In memory of Tadek Pytlik,
our teacher and friend

SOBOLEV SPACES RELATED TO SCHRÖDINGER OPERATORS WITH POLYNOMIAL POTENTIALS

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ABSTRACT. The aim of this presentation is to prove the following theorem. Let

$$Af(x) = P(D)f(x) + V(x)f(x),$$

where $P(ix)$ is a nonnegative homogeneous elliptic polynomial on $\mathbf{R}^d$ and V is a nonnegative polynomial potential. Then for every $1 < p < \infty$ and every $\alpha > 0$ there exist constants $C_1, C_2 > 0$ such that

$$\|P(D)^\alpha f\|_{L^p} + \|V^\alpha f\|_{L^p} \leq C_1 \|A^\alpha f\|_{L^p}$$

and

$$\|A^\alpha f\|_{L^p} \leq C_2 \|\left(P(D)^\alpha + V^\alpha\right) f\|_{L^p}$$

for f in the Schwartz class $\mathcal{S}(\mathbf{R}^d)$.

We take advantage of the Christ inversion theorem for singular integral operators with a small amount of smoothness on nilpotent Lie groups, the maximal subelliptic L^2-estimates for the generators of stable semi-groups of measures, and the principle of transference of Coifman-Weiss.

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