Holomorphy of spectral multipliers of the Ornstein-Uhlenbeck operator.

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Consider a nonnegative self-adjoint operator A on $L^2(M,\mu)$, where (M,μ) is a measure space. A bounded Borel function m on $[0,\infty)$ is called a L^p *-multiplier for* A ($1 \le p < \infty$), if the operator $m(A)$, defined spectrally, extends from $L^p \cap L^2(\mu)$ to a bounded operator on $L^p(\mu)$. The set $\mathcal{M}_p(A)$ of L^p-multipliers forms a Banach algebra. Necessary and sufficients conditions for membership in $\mathcal{M}_p(A)$ have useful applications to partial differential equations, spectral theory, potential theory.

In the last thirty-odd years this problem has been investigated for several operators: Laplace-Beltrami operators on Riemannian manifolds, sums of squares of vector fields, Schrödinger operators, pseudodifferential operators.

I shall present some sufficient and necessary conditions for the Ornstein-Uhlenbeck operator, a "natural" Laplacian on the Euclidean space with Gauss measure, discussing in particular the necessity of the holomorphy of the multiplier in a sector.