# Comparison of spaces of Hardy type on some metric measure spaces of exponential growth

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In this talk I shall compare two atomic Hardy spaces $H^1$ and $\mathfrak{h}^1$ on a class of metric measure spaces of exponential growth. Examples of spaces in this class are $\mathbb{R}^n$ with the Gauss measure and a natural metric associated to the Ornstein-Uhlenbeck operator, Riemannian manifolds with Ricci curvature bounded from below and spectral gap and homogeneous trees. The space $H^1$ is defined as in the classical case, but with atoms that are supported only on small balls and $\mathfrak{h}^1$ is its local version 'a la Goldberg'. Singular integrals which satisfy a Hörmander integral condition on small balls are always bounded from $H^1$ to $L^1$. As far the $\mathfrak{h}^1 - L^1$ boundedness is concerned, the situation is different. I shall contrast what happens on Gauss space with convolution operators on homogeneous trees, on non compact unimodular Lie groups and Riemannian manifolds. These results are joint work with A. Carbonaro and S. Meda.