# Hardy spaces adapted to divergence form elliptic operators

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**Abstract**

The classical Stein-Weiss $H^p$ spaces in $\mathbb{R}^n$ are closely linked to the Laplacian

$$\Delta := -\left(\left(\frac{\partial}{\partial x_1}\right)^2 + ... + \left(\frac{\partial}{\partial x_n}\right)^2\right).$$

For example, these spaces may be characterized in terms of the $L^p$ behavior of appropriate non-tangential maximal functions or square functions associated either to the Poisson semigroup $e^{-t\sqrt{\Delta}}$ or to the heat semigroup $e^{-t\Delta}$. Moreover, at least for some range of $p$, they may be characterized in terms of the $L^p$ behavior of the Riesz Transforms $\nabla(\Delta)^{-1/2}$.

Auscher, Duong, McIntosh and Yan have developed certain aspects of Hardy space theory, paralleling the classical results, in which the Laplacian is replaced by another operator $L$ which enjoys a pointwise Gaussian heat kernel bound.

In this talk, we shall discuss joint work with S. Mayboroda and A. McIntosh, in which we develop Hardy space theory corresponding to a second order divergence form operator

$$L := -\mathrm{div} A \nabla$$

(where $A$ is an $n \times n$ complex elliptic matrix of bounded measurable coefficients), for which pointwise heat kernel bounds may be lacking. Much of this theory still runs parallel to the classical theory, but there are now certain differences, and it is these differences that we shall emphasize. In particular, characterizing these spaces in terms of the Riesz transforms $\nabla L^{-1/2}$, which amounts to extending the Kato problem to spaces beyond $L^p$, is problematic, and as yet we have only partial results to report.