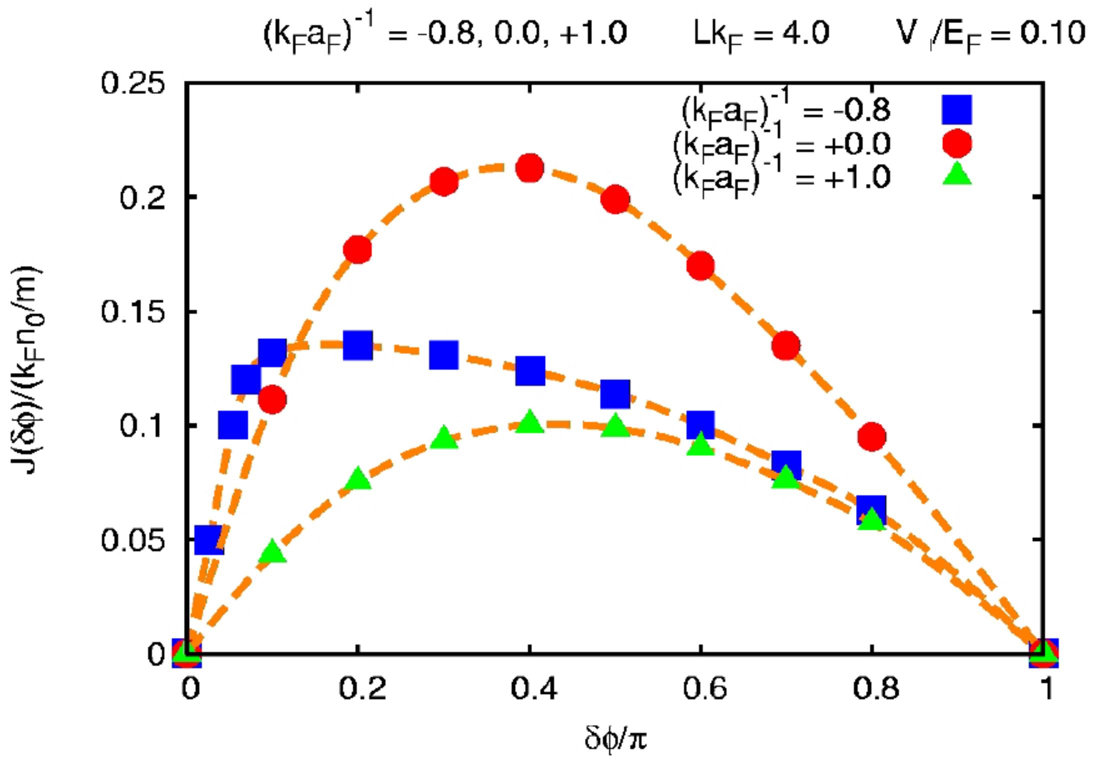
Gap and phase profile for different couplings



Friedel oscillations are washed out when evolving from the BCS to the BEC limit.

Suppression of the gap due to the barrier and phase difference increase monotonically from BCS to BEC limit.

Current vs phase relation through the crossover



At unitarity (crossover region) the Josephson current is enhanced.

Curves deviate from $J \propto \sin(\delta\varphi)$ in the BEC limit to (about) $J \propto \cos(\delta\varphi/2)$ in the BCS limit (for relatively small barriers with respect to the Fermi energy).

Critical Josephson current through the crossover

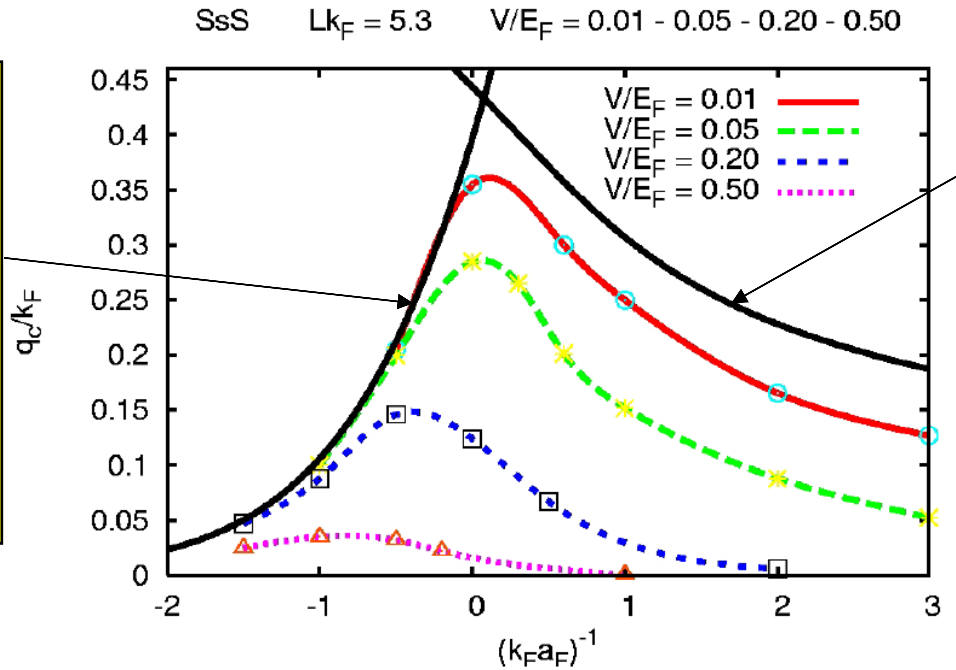
Depairing velocity:
Landau criterion applied to pair-breaking excitations.

$$\frac{q_c^2}{m} = \sqrt{\mu^2 + \Delta_0^2} - \mu$$

It reduces to

$$q_c = m\Delta_0 / k_F$$

in the BCS limit.



