Sub-Laplacians of holomorphic L^p-type on Lie groups

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Abstract

We study the problem of determining all connected Lie groups G which have the following property (hlp): every sub-Laplacian L on G is of holomorphic L^p-type for $1 \leq p < \infty,\ p \neq 2$. First we show that semi-simple non-compact Lie groups with finite center have this property, by using holomorphic families of representations in the class one principal series of G and the Kunze-Stein phenomenon. We then apply an L^p-transference principle for induced representations, essentially due to Anker, to show that every connected Lie group G whose semi-simple quotient by its radical is non-compact has property (hlp). One is thus reduced to studying those groups for which the semi-simple quotient is compact, i.e. to compact extensions of solvable Lie groups. We consider semi-direct extensions of exponential solvable Lie groups by connected compact Lie groups. It had been proved in previous joint work with Hebisch and Ludwig that every exponential solvable Lie group S, which has a non-$*$ regular co-adjoint orbit whose restriction to the nilradical is closed, has property (hlp), and we show here that (hlp) remains valid for compact extensions of these groups.