

# Regularity results for fractional filtration equations

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## Abstract

In this talk we will discuss regularity issues for the solutions to nonlinear nonlocal diffusion problems of the form

$$u_t + (-\Delta)^{\sigma/2}\phi(u) = 0,$$

with integrable nonnegative initial data. We will devote special attention to the cases  $\phi(u) = u^m$ ,  $m > 0$ ,  $\sigma \in (0, 2)$ ,  $N \geq 1$ , studied in the papers [PQRV1] and [PQRV2], and to the “limit” case  $\phi(u) = \log(1 + u)$ ,  $N = 1$ ,  $\sigma = 1$ ; see [PQRV3].

When dealing with this kind of problems, the availability of adequate functional inequalities is crucial. We will start by discussing the appropriate functional framework, as well as some of the required inequalities.

We will next turn our attention to the regularity of solutions. Under certain conditions for the nonlinearity, we prove that solutions that are  $C^\alpha$  for some  $\alpha \in (0, 1)$  are in fact  $C^{1,\beta}$  for all  $\beta \in (0, 1)$ . Hence they are classical solutions. Since bounded solutions are  $C^\alpha$  for some  $\alpha \in (0, 1)$  [AC], the first step towards regularity is therefore to prove that solutions become immediately bounded. This is indeed true if the initial data belong to a suitable class (which depends on the nonlinearity).

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## References

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