Sound velocity:
Landau criterion applied to Bogoliubov-Anderson mode.

It reduces to

$$2q_c = \sqrt{m_B \mu_B}$$

in the BEC limit.

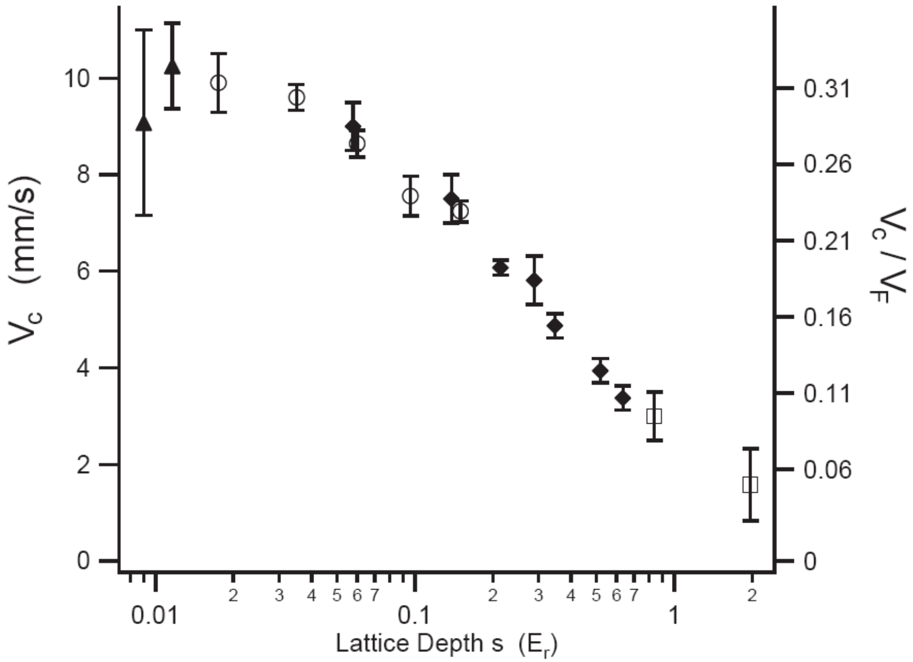
Josephson critical current controlled by Landau critical velocity (+ barrier details).

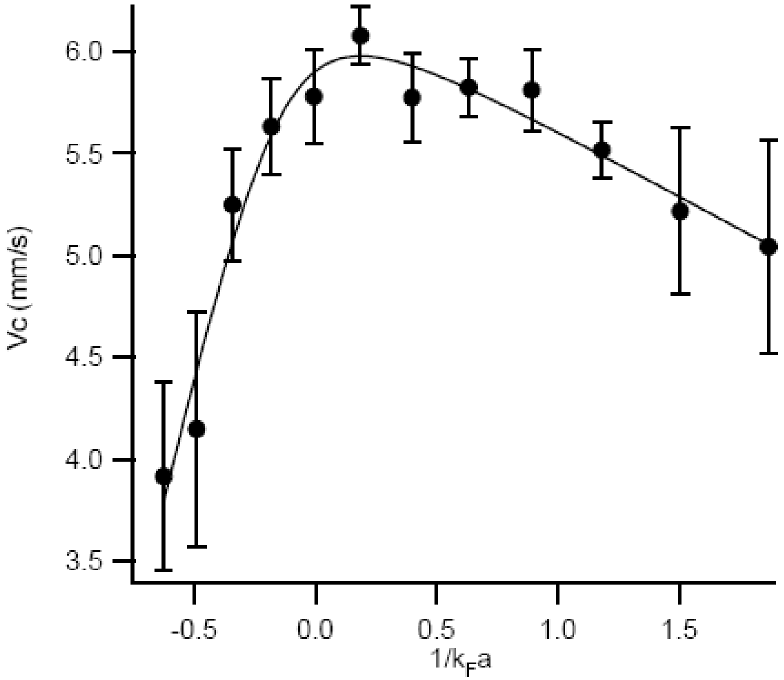
$J_c \propto \Delta$ only when the critical velocity is determined by pair-breaking (BCS to crossover region)

$J_c \propto c$ from crossover region to BEC

SUPERFLUID FLOW IS MOST ROBUST AT (ABOUT) UNITARITY

Recent experimental results





Courtesy of W. Ketterle's group

(now on arXiv:0707.2354)

A. Spuntarelli, P. Pieri, and G.C. Strinati, PRL **99**, 040401 (2007)

http://fisica.unicam.it/bcsbec