

## Keakeya-Nikodym averages and $L^p$ -norms of eigenfunctions

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We provide a necessary and sufficient condition that  $L^p$ -norms,  $2 < p < 6$ , of eigenfunctions of the square root of minus the Laplacian on 2-dimensional compact boundaryless Riemannian manifolds  $M$  are small compared to a natural power of the eigenvalue  $\lambda$ . The condition that ensures this is that their  $L^2$  norms over  $O(\lambda^{-1/2})$  neighborhoods of arbitrary unit geodesics are small when  $\lambda$  is large (which is not the case for the highest weight spherical harmonics on  $S^2$  for instance). The proof exploits Gauss' lemma and the fact that the bilinear oscillatory integrals in Hrmander's proof of the Carleson-Sjlin theorem become better and better behaved away from the diagonal. Our results imply that QUE cannot hold on a compact boundaryless Riemannian manifold  $(M, g)$  of dimension two if  $L^p$ -norms are saturated for a given  $2 < p < 6$ .