Two-Scale Homogenization of Visco-Elasticity

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Abstract. This talk deals with processes in nonlinear inelastic materials, whose constitutive behaviour is represented by the inclusion

$$\frac{\partial}{\partial t}[\varepsilon - B(x){:}\sigma] \in \partial\varphi(\sigma, x); \tag{1}$$

here by σ we denote the stress tensor, by ε (= $\nabla_s \vec{u}$) the linearized strain tensor, by $B(x)$ the compliance tensor, and by $\partial\varphi(\cdot, x)$ the subdifferential of a convex function $\varphi(\cdot, x)$. This relation accounts for elasto-visco-plasticity, including a nonlinear version of the classical Maxwell model of visco-elasticity and the Prandtl-Reuss model of elasto-plasticity.

The constitutive law is coupled with the equation of continuum dynamics

$$\rho\frac{\partial^2 \vec{u}}{\partial t^2} - \nabla\cdot\sigma = \vec{f} \qquad (\rho : \text{density}, \ \vec{f} : \text{prescribed load}), \tag{2}$$

and well-posedness is proved for an initial- and boundary-value problem. The function φ and the tensor B are then assumed to oscillate periodically with respect to x, and as this period vanishes a two-scale model of the asymptoptic behaviour is derived via Nguetseng's notion of *two-scale convergence.* A fully homogenized single-scale model is also retrieved, and its equivalence with the two-scale problem is proved. This formulation is nonlocal in time, and is at variance with that based on so-called *analogical models,* that rest on a mean-field-type hypothesis.