Uniqueness Results for Multi-spectral Bioluminescence Tomography

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Bioluminescence tomography (BLT) is a rapidly developing optical technique for molecular imaging of small animals. One major issue with this technique is that the solution uniqueness is not well solved up to now. In our previous studies, it has been found that the solution uniqueness does not hold for BLT under a single wavelength setting. Extra *a priori* knowledge must be utilized in the image reconstruction for mono-spectral BLT. In this work, this issue is studied under a multi-spectral setting. We first establish the forward process as the multi-spectral diffusion approximation to the multi-spectral radiative transport equation under the coherent scattering condition. Multi-spectral bioluminescence tomography (MSBLT) is then formulated as an inverse source problem for the multi-spectral diffusion approximation subject to multi-spectral Cauchy data. Then we provide the characterization of the solution structure for MSBLT as in our previous studies. Because any source distribution can be well approximated by radial basis functions (RBF), the solution uniqueness for MSBLT is studied in details for RBF sources. When a source is of at least two wavelengths and has a RBF distribution, the solution uniqueness for BLT is established for the first time under the following practical conditions: a) the object is piece-wisely homogeneous; b) there are measurements of those two wave lengths on one part of the object surface; c) the effective attenuation coefficients are different for those two wavelengths; d) the light source distributions of those two wavelengths are of the same support and proportional everywhere on their supports. Our previous reconstruction algorithms are also extended to MSBLT